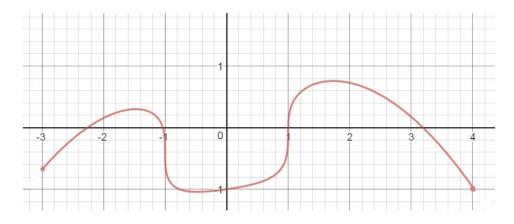
## Practice Problems for Final Exam

- 1. Find the distance between the points (6, -2) and (-4, 5).
- 2. Find the midpoint of the segment that joins the points (5,-1) and (3,5).
- 3. Find an equation of the circle of radius 5 centered at (-2,3).
- 4. Find the center and radius of the circle with equation  $x^2 + (y-2)^2 = 36$ .
- 5. Find the center and radius of the circle with equation  $x^2 + y^2 + 6y + 2 = 0$ .
- 6. Find an equation of the line with slope 7 that passes through (4, -1).
- 7. Find an equation of the line that passes through (6,2) and (-4,3).
- 8. Find an equation of the line that passes through (3,-1) that is parallel to y=6x+1.
- 9. Find an equation of the line that passes through (1,1) that is perpendicular to 2x + y = 4.
- 10. Find all real and complex solutions to  $x^2 + 14x = 32$ .
- 11. Find all real and complex solutions to  $2x^2 + 6x 5 = 0$ .
- 12. Find all real and complex solutions to  $3x^2 2x + 1 = 0$ .

- 13. Find all real solutions to  $\frac{6}{x^2-1}-\frac{3}{2}=\frac{3}{x-1}$ .
- 14. Find all real solutions to  $\frac{2}{x+3} + \frac{3}{8} = \frac{5}{4x+12}$ .
- 15. Find all real solutions to  $x^6 2x^3 3 = 0$ .
- 16. Find all real solutions to  $x^{3/2} 10x^{1/2} + 25x^{-1/2} = 0$ .
- 17. Find all real solutions to  $x^2\sqrt{x+3} = (x+3)^{3/2}$ .
- 18. Find all real solutions to  $x^5 x^3 2x = 0$ .
- 19. Solve the inequality 2 5x < 7.
- 20. Solve the inequality  $-4 < 2x 4 \le -2$ .
- 21. Solve the equation |8 3x| = 1.
- 22. Solve the inequality  $|4x + 1| \ge 21$ .
- 23. Solve the inequality  $x^2 + 5x + 6 > 0$ .
- 24. Solve the inequality  $2x^2 + x \ge 1$ .

- 25. Consider the function  $f(x) = x^2 4x$ . Evaluate f(x-3) and simplify.
- 26. Find the domain of the function  $f(x) = \sqrt{4 x^2}$ .
- 27. Find the domain of the function  $f(x) = \frac{x-1}{x^2 + 3x 10}$
- 28. Find the domain of the function  $f(x) = \frac{5x}{\sqrt{x-1}}$ .
- 29. Find the domain of the function  $f(x) = \ln(8 2x)$ .
- 30. Find the domain of the function  $f(x) = \frac{1}{\log_2(x)}$ .
- 31. If  $f(x) = 3\sqrt{x-4}$  and  $g(x) = x^2 1$ , find the formula for  $(f \circ g)(x)$ .
- 32. If  $f(x) = 3\sqrt{x-4}$  and  $g(x) = x^2 1$ , find the formula for  $(g \circ f)(x)$ .
- 33. If  $f(x) = 13x^{5/3} 1$ , find the formula for  $f^{-1}(x)$ .
- 34. If  $f(x) = \frac{2x+1}{3x-7}$ , find the formula for  $f^{-1}(x)$ .
- 35. Sketch a graph of the function  $f(x) = \sqrt[3]{x^2 1}$  by making a table of values and plotting some points.

Consider the following graph of a function, y = f(x).



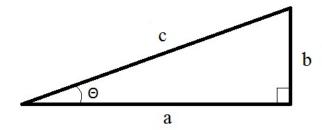
- 36. Find the domain of f.
- 37. Find, approximately, the range of f.
- 38. Find, approximately, the intervals where f is increasing.
- 39. Find, approximately, the intervals where f is decreasing.
- 40. Find, approximately, the intervals on which f(x) > 0.
- 41. Find the approximate coordinates of any local maxima of f.
- 42. Find the approximate coordinates of any local minima of f.
- 43. Is f a one-to-one function?
- 44. Sketch the graph of y = f(2 x) + 1.

- 45. Write the standard form of the quadratic function  $f(x) = 2x^2 8x + 4$ .
- 46. Find the coordinates of the vertex of the graph of  $y = x^2 5x + 2$ .
- 47. Find the maximum or minimum value of  $f(x) = 3x^2 8x + 4$ .
- 48. Determine the end behavior of the function  $f(x) = 3x^4 4x^3 10x 1$ .
- 49. Consider the function  $f(x) = x^4 + x^3 2x^2$ . Find all real zeros of f, state their multiplicities, and sketch the graph of f.
- 50. Find the quotient and remainder of the division  $\frac{x^4 2x^2 + 7x}{x^2 x + 3}$ .
- 51. Find the quotient and remainder of the division  $\frac{x^2 5x + 4}{x 3}$ .
- 52. Let  $f(x) = x^5 2x^4 9x^3 + 22x^2 + 4x 24$ . Suppose that you know that 2 is a zero of f of multiplicity 3. Use this information to completely factor f.
- 53. Find a polynomial of degree 3 with integer coefficients and zeros at  $\frac{1}{2}$ , -1, and 2.
- 54. Let  $f(x) = \frac{1}{(x+2)^2}$ . Find all zeros of f, vertical asymptotes of f, and horizontal asymptotes of f. Find the behavior of the graph near the vertical asymptotes, and use this to sketch a graph of f.
- 55. Let  $f(x) = \frac{x^2 1}{x^2 2x 8}$ . Find all zeros of f, vertical asymptotes of f, and horizontal asymptotes of f. Find the behavior of the graph near the vertical asymptotes, and use this to sketch a graph of f.
- 56. Solve the inequality  $\frac{x^2 9}{x^3 + x^2 4x 4} > 0$ .

- 57. Let  $f(x) = 4e^{4-x}$ . Use a calculator to find f(-1), rounded to three decimal places.
- 58. Write the equation  $\log_6(36) = 2$  in exponential form.
- 59. Write the equation  $4^x = 20$  in logarithmic form.
- 60. Let  $f(x) = 7\log_3(x+2)$ . Use the change of base formula and a calculator to find f(2), rounded to three decimal places.
- 61. Use the log laws to expand  $\log_3\left(\frac{(x+4)\sqrt{2x}}{(x+1)^7}\right)$ .
- 62. Use the log laws to expand  $\log_{10} \left( \frac{x^2 y^4}{\sqrt{x^3 6}} \right)$ .
- 63. Write as a single logarithm using the log laws:  $\ln(4x) 2\ln(x-1) 6\ln(x+2)$ .
- 64. Solve the equation  $3^{x-4} = 27$ .
- 65. Solve the equation  $e^{4x} + 4 = 9$ .
- 66. Solve the equation  $2^{2x} 2^x 12 = 0$ .
- 67. Solve the equation  $\log_{10}(2x-3) + 1 = 0$ .
- 68. Solve the equation  $\log_3(x^2 4) + \log_3(x) = \log_3(x 2)$ .
- 69. Solve the equation  $\log_8(x+5) \log_8(x-2) = 1$ .

- 70. Convert the angle  $160^{\circ}$  to radians.
- 71. Convert the angle  $\frac{\pi}{18}$  to degrees.
- 72. Find an angle between 0 and  $2\pi$  that is coterminal to  $-\frac{19\pi}{7}$ .
- 73. Find the reference angle of  $-\frac{19\pi}{7}$ .
- 74. If  $\theta$  is an angle such that  $\sin \theta < 0$  and  $\tan \theta > 0$ , in what quadrant must  $\theta$  lie?
- 75. A circle with a radius 4 has a sector with central angle 30°. Find the area of this sector.
- 76. A circle with radius 3 has a sector with area  $6\pi$ . Find the length of the arc surrounding this sector.
- 77. What is the domain of the function  $f(x) = \sin^{-1}(x)$ ?
- 78. What is the range of the function  $f(x) = \cos^{-1}(x)$ ?
- 79. What is the range of the function  $f(x) = \tan^{-1}(x)$ ?
- 80. What is the amplitude of the function  $f(x) = 3\cos(7x 2) + 5$ ?
- 81. What is the period of the function  $f(x) = 3\cos(7x 2) + 5$ ?

The following questions are based on a triangle of this form:



82. If b = 3 and c = 4, find a.

83. If a = 5 and c = 7, find  $\tan \theta$ .

84. If a=4 and  $\theta=45^{\circ}$ , find c.

85. If b = 1 and c = 2, find the value of  $\theta$ .

86. If b = 1 and c = 3, find the area of the triangle.

In these problems, you are given three parts of an oblique triangle, where side a is opposite angle A, side b is opposite angle B, and side c is opposite angle C. In each problem, follow the instructions to give the desired information.

87. Suppose that  $A = 30^{\circ}$ ,  $C = 80^{\circ}$ , and b = 10. Find the length of side a.

88. Suppose that  $A=40^{\circ}$ ,  $B=70^{\circ}$ , and a=2. Find the length of side b.

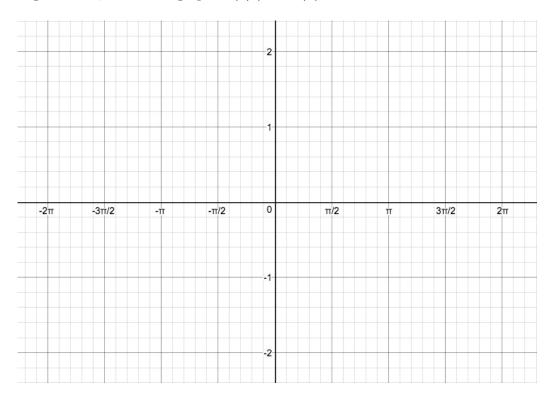
89. Suppose that a = 8, b = 10, and c = 12. Find the measure of angle A.

90. Suppose that  $A = 120^{\circ}$ , b = 8, c = 2. Find the length of side a.

91. Suppose that  $A = 35^{\circ}$ , b = 2, c = 7. Find the area of the triangle.

92. Suppose that  $A=25^{\circ}$ , a=12, and c=23. How many possible solutions are there to this triangle?

93. On the grid below, sketch the graph of  $f(x) = \sin(x)$ .



- 94. Review what the graphs of tan(x), cot(x), sec(x), and csc(x) look like. (For example, redo the graph-matching problem from Exam 4.)
- 95. For each of the following, find the exact value of the expression (that is, a rounded answer from your calculator will NOT be good enough).

  (a)  $\sin^{-1}(\sin(\frac{2\pi}{3}))$ 
  - (b)  $\tan(\tan^{-1}(-4))$
  - (c)  $\csc(\sin^{-1}(0))$
  - (d)  $\sin(\tan^{-1}(-1))$
- 96. Write the expression  $\cos(\tan^{-1}(x))$  as an algebraic expression in terms of x.
- 97. Write the expression  $\cot(\sin^{-1}(x))$  as an algebraic expression in terms of x.

- 98. Use a half-angle formula to find the exact value of  $\tan(15^{\circ})$ . (A rounded answer from your calculator will NOT be good enough.)
- 99. Use a half-angle formula to find the exact value of  $\cos\left(\frac{\pi}{8}\right)$ . (A rounded answer from your calculator will NOT be good enough.)
- 100. Verify the identity  $(1 \tan x)(1 \cot x) = 2 \sec x \csc x$ .
- 101. Verify the identity  $\frac{\sin 2x}{\sin x} \frac{\cos 2x}{\cos x} = \sec x$ .
- 102. Verify the identity  $(\cos x + \cos y)^2 + (\sin x \sin y)^2 = 2 + 2\cos(x + y)$ .
- 103. Find the exact value of  $\tan(\csc^{-1}(4) \cos^{-1}(\frac{1}{8}))$ .
- 104. Find the exact value of  $\cos(2\sin^{-1}(\frac{1}{3}) \frac{\pi}{4})$ .
- 105. Write the expression  $\cos(\sin^{-1} x + \cos^{-1} y)$  as an algebraic expression in terms of x and y.
- 106. Write the expression  $\sin(2\tan^{-1}x)$  as an algebraic expression in terms of x.
- 107. Solve the equation  $4\cos(\theta) = 1$ .
- 108. Solve the equation  $tan(\theta) = -6$ .
- 109. Solve the equation  $\cos(\theta) \tan(\theta) \cos(\theta) = 0$ .
- 110. Solve the equation  $3\sin^2(\theta) + 2\sin(\theta) = 1$ .

- 111. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be 32°. One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is 35°. Find the height of the mountain, to the nearest foot.
- 112. A 96-ft tree casts a shadow that is 120 ft long. What is the angle of elevation of the sun?
- 113. The Leaning Tower of Pisa leans 5.6° from the vertical. A tourist stands 105 m from its base with the tower leaning directly towards her. She measures the angle of elevation to the top of the tower to be 29.2°. Find the length of the tower, to the nearest meter.
- 114. Find two integers whose sum is 4542 and whose product is 1366632.
- 115. An apartment has a rectangular bedroom with an area of 228 ft<sup>2</sup> that is 7 ft longer than it is wide. What is the width of the room?
- 116. A large box has a volume of 180 ft<sup>3</sup>. Its length is 9 ft greater than its height, and its width is 4 ft less than its height. What are the dimensions of the box?
- 117. A ball is thrown into the air, and its height h after t seconds is given by  $h(t) = 128 + 16t 16t^2$ . Find the maximum height achieved by the ball.
- 118. You invest \$500 into an account with an annual interest rate of 8% that compounds monthly. How much money will be in your account after 2 years?
- 119. You invest \$800 into an account with an annual interest rate of 10% that compounds continuously. How much money will be in your account after 3 years?
- 120. You and your friend are each investing \$1000 into bank accounts. Your account has an annual interest rate of 6% and compounds continuously. Your friend's account compounds monthly, but you don't know the interest rate. At the end of 1 year, you end up with the exact same amount of money in your account as your friend has in her account. Find the annual interest rate of your friend's account.