Math 128: Plane Trigonometry **Name (Print):** ______________________
Spring 2023
Practice Problems for Final Exam

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1. Convert the angle $160°$ to radians.

2. Convert the angle $\frac{\pi}{18}$ to degrees.

3. Find an angle between 0 and 2π that is coterminal to $-\frac{19\pi}{7}$.

4. Find the reference angle of $-\frac{19\pi}{7}$.

5. If θ is an angle such that $\sin\theta < 0$ and $\tan\theta > 0$, in what quadrant must θ lie?

6. A circle with a radius 4 has a sector with central angle $30°$. Find the area of this sector.

7. A circle with radius 3 has a sector with area 6π. Find the length of the arc surrounding this sector.

8. What is the domain of the function $f(x) = \sin^{-1}(x)$?

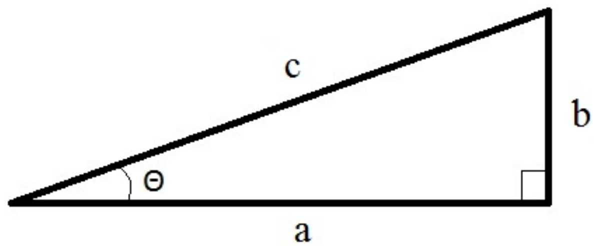
9. What is the range of the function $f(x) = \cos^{-1}(x)$?

10. What is the range of the function $f(x) = \tan^{-1}(x)$?

11. What is the amplitude of the function $f(x) = 3\cos(7x - 2) + 5$?

12. What is the period of the function $f(x) = 3\cos(7x - 2) + 5$?

The following questions are based on a triangle of this form:



13. If $b = 3$ and $c = 4$, find a.

14. If $a = 5$ and $c = 7$, find $\tan\theta$.

15. If $a = 4$ and $\theta = 45°$, find c.

16. If $b = 1$ and $c = 2$, find the value of θ.

17. If $b = 1$ and $c = 3$, find the area of the triangle.

In these problems, you are given three parts of an oblique triangle, where side a is opposite angle A, side b is opposite angle B, and side c is opposite angle C. In each problem, follow the instructions to give the desired information.

18. Suppose that $A = 30°$, $C = 80°$, and $b = 10$. Find the length of side a.

19. Suppose that $A = 40°$, $B = 70°$, and $a = 2$. Find the length of side b.

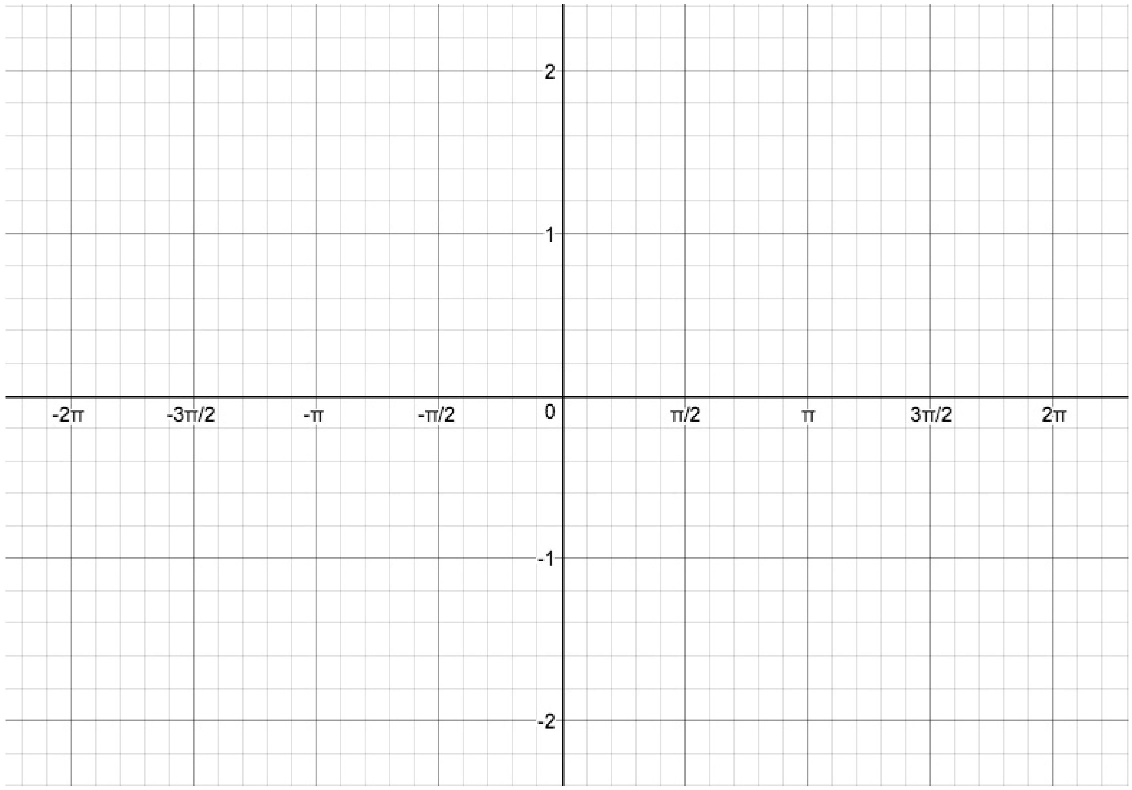
20. Suppose that $a = 8$, $b = 10$, and $c = 12$. Find the measure of angle A.

21. Suppose that $A = 120°$, $b = 8$, $c = 2$. Find the length of side a.

22. Suppose that $A = 35°$, $b = 2$, $c = 7$. Find the area of the triangle.

23. Suppose that $A = 25°$, $a = 12$, and $c = 23$. How many possible solutions are there to this triangle?

24. On the grid below, sketch the graphs of $f(x) = 2\sin(3x)$ and $g(x) = \frac{1}{2}\cos(2x)$.



25. Review what the graphs of $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$ look like. (For example, redo the graph-matching problem from Exam 1.)

26. Consider each of the following angles, expressed with inverse trig functions. Determine the quadrant of each angle.
(a) $\sin^{-1}(-\frac{2}{3})$

(b) $\tan^{-1}(-\frac{2}{3})$

(c) $\cos^{-1}(-\frac{2}{3})$

27. For each of the following, find the exact value of the expression (that is, a rounded answer from your calculator will NOT be good enough).
(a) $\sin^{-1}(\sin(\frac{2\pi}{3}))$

(b) $\tan(\tan^{-1}(-4))$

(c) $\csc(\sin^{-1}(0))$

(d) $\sin(\tan^{-1}(-1))$

28. Verify the identity $(1 - \tan x)(1 - \cot x) = 2 - \sec x \csc x$.

29. Verify the identity $\dfrac{\sin 2x}{\sin x} - \dfrac{\cos 2x}{\cos x} = \sec x$.

30. Verify the identity $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 2 + 2\cos(x + y)$.

31. Use a half-angle formula to find the exact value of $\tan 15°$. (A rounded answer from your calculator will NOT be good enough.)

32. Use a half-angle formula to find the exact value of $\cos \dfrac{\pi}{8}$. (A rounded answer from your calculator will NOT be good enough.)

33. Write the expression $\csc(\tan^{-1} x)$ as an algebraic expression in terms of x.

34. Write the expression $\tan(\cos^{-1} x)$ as an algebraic expression in terms of x.

35. Write the expression $\cos(\sin^{-1} x + \cos^{-1} y)$ as an algebraic expression in terms of x and y.

36. Write the expression $\sin(2\tan^{-1} x)$ as an algebraic expression in terms of x.

37. Find the general solution of the equation $6\sin(\theta) + 10 = 7$.

38. Find the general solution of the equation $4\cos\theta = 1$.

39. Find the general solution of the equation $11 - \tan(\theta) = 3$.

40. Find the general solution of the equation $\cos\theta\sin\theta - \cos\theta = 0$.

41. Find the general solution of the equation $\sin\theta = \cos 2\theta$.

42. Let $z = 1 + i$, let $w = 1 - \sqrt{3}i$.
(a) Write z and w in polar form.

(b) Compute zw.

(c) Compute $(zw)^7$.

(d) Compute $\frac{z^2}{w^3}$.

43. Consider the point $P = (-\sqrt{6}, \sqrt{2})$ in rectangular coordinates. Convert P to polar coordinates.

44. Consider the point $Q = (3, \pi/6)$ in polar coordinates. Convert Q to rectangular coordinates.

45. Using the variables x and y, convert the polar equation $r = 6\sec\theta$ to rectangular coordinates.

46. Using the variables x and y, convert the polar equation $r = 2\cos\theta$ to rectangular coordinates.

47. Using the variables x and y, convert the polar equation $r = 1+\cos\theta$ to rectangular coordinates.

48. Consider the vector $\mathbf{v} = \langle 7, -2 \rangle$, and let $\mathbf{u}$ be the vector with magnitude $\sqrt{8}$ and direction $135°$.
(a) Write $\mathbf{v}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

(b) Compute the magnitude of $\mathbf{v}$.

(c) Compute the direction of $\mathbf{v}$.

(d) Write $\mathbf{u}$ in component form.

(e) Compute the dot product $\mathbf{u} \cdot \mathbf{v}$.

(f) Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.

(g) Compute the vector $9\mathbf{u} + 4\mathbf{v}$ in component form.

(h) Determine whether $9\mathbf{u} + 4\mathbf{v}$ is orthogonal to $\mathbf{u}$.

49. Write an equation for the parabola with vertex at the origin whose focus is the point $(0, -2)$.

50. Write an equation for an ellipse centered at the origin that has a focus at $(1, 0)$ and a vertex at $(3, 0)$.

51. Write an equation for a hyperbola centered at the origin that has a focus at $(0, 1)$ and asymptotes at $y = 2x$ and $y = -2x$.

52. Suppose a parabola has the equation $y^2 = 8x$. Find the focus and the directrix of this parabola.

53. Suppose an ellipse has the equation $4x^2 + 25y^2 = 100$. Find the foci of this ellipse and the length of its major axis.

54. Suppose a hyperbola has the equation $16x^2 - 4y^2 = 64$. Find the foci, vertices, and asymptotes of this hyperbola.

55. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be $32°$. One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is $35°$. Find the height of the mountain, to the nearest foot.

56. A 96-ft tree casts a shadow that is 120 ft long. What is the angle of elevation of the sun?

57. The Leaning Tower of Pisa leans $5.6°$ from the vertical. A tourist stands 105 m from its base with the tower leaning directly towards her. She measures the angle of elevation to the top of the tower to be $29.2°$. Find the length of the tower, to the nearest meter.

58. A lawn mower is pushed a distance of 200 ft along a horizontal path by a constant force of 50 lb. The handle of the lawn mower is at an angle of $30°$ from the horizontal. Find the work done.