## Final Exam Material and Expectations

For the final exam, you should be able to do the following things:

**Chapter 2.**

- Given a graph, find one-sided and two-sided limits
- Evaluate easy limits by plugging in the value
- Use limit laws to rewrite limits as necessary
- Evaluate harder limits by using some tricks, which include:
  - factoring polynomials and canceling common factors
  - expanding out polynomials, combining like terms, and refactoring
  - simplifying radicals by multiplying by the conjugate
  - simplifying complex fractions by simplifying and rewriting as a single fraction
  - considering piecewise-defined functions by looking at left and right limits separately
  - rewriting absolute value as a piecewise-defined function
- Use the Squeeze Theorem to find limits
- Determine if a function is continuous using limits
- Determine whether a discontinuity is removable, jump, or infinite
- Use $\lim_{x\to 0} \dfrac{\sin(x)}{x}$ to compute limits

**Chapter 3.**

- Use the limit definition of the derivative to compute the derivative of a function at a point
- Use the basic differentiation rules, including:
  - derivative of a sum is the sum of the derivatives
  - derivative of a constant multiple is the constant multiple of the derivative
  - product rule
  - quotient rule
  - chain rule
- Know the derivatives of powers of $x$ and trig functions (full list of derivatives to know is at the end)
- Find the equation of a tangent line to a curve
- Use implicit differentiation to find $\frac{dy}{dx}$ for an implicitly defined curve
- Compute second derivatives, and general higher derivatives
- Use logarithmic differentiation to differentiate messy functions

**Chapter 4.**

- Solve related rates problems
- Use the linearization formula to estimate values of functions
- Use $f'$ to get where $f$ is increasing or decreasing and find local maxima and minima
- Use $f''$ to get where $f$ is concave up or concave down and find inflection points
- Compute limits at infinity and negative infinity
- Use limit info to check for vertical and horizontal asymptotes
- Sketch the graph of a function using the previous ingredients

- Solve optimization problems
- Use l'Hôpital's Rule to evaluate limits of indeterminate forms, including
  - $\frac{0}{0}$ form
  - $\frac{\infty}{\infty}$ form
  - $0 \cdot \infty$ form
  - $0^0$ form
  - $\infty^0$ form
  - $1^\infty$ form

**Chapter 5.**

- Consider the integral as a measure of signed area between curve and $x$-axis
- Use basic integral rules, including:
  - integral of a sum is the sum of the integrals
  - integral of a constant multiple is the constant multiple of the integral
  - $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
- Use antiderivatives to evaluate definite integrals and express indefinite integrals
- Use integration by substitution to solve definite and indefinite integrals
- Find the average value of a function on an interval

**Chapter 6.**

- Find the area between two curves
- Find the volume of a solid with a flat base and info about cross-sections
- Find the volume of a solid of revolution using either disk/washer method or cylindrical shells method
- Use integration to solve problems involving work or density
- Recall the definitions of sinh and cosh in terms of $e^x$ and $e^{-x}$, and know their derivatives and antiderivatives

**Derivatives and Integrals To Know.**

- You should definitely have the following memorized:

| | | |
|---|---|---|
| (i) | $\frac{d}{dx}\left(x^n\right) = nx^{n-1}$ | $\int x^n \ dx = \frac{x^{n+1}}{n+1} + C$ |
| (ii) | $\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$ | $\int \cos(x) \ dx = \sin(x) + C$ |
| (iii) | $\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$ | $\int \sin(x) \ dx = -\cos(x) + C$ |
| (iv) | $\frac{d}{dx}\left(e^x\right) = e^x$ | $\int e^x \ dx = e^x + C$ |
| (v) | $\frac{d}{dx}\left(\ln(x)\right) = \frac{1}{x}$ | $\int \frac{1}{x} \ dx = \ln(\lvert x\rvert) + C$ |
| (vi) | $\frac{d}{dx}\left(\tan^{-1}(x)\right) = \frac{1}{1+x^2}$ | $\int \frac{1}{1+x^2} \ dx = \tan^{-1}(x) + C$ |
| (vii) | $\frac{d}{dx}\left(\sinh(x)\right) = \cosh(x)$ | $\int \cosh(x) \ dx = \sinh(x) + C$ |
| (viii) | $\frac{d}{dx}\left(\cosh(x)\right) = \sinh(x)$ | $\int \sinh(x) \ dx = \cosh(x) + C$ |

- Additionally, you may want to either memorize or remember the process of finding the derivatives and/or integrals of the following functions: $\tan(x)$, $\sec(x)$, $b^x$, $\log_b(x)$, $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tanh(x)$, $\operatorname{sech}(x)$.