Math 129: Pre-Calculus **Name (Print):** _____
Spring 2018
Practice Problems for Final Exam

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1. Find the distance between the points $(6, -2)$ and $(-4, 5)$.

2. Find the midpoint of the segment that joins the points $(5, -1)$ and $(3, 5)$.

3. Find an equation of the circle of radius 5 centered at $(-2, 3)$.

4. Find the center and radius of the circle with equation $x^2 + (y - 2)^2 = 36$.

5. Find the center and radius of the circle with equation $x^2 + y^2 + 6y + 2 = 0$.

6. Find an equation of the line with slope 7 that passes through $(4, -1)$.

7. Find an equation of the line that passes through $(6, 2)$ and $(-4, 3)$.

8. Find an equation of the line that passes through $(3, -1)$ that is parallel to $y = 6x + 1$.

9. Find an equation of the line that passes through $(1, 1)$ that is perpendicular to $2x + y = 4$.

10. Find all real and complex solutions to $x^2 + 14x = 32$.

11. Find all real and complex solutions to $2x^2 + 6x - 5 = 0$.

12. Find all real and complex solutions to $3x^2 - 2x + 1 = 0$.

13. Find all real solutions to $\dfrac{6}{x^2-1} - \dfrac{3}{2} = \dfrac{3}{x-1}$.

14. Find all real solutions to $\dfrac{2}{x+3} + \dfrac{3}{8} = \dfrac{5}{4x+12}$.

15. Find all real solutions to $x^6 - 2x^3 - 3 = 0$.

16. Find all real solutions to $x^{3/2} - 10x^{1/2} + 25x^{-1/2} = 0$.

17. Find all real solutions to $x^2\sqrt{x+3} = (x+3)^{3/2}$.

18. Find all real solutions to $x^5 - x^3 - 2x = 0$.

19. Solve the inequality $2 - 5x < 7$.

20. Solve the inequality $-4 < 2x - 4 \leq -2$.

21. Solve the equation $|8 - 3x| = 1$.

22. Solve the inequality $|4x + 1| \geq 21$.

23. Solve the inequality $x^2 + 5x + 6 > 0$.

24. Solve the inequality $2x^2 + x \geq 1$.

25. Consider the function $f(x) = x^2 - 4x$. Evaluate $f(x-3)$ and simplify.

26. Find the domain of the function $f(x) = \sqrt{4 - x^2}$.

27. Find the domain of the function $f(x) = \dfrac{x-1}{x^2 + 3x - 10}$

28. Find the domain of the function $f(x) = \dfrac{5x}{\sqrt{x-1}}$.

29. Find the domain of the function $f(x) = \ln(8 - 2x)$.

30. Find the domain of the function $f(x) = \dfrac{1}{\log_2(x)}$.

31. If $f(x) = 3\sqrt{x-4}$ and $g(x) = x^2 - 1$, find the formula for $(f \circ g)(x)$.

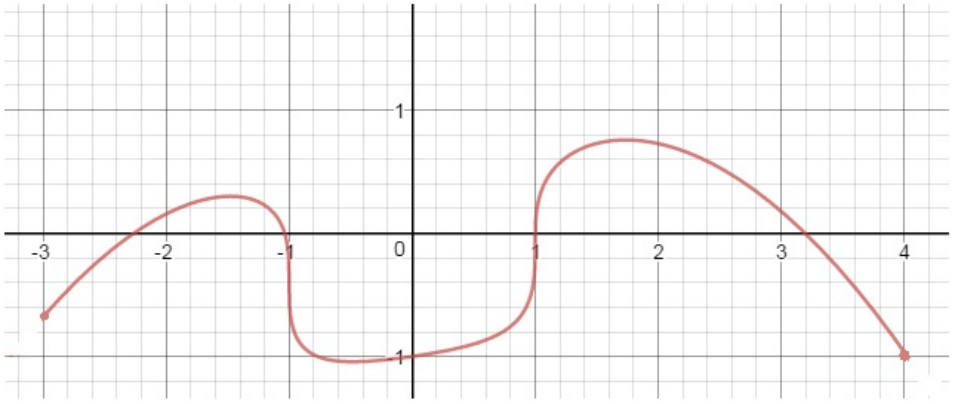
32. If $f(x) = 3\sqrt{x-4}$ and $g(x) = x^2 - 1$, find the formula for $(g \circ f)(x)$.

33. If $f(x) = 13x^{5/3} - 1$, find the formula for $f^{-1}(x)$.

34. If $f(x) = \dfrac{2x+1}{3x-7}$, find the formula for $f^{-1}(x)$.

35. Sketch a graph of the function $f(x) = \sqrt[3]{x^2 - 1}$ by making a table of values and plotting some points.

Consider the following graph of a function, $y = f(x)$.



36. Find the domain of f.

37. Find, approximately, the range of f.

38. Find, approximately, the intervals where f is increasing.

39. Find, approximately, the intervals where f is decreasing.

40. Find, approximately, the intervals on which $f(x) > 0$.

41. Find the approximate coordinates of any local maxima of f.

42. Find the approximate coordinates of any local minima of f.

43. Is f a one-to-one function?

44. Sketch the graph of $y = f(2 - x) + 1$.

45. Write the standard form of the quadratic function $f(x) = 2x^2 - 8x + 4$.

46. Find the coordinates of the vertex of the graph of $y = x^2 - 5x + 2$.

47. Find the maximum or minimum value of $f(x) = 3x^2 - 8x + 4$.

48. Determine the end behavior of the function $f(x) = 3x^4 - 4x^3 - 10x - 1$.

49. Consider the function $f(x) = x^4 + x^3 - 2x^2$. Find all real zeros of f, state their multiplicities, and sketch the graph of f.

50. Find the quotient and remainder of the division $\dfrac{x^4 - 2x^2 + 7x}{x^2 - x + 3}$.

51. Find the quotient and remainder of the division $\dfrac{x^2 - 5x + 4}{x - 3}$.

52. Let $f(x) = x^5 - 2x^4 - 9x^3 + 22x^2 + 4x - 24$. Suppose that you know that 2 is a zero of f of multiplicity 3. Use this information to completely factor f.

53. Find a polynomial of degree 3 with integer coefficients and zeros at $\frac{1}{2}$, -1, and 2.

54. Let $f(x) = \dfrac{1}{(x+2)^2}$. Find all zeros of f, vertical asymptotes of f, and horizontal asymptotes of f. Find the behavior of the graph near the vertical asymptotes, and use this to sketch a graph of f.

55. Let $f(x) = \dfrac{x^2 - 1}{x^2 - 2x - 8}$. Find all zeros of f, vertical asymptotes of f, and horizontal asymptotes of f. Find the behavior of the graph near the vertical asymptotes, and use this to sketch a graph of f.

56. Solve the inequality $\dfrac{x^2 - 9}{x^3 + x^2 - 4x - 4} > 0$.

57. Let $f(x) = 4e^{4-x}$. Use a calculator to find $f(-1)$, rounded to three decimal places.

58. Write the equation $\log_6(36) = 2$ in exponential form.

59. Write the equation $4^x = 20$ in logarithmic form.

60. Let $f(x) = 7\log_3(x+2)$. Use the change of base formula and a calculator to find $f(2)$, rounded to three decimal places.

61. Use the log laws to expand $\log_3\left(\dfrac{(x+4)\sqrt{2x}}{(x+1)^7}\right)$.

62. Use the log laws to expand $\log_{10}\left(\dfrac{x^2y^4}{\sqrt{x^3-6}}\right)$.

63. Write as a single logarithm using the log laws: $\ln(4x) - 2\ln(x-1) - 6\ln(x+2)$.

64. Solve the equation $3^{x-4} = 27$.

65. Solve the equation $e^{4x} + 4 = 9$.

66. Solve the equation $2^{2x} - 2^x - 12 = 0$.

67. Solve the equation $\log_{10}(2x-3) + 1 = 0$.

68. Solve the equation $\log_3(x^2-4) + \log_3(x) = \log_3(x-2)$.

69. Solve the equation $\log_8(x+5) - \log_8(x-2) = 1$.

70. Convert the angle $160°$ to radians.

71. Convert the angle $\frac{\pi}{18}$ to degrees.

72. Find an angle between 0 and 2π that is coterminal to $-\frac{19\pi}{7}$.

73. Find the reference angle of $-\frac{19\pi}{7}$.

74. If θ is an angle such that $\sin\theta < 0$ and $\tan\theta > 0$, in what quadrant must θ lie?

75. A circle with a radius 4 has a sector with central angle $30°$. Find the area of this sector.

76. A circle with radius 3 has a sector with area 6π. Find the length of the arc surrounding this sector.

77. What is the domain of the function $f(x) = \sin^{-1}(x)$?

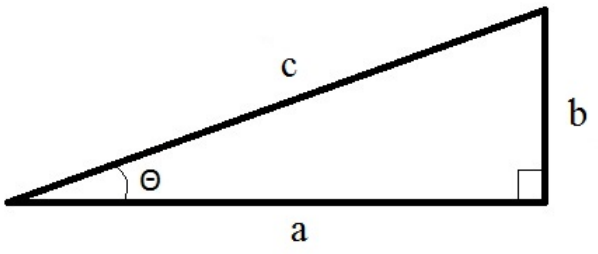
78. What is the range of the function $f(x) = \cos^{-1}(x)$?

79. What is the range of the function $f(x) = \tan^{-1}(x)$?

80. What is the amplitude of the function $f(x) = 3\cos(7x - 2) + 5$?

81. What is the period of the function $f(x) = 3\cos(7x - 2) + 5$?

The following questions are based on a triangle of this form:



82. If $b = 3$ and $c = 4$, find a.

83. If $a = 5$ and $c = 7$, find $\tan\theta$.

84. If $a = 4$ and $\theta = 45°$, find c.

85. If $b = 1$ and $c = 2$, find the value of θ.

86. If $b = 1$ and $c = 3$, find the area of the triangle.

In these problems, you are given three parts of an oblique triangle, where side a is opposite angle A, side b is opposite angle B, and side c is opposite angle C. In each problem, follow the instructions to give the desired information.

87. Suppose that $A = 30°$, $C = 80°$, and $b = 10$. Find the length of side a.

88. Suppose that $A = 40°$, $B = 70°$, and $a = 2$. Find the length of side b.

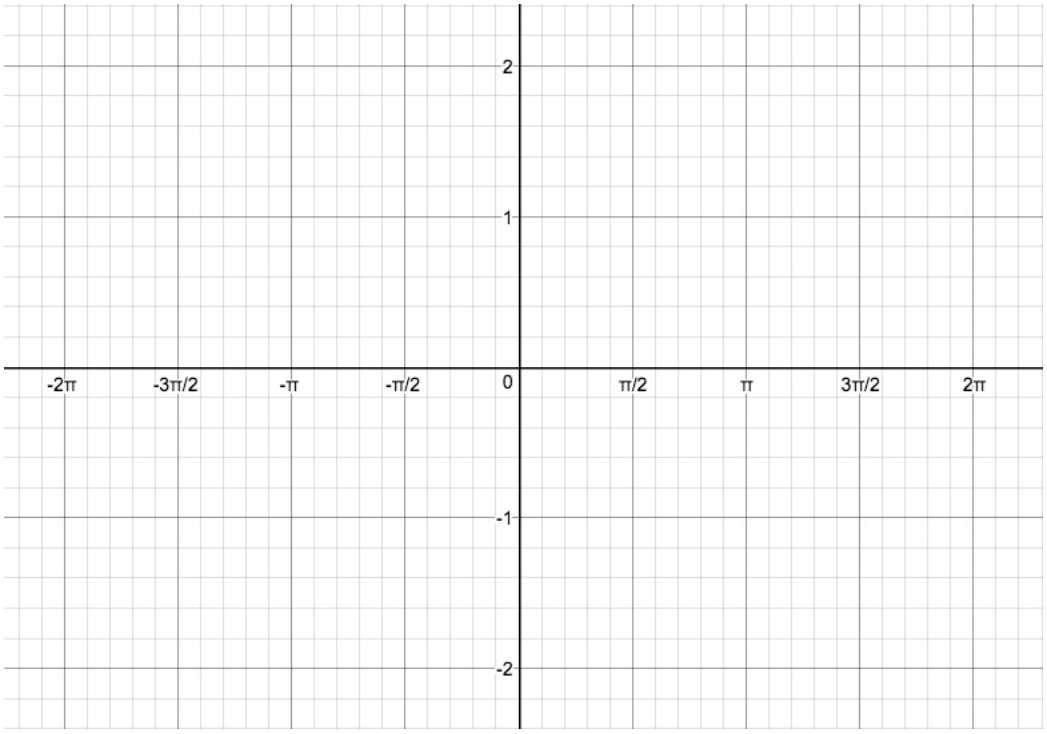
89. Suppose that $a = 8$, $b = 10$, and $c = 12$. Find the measure of angle A.

90. Suppose that $A = 120°$, $b = 8$, $c = 2$. Find the length of side a.

91. Suppose that $A = 35°$, $b = 2$, $c = 7$. Find the area of the triangle.

92. Suppose that $A = 25°$, $a = 12$, and $c = 23$. How many possible solutions are there to this triangle?

93. On the grid below, sketch the graph of $f(x) = \sin(x)$.



94. Review what the graphs of $\tan(x)$, $\cot(x)$, $\sec(x)$, and $\csc(x)$ look like. (For example, redo the graph-matching problem from Exam 4.)

95. For each of the following, find the exact value of the expression (that is, a rounded answer from your calculator will NOT be good enough).
(a) $\sin^{-1}(\sin(\frac{2\pi}{3}))$

(b) $\tan(\tan^{-1}(-4))$

(c) $\csc(\sin^{-1}(0))$

(d) $\sin(\tan^{-1}(-1))$

96. Write the expression $\cos(\tan^{-1}(x))$ as an algebraic expression in terms of x.

97. Write the expression $\cot(\sin^{-1}(x))$ as an algebraic expression in terms of x.

98. Use a half-angle formula to find the exact value of $\tan(15°)$. (A rounded answer from your calculator will NOT be good enough.)

99. Use a half-angle formula to find the exact value of $\cos\left(\frac{\pi}{8}\right)$. (A rounded answer from your calculator will NOT be good enough.)

100. Verify the identity $(1 - \tan x)(1 - \cot x) = 2 - \sec x \csc x$.

101. Verify the identity $\dfrac{\sin 2x}{\sin x} - \dfrac{\cos 2x}{\cos x} = \sec x$.

102. Verify the identity $(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 2 + 2\cos(x + y)$.

103. Find the exact value of $\tan(\csc^{-1}(4) - \cos^{-1}(\frac{1}{8}))$.

104. Find the exact value of $\cos(2\sin^{-1}(\frac{1}{3}) - \frac{\pi}{4})$.

105. Write the expression $\cos(\sin^{-1} x + \cos^{-1} y)$ as an algebraic expression in terms of x and y.

106. Write the expression $\sin(2\tan^{-1} x)$ as an algebraic expression in terms of x.

107. Solve the equation $4\cos(\theta) = 1$.

108. Solve the equation $\tan(\theta) = -6$.

109. Solve the equation $\cos(\theta)\tan(\theta) - \cos(\theta) = 0$.

110. Solve the equation $3\sin^2(\theta) + 2\sin(\theta) = 1$.

111. Let $z = 1 + i$, let $w = 1 - \sqrt{3}i$.
(a) Write z and w in polar form.

(b) Compute zw.

(c) Compute $(zw)^7$.

(d) Compute $\dfrac{z^2}{w^3}$.

112. Consider the point $P = (-\sqrt{6}, \sqrt{2})$ in rectangular coordinates. Convert P to polar coordinates.

113. Consider the point $Q = (3, \pi/6)$ in polar coordinates. Convert Q to rectangular coordinates.

114. Using the variables x and y, convert the polar equation $r = 6\sec\theta$ to rectangular coordinates.

115. Using the variables x and y, convert the polar equation $r = 2\cos\theta$ to rectangular coordinates.

116. Using the variables x and y, convert the polar equation $r = 1+\cos\theta$ to rectangular coordinates.

117. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be 32°. One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is 35°. Find the height of the mountain, to the nearest foot.

118. A 96-ft tree casts a shadow that is 120 ft long. What is the angle of elevation of the sun?

119. The Leaning Tower of Pisa leans 5.6° from the vertical. A tourist stands 105 m from its base with the tower leaning directly towards her. She measures the angle of elevation to the top of the tower to be 29.2°. Find the length of the tower, to the nearest meter.

120. Find two integers whose sum is 4542 and whose product is 1366632.

121. An apartment has a rectangular bedroom with an area of 228 ft^2 that is 7 ft longer than it is wide. What is the width of the room?

122. A large box has a volume of 180 ft^3. Its length is 9 ft greater than its height, and its width is 4 ft less than its height. What are the dimensions of the box?

123. A ball is thrown into the air, and its height h after t seconds is given by $h(t) = 128+16t-16t^2$. Find the maximum height achieved by the ball.

124. You invest \$500 into an account with an annual interest rate of 8% that compounds monthly. How much money will be in your account after 2 years?

125. You invest \$800 into an account with an annual interest rate of 10% that compounds continuously. How much money will be in your account after 3 years?

126. You and your friend are each investing \$1000 into bank accounts. Your account has an annual interest rate of 6% and compounds continuously. Your friend's account compounds monthly, but you don't know the interest rate. At the end of 1 year, you end up with the exact same amount of money in your account as your friend has in her account. Find the annual interest rate of your friend's account.