# FINAL EXAM MATERIAL AND EXPECTATIONS

For the final exam, you should be able to do the following things:

### Chapter 2.

- Given a graph, find one-sided and two-sided limits
- Evaluate easy limits by plugging in the value
- Use limit laws to rewrite limits as necessary
- Evaluate harder limits by using some tricks, which include:
  - factoring polynomials and canceling common factors
  - expanding out polynomials, combining like terms, and refactoring
  - simplifying radicals by multiplying by the conjugate
  - simplifying complex fractions by simplifying and rewriting as a single fraction
  - considering piecewise-defined functions by looking at left and right limits separately
  - rewriting absolute value as a piecewise-defined function
- Use the Squeeze Theorem to find limits
- Determine if a function is continuous using limits
- Determine whether a discontinuity is removable, jump, or infinite
- Use  $\lim_{x \to 0} \frac{\sin(x)}{x}$  to compute limits

#### Chapter 3.

- Use the limit definition of the derivative to compute the derivative of a function at a point
- Use the basic differentiation rules, including:
  - derivative of a sum is the sum of the derivatives
  - derivative of a constant multiple is the constant multiple of the derivative
  - product rule
  - quotient rule
  - chain rule
- Know the derivatives of powers of x and trig functions (full list of derivatives to know is at the end)
- Find the equation of a tangent line to a curve
- Use implicit differentiation to find  $\frac{dy}{dx}$  for an implicitly defined curve
- Compute second derivatives, and general higher derivatives
- Use logarithmic differentiation to differentiate messy functions

### Chapter 4.

- Solve related rates problems
- Use the linearization formula to estimate values of functions
- Use f' to get where f is increasing or decreasing and find local maxima and minima
- Use f'' to get where f is concave up or concave down and find inflection points
- Compute limits at infinity and negative infinity
- Use limit info to check for vertical and horizontal asymptotes
- Sketch the graph of a function using the previous ingredients

- Solve optimization problems
- Use l'Hôpital's Rule to evaluate limits of indeterminate forms, including

| $-\frac{0}{0}$ form           | $- 0 \cdot \infty$ form $- 0^0$ form |
|-------------------------------|--------------------------------------|
| $-\frac{\infty}{\infty}$ form | $-\infty^0$ form $-1^\infty$ form    |

## Chapter 5.

- Consider the integral as a measure of signed area between curve and x-axis
- Use basic integral rules, including:
  - integral of a sum is the sum of the integrals
- integral of a constant multiple is the constant multiple of the integral  $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$  Use antiderivatives to evaluate definite integrals and express indefinite in-
- tegrals
- Use integration by substitution to solve definite and indefinite integrals
- Find the average value of a function on an interval

# Chapter 6.

- Find the area between two curves
- Find the volume of a solid with a flat base and info about cross-sections
- Find the volume of a solid of revolution using either disk/washer method or cylindrical shells method
- Use integration to solve problems involving work or density
- Recall the definitions of sinh and cosh in terms of  $e^x$  and  $e^{-x}$ , and know their derivatives and antiderivatives

### Derivatives and Integrals To Know.

• You should definitely have the following memorized:

(i) 
$$\frac{d}{dx}(x^{n}) = nx^{n-1}$$
$$\int x^{n} dx = \frac{x^{n+1}}{n+1} + C$$
$$(ii) \frac{d}{dx}(\sin(x)) = \cos(x)$$
$$\int \cos(x) dx = \sin(x) + C$$
$$(iii) \frac{d}{dx}(\cos(x)) = -\sin(x)$$
$$\int \sin(x) dx = -\cos(x) + C$$
$$(iv) \frac{d}{dx}(e^{x}) = e^{x}$$
$$\int e^{x} dx = e^{x} + C$$
$$(v) \frac{d}{dx}(\ln(x)) = \frac{1}{x}$$
$$\int \frac{1}{x} dx = \ln(|x|) + C$$
$$(vi) \frac{d}{dx}(\tan^{-1}(x)) = \frac{1}{1+x^{2}}$$
$$\int \frac{1}{1+x^{2}} dx = \tan^{-1}(x) + C$$
$$(vii) \frac{d}{dx}(\sinh(x)) = \cosh(x)$$
$$\int \cosh(x) dx = \sinh(x) + C$$
$$(viii) \frac{d}{dx}(\cosh(x)) = \sinh(x)$$
$$\int \sinh(x) dx = \cosh(x) + C$$

• Additionally, you may want to either memorize or remember the process of finding the derivatives and/or integrals of the following functions:  $\tan(x), \sec(x), b^x, \log_b(x), \sin^{-1}(x), \cos^{-1}(x), \tanh(x), \operatorname{sech}(x).$ 

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