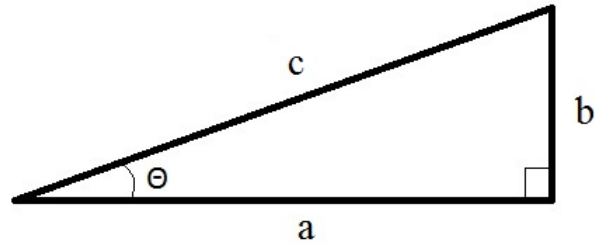


Math 128: Plane Trigonometry
Spring 2019
Practice Problems for Final Exam

Name (Print): _____

-
1. Convert the angle 160° to radians.
 2. Convert the angle $\frac{\pi}{18}$ to degrees.
 3. Find an angle between 0 and 2π that is coterminal to $-\frac{19\pi}{7}$.
 4. Find the reference angle of $-\frac{19\pi}{7}$.
 5. If θ is an angle such that $\sin \theta < 0$ and $\tan \theta > 0$, in what quadrant must θ lie?
 6. A circle with a radius 4 has a sector with central angle 30° . Find the area of this sector.
 7. A circle with radius 3 has a sector with area 6π . Find the length of the arc surrounding this sector.
 8. What is the domain of the function $f(x) = \sin^{-1}(x)$?
 9. What is the range of the function $f(x) = \cos^{-1}(x)$?
 10. What is the range of the function $f(x) = \tan^{-1}(x)$?
 11. What is the amplitude of the function $f(x) = 3 \cos(7x - 2) + 5$?
 12. What is the period of the function $f(x) = 3 \cos(7x - 2) + 5$?

The following questions are based on a triangle of this form:



13. If $b = 3$ and $c = 4$, find a .

14. If $a = 5$ and $c = 7$, find $\tan \theta$.

15. If $a = 4$ and $\theta = 45^\circ$, find c .

16. If $b = 1$ and $c = 2$, find the value of θ .

17. If $b = 1$ and $c = 3$, find the area of the triangle.

In these problems, you are given three parts of an oblique triangle, where side a is opposite angle A , side b is opposite angle B , and side c is opposite angle C . In each problem, follow the instructions to give the desired information.

18. Suppose that $A = 30^\circ$, $C = 80^\circ$, and $b = 10$. Find the length of side a .

19. Suppose that $A = 40^\circ$, $B = 70^\circ$, and $a = 2$. Find the length of side b .

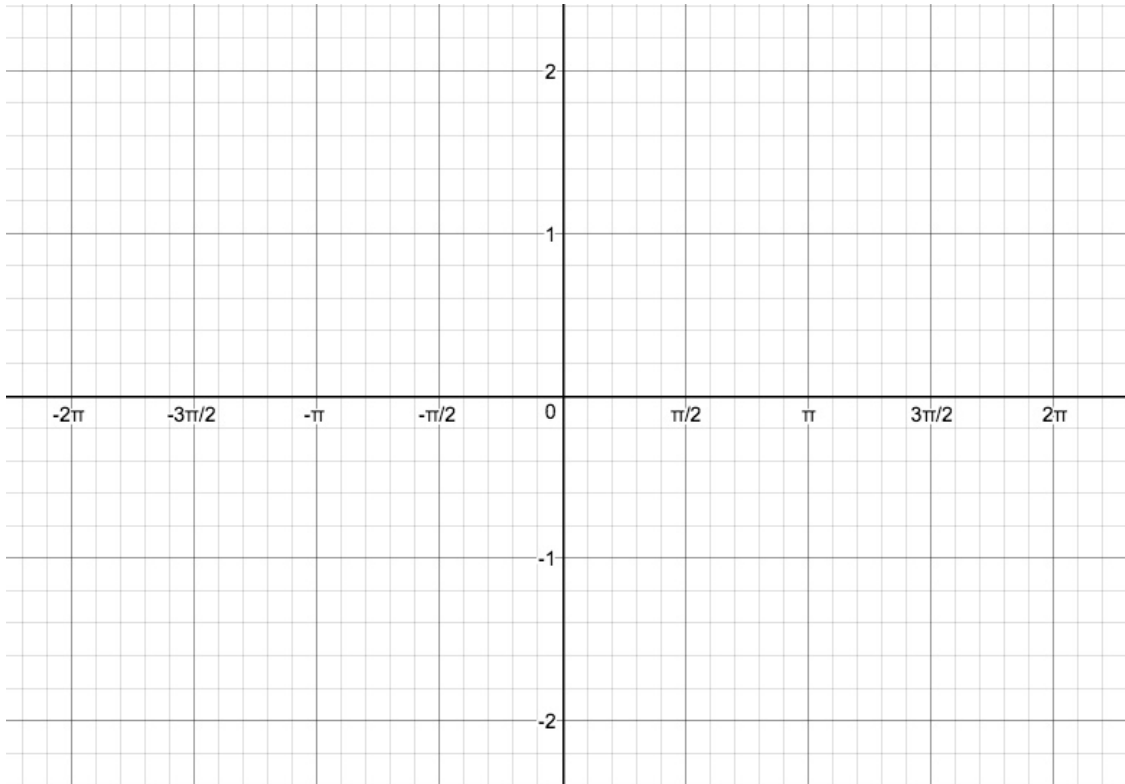
20. Suppose that $a = 8$, $b = 10$, and $c = 12$. Find the measure of angle A .

21. Suppose that $A = 120^\circ$, $b = 8$, $c = 2$. Find the length of side a .

22. Suppose that $A = 35^\circ$, $b = 2$, $c = 7$. Find the area of the triangle.

23. Suppose that $A = 25^\circ$, $a = 12$, and $c = 23$. How many possible solutions are there to this triangle?

24. On the grid below, sketch the graph of $f(x) = \sin(x)$.



25. Review what the graphs of $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$ look like. (For example, redo the graph-matching problem from Exam 1.)

26. For each of the following, find the exact value of the expression (that is, a rounded answer from your calculator will NOT be good enough).

(a) $\sin^{-1}(\sin(\frac{2\pi}{3}))$

(b) $\tan(\tan^{-1}(-4))$

(c) $\csc(\sin^{-1}(0))$

(d) $\sin(\tan^{-1}(-1))$

27. Verify the identity $(1 - \tan x)(1 - \cot x) = 2 - \sec x \csc x$.
28. Verify the identity $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x} = \sec x$.
29. Verify the identity $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 2 + 2 \cos(x + y)$.
30. Use a half-angle formula to find the exact value of $\tan 15^\circ$. (A rounded answer from your calculator will NOT be good enough.)
31. Use a half-angle formula to find the exact value of $\cos \frac{\pi}{8}$. (A rounded answer from your calculator will NOT be good enough.)
32. Write the expression $\cos(\sin^{-1} x + \cos^{-1} y)$ as an algebraic expression in terms of x and y .
33. Write the expression $\sin(2 \tan^{-1} x)$ as an algebraic expression in terms of x .
34. Solve the equation $4 \cos \theta = 1$.
35. Solve the equation $\cos \theta \sin \theta - \cos \theta = 0$.
36. Solve the equation $\sin \theta = \cos 2\theta$.
37. Solve the equation $\sin \theta - \cos \theta - \tan \theta + 1 = 0$.
38. Let $z = 1 + i$, let $w = 1 - \sqrt{3}i$.
- Write z and w in polar form.
 - Compute zw .
 - Compute $(zw)^7$.
 - Compute $\frac{z^2}{w^3}$.

39. Consider the point $P = (-\sqrt{6}, \sqrt{2})$ in rectangular coordinates. Convert P to polar coordinates.
40. Consider the point $Q = (3, \pi/6)$ in polar coordinates. Convert Q to rectangular coordinates.
41. Using the variables x and y , convert the polar equation $r = 6 \sec \theta$ to rectangular coordinates.
42. Using the variables x and y , convert the polar equation $r = 2 \cos \theta$ to rectangular coordinates.
43. Using the variables x and y , convert the polar equation $r = 1 + \cos \theta$ to rectangular coordinates.
44. Consider the vector $\mathbf{v} = \langle 7, -2 \rangle$, and let \mathbf{u} be the vector with magnitude $\sqrt{8}$ and direction 135° .
- Write \mathbf{v} in terms of \mathbf{i} and \mathbf{j} .
 - Compute the magnitude of \mathbf{v} .
 - Compute the direction of \mathbf{v} .
 - Write \mathbf{u} in component form.
 - Compute the dot product $\mathbf{u} \cdot \mathbf{v}$.
 - Compute the angle between \mathbf{u} and \mathbf{v} .
 - Compute the vector $9\mathbf{u} + 4\mathbf{v}$ in component form.
 - Determine whether $9\mathbf{u} + 4\mathbf{v}$ is orthogonal to \mathbf{u} .
 - Calculate the component of \mathbf{u} along \mathbf{v} .
 - Calculate the projection of \mathbf{u} onto \mathbf{v} , $\text{proj}_{\mathbf{v}}\mathbf{u}$.

45. Write an equation for the parabola with vertex at the origin whose focus is the point $(0, -2)$.

46. Write an equation for an ellipse centered at the origin that has a focus at $(1, 0)$ and a vertex at $(3, 0)$.

47. Write an equation for a hyperbola centered at the origin that has a focus at $(0, 1)$ and asymptotes at $y = 2x$ and $y = -2x$.

48. Suppose a parabola has the equation $y^2 = 8x$. Find the focus and the directrix of this parabola.

49. Suppose an ellipse has the equation $4x^2 + 25y^2 = 100$. Find the foci of this ellipse and the length of its major axis.

50. Suppose a hyperbola has the equation $16x^2 - 4y^2 = 64$. Find the foci, vertices, and asymptotes of this hyperbola.

51. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be 32° . One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is 35° . Find the height of the mountain, to the nearest foot.

52. A 96-ft tree casts a shadow that is 120 ft long. What is the angle of elevation of the sun?

53. The Leaning Tower of Pisa leans 5.6° from the vertical. A tourist stands 105 m from its base with the tower leaning directly towards her. She measures the angle of elevation to the top of the tower to be 29.2° . Find the length of the tower, to the nearest meter.
54. A pilot heads his jet due east. The jet has a speed of 425 miles per hour relative to the air. The wind is blowing due north with a speed of 40 miles per hour. Find the true velocity of the jet as a vector.
55. A boat heads in the direction $N 72^\circ E$. The speed of the boat relative to the water is 24 miles per hour. The water is flowing directly south at an unknown speed. It is observed that the true direction of the boat is directly east. Find the speed of the flow of the water.
56. A lawn mower is pushed a distance of 200 ft along a horizontal path by a constant force of 50 lb. The handle of the lawn mower is at an angle of 30° from the horizontal. Find the work done.