Math 129: Pre-Calculus
Practice Problems for Trig Exam
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1. Convert the angle $160^{\circ}$ to radians.
2. Convert the angle $\frac{\pi}{18}$ to degrees.
3. Find an angle between 0 and $2 \pi$ that is coterminal to $-\frac{19 \pi}{7}$.
4. Find the reference angle of $-\frac{19 \pi}{7}$.
5. If $\theta$ is an angle such that $\sin (\theta)<0$ and $\tan (\theta)>0$, in what quadrant must $\theta$ lie?
6. Find the terminal point on the unit circle determined by $t=\frac{\pi}{6}$.
7. What is the domain of the function $f(x)=\sin ^{-1}(x)$ ?
8. What is the range of the function $f(x)=\cos ^{-1}(x)$ ?
9. What is the range of the function $f(x)=\tan ^{-1}(x)$ ?
10. What is the amplitude of the function $f(x)=3 \cos (7 x-2)+5$ ?
11. What is the period of the function $f(x)=3 \cos (7 x-2)+5$ ?

The following questions are based on a triangle of this form:

12. If $b=3$ and $c=4$, find $a$.
13. If $a=5$ and $c=7$, find $\tan \theta$.
14. If $a=4$ and $\theta=45^{\circ}$, find $c$.
15. If $b=1$ and $c=2$, find the value of $\theta$.
16. If $b=1$ and $c=3$, find the area of the triangle.

In these problems, you are given three parts of an oblique triangle, where side $a$ is opposite angle $A$, side $b$ is opposite angle $B$, and side $c$ is opposite angle $C$. In each problem, follow the instructions to give the desired information.
17. Suppose that $A=30^{\circ}, C=80^{\circ}$, and $b=10$. Find the length of side $a$.
18. Suppose that $A=40^{\circ}, B=70^{\circ}$, and $a=2$. Find the length of side $b$.
19. Suppose that $a=8, b=10$, and $c=12$. Find the measure of angle $A$.
20. Suppose that $A=120^{\circ}, b=8, c=2$. Find the length of side $a$.
21. Suppose that $A=35^{\circ}, b=2, c=7$. Find the area of the triangle.
22. Suppose that $A=25^{\circ}, a=12$, and $c=23$. How many possible solutions are there to this triangle?
23. On the grid below, sketch the graphs of $f(x)=\sin (x)$ and $g(x)=\cos (x)$.

24. Review what the graphs of $\tan (x), \cot (x), \sec (x)$, and $\csc (x)$ look like. (For example, redo the graph-matching problem from Exam 3.)
25. Consider each of the following angles, expressed with inverse trig functions. Determine the quadrant of each angle.
(a) $\sin ^{-1}\left(-\frac{2}{3}\right)$
(b) $\tan ^{-1}\left(-\frac{2}{3}\right)$
(c) $\cos ^{-1}\left(-\frac{2}{3}\right)$
26. For each of the following, find the exact value of the expression (that is, a rounded answer from your calculator will NOT be good enough).
(a) $\sin ^{-1}\left(\sin \left(\frac{2 \pi}{3}\right)\right)$
(b) $\tan \left(\tan ^{-1}(-4)\right)$
(c) $\csc \left(\sin ^{-1}(0)\right)$
(d) $\sin \left(\tan ^{-1}(-1)\right)$
27. Verify the identity $(1-\tan x)(1-\cot x)=2-\sec x \csc x$.
28. Verify the identity $\frac{\sin 2 x}{\sin x}-\frac{\cos 2 x}{\cos x}=\sec x$.
29. Verify the identity $(\cos x+\cos y)^{2}+(\sin x-\sin y)^{2}=2+2 \cos (x+y)$.
30. Use a half-angle formula to find the exact value of $\tan \left(15^{\circ}\right)$. (A rounded answer from your calculator will NOT be good enough.)
31. Use a half-angle formula to find the exact value of $\cos \left(\frac{\pi}{8}\right)$. (A rounded answer from your calculator will NOT be good enough.)
32. Write the expression $\csc \left(\tan ^{-1} x\right)$ as an algebraic expression in terms of $x$.
33. Write the expression $\tan \left(\cos ^{-1} x\right)$ as an algebraic expression in terms of $x$.
34. Write the expression $\cos \left(\sin ^{-1} x+\cos ^{-1} y\right)$ as an algebraic expression in terms of $x$ and $y$.
35. Write the expression $\sin \left(2 \tan ^{-1} x\right)$ as an algebraic expression in terms of $x$.
36. Solve the equation $4 \cos \theta=1$.
37. Solve the equation $\cos \theta \sin \theta-\cos \theta=0$.
38. Solve the equation $\tan (3 \theta)=5$
39. Solve the equation $\sin \theta=\cos 2 \theta$.
40. Let $z=1+i$, let $w=1-\sqrt{3} i$.
(a) Write $z$ and $w$ in polar form.
(b) Compute $z w$.
(c) Compute $(z w)^{7}$.
(d) Compute $\frac{z^{2}}{w^{3}}$.
41. Consider the point $P=(-\sqrt{6}, \sqrt{2})$ in rectangular coordinates. Convert $P$ to polar coordinates.
42. Consider the point $Q=(3, \pi / 6)$ in polar coordinates. Convert $Q$ to rectangular coordinates.
43. Using the variables $x$ and $y$, convert the polar equation $r=6 \sec \theta$ to rectangular coordinates.
44. Using the variables $x$ and $y$, convert the polar equation $r=2 \cos \theta$ to rectangular coordinates.
45. Using the variables $x$ and $y$, convert the polar equation $r=1+\cos \theta$ to rectangular coordinates.
46. Consider the vector $\mathbf{v}=\langle 7,-2\rangle$, and let $\mathbf{u}$ be the vector with magnitude $\sqrt{8}$ and direction $135^{\circ}$.
(a) Write $\mathbf{v}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.
(b) Compute the magnitude of $\mathbf{v}$.
(c) Compute the direction of $\mathbf{v}$.
(d) Write $\mathbf{u}$ in component form.
(e) Compute the dot product $\mathbf{u} \cdot \mathbf{v}$.
(f) Compute the angle between $\mathbf{u}$ and $\mathbf{v}$.
(g) Compute the vector $9 \mathbf{u}+4 \mathbf{v}$ in component form.
(h) Determine whether $9 \mathbf{u}+4 \mathbf{v}$ is orthogonal to $\mathbf{u}$.
47. To estimate the height of a mountain above a level plain, the angle of elevation to the top of the mountain is measured to be $32^{\circ}$. One thousand feet closer to the mountain along the plain, it is found that the angle of elevation is $35^{\circ}$. Find the height of the mountain, to the nearest foot.
48. A 96 - ft tree casts a shadow that is 120 ft long. What is the angle of elevation of the sun?
49. The Leaning Tower of Pisa leans $5.6^{\circ}$ from the vertical. A tourist stands 105 m from its base with the tower leaning directly towards her. She measures the angle of elevation to the top of the tower to be $29.2^{\circ}$. Find the length of the tower, to the nearest meter.
50. A lawn mower is pushed a distance of 200 ft along a horizontal path by a constant force of 50 lb . The handle of the lawn mower is at an angle of $30^{\circ}$ from the horizontal. Find the work done.

