Math 155: Calculus I
Spring 2024
Practice Problems for Final Exam

Name (Print): ____________________

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1. Compute $\lim_{x\to 2} \dfrac{x^2-2}{x^3+4x+3}$.

2. Compute $\lim_{x\to 0} e^x$.

3. Compute $\lim_{x\to 4} \log_2(x)$.

4. Compute $\lim_{x\to 3} \dfrac{x^2-6x+9}{x^2-9}$.

5. Compute $\lim_{x\to -1} \dfrac{\sqrt{2x+3}-1}{x+1}$.

6. Compute $\lim_{x\to 0} \dfrac{(x+2)^3-8}{x}$.

7. Compute $\lim_{x\to 0} \left(\dfrac{1}{x\sqrt{x+1}} - \dfrac{1}{x}\right)$.

8. Compute $\lim_{x\to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$.

9. Compute $\lim_{x\to 0^+} \ln(x)$.

10. Compute $\lim_{x\to 0} \dfrac{\sin(2x)}{x}$.

11. Compute $\lim_{x\to \infty} \dfrac{2x^3-4x+18}{x^3+20x^2+11x+13}$.

12. Compute $\lim_{x\to -\infty} \dfrac{5x^2+10x-1}{x^3+2}$.

13. Compute $\lim\limits_{x\to 1} \dfrac{x^3 - 2x^2 + 1}{x^3 - 1}$.

14. Compute $\lim\limits_{x\to 0} \dfrac{e^{2x} - 1}{\sin(x)}$.

15. Compute $\lim\limits_{x\to \infty} \dfrac{\ln(x)}{\sqrt{x}}$.

16. Compute $\lim\limits_{x\to 0^+} \dfrac{\ln(x)}{\sqrt{x}}$.

17. Compute $\lim\limits_{x\to -1} \dfrac{x^9 + 1}{x^5 + 1}$.

18. Compute $\lim\limits_{x\to 0} \dfrac{\sqrt{1+2x} - \sqrt{1-4x}}{x}$.

19. Compute $\lim\limits_{x\to 0} \dfrac{e^x - 1 - x}{x^2}$.

20. Compute $\lim\limits_{x\to 0} \dfrac{\ln(1+x)}{\sin(x) + e^x - 1}$.

21. Compute $\lim\limits_{x\to \infty} x^3 e^{-x^2}$.

22. Compute $\lim\limits_{x\to 0^+} x^{\sqrt{x}}$.

23. Compute $\lim\limits_{x\to \infty} x^{1/x}$.

24. Compute $\lim\limits_{x\to 0^+} (1-x)^{1/x}$.

25. Use the Squeeze Theorem to find $\lim_{x\to 0} x^2 \sin(\frac{\pi}{x})$.

26. Let $f(x) = \dfrac{x^2-16}{2x^2+8x}$. Find any discontinuities and determine whether they are removable, jump, or infinite.

27. Let $f(x) = \dfrac{x^2+2x+1}{x^2-1}$. Find any discontinuities and determine whether they are removable, jump, or infinite.

28. Let $f(x) = x^2 + 3x - 2$. Use the limit definition of the derivative to compute $f'(4)$.

29. Let $f(x) = \dfrac{2}{x+3}$. Use the limit definition of the derivative to compute $f'(x)$.

30. Let $f(x) = x\ln(x^2)$. Find $f'(x)$.

31. Let $f(x) = e^{-x^2}$. Find $f'(x)$.

32. Let $f(x) = \sin(x)e^{-\cos(x)}$. Find $f'(x)$.

33. Let $f(x) = \dfrac{6x}{\ln(x)}$. Find $f'(x)$.

34. Let $f(x) = \sqrt{\sec(x)}$. Find $f'(x)$.

35. Let $f(x) = \ln(x)\ln(\ln(x))$. Find $f'(x)$.

36. Let $f(x) = e^x e^{(e^x)}$. Find $f'(x)$.

37. Let $f(x) = \tan(e^x)e^{-x}$. Find $f'(x)$.

38. Let $f(x) = \cosh(x^2)$. Find $f'(x)$.

39. Find the equation of the line tangent to the curve $y = e^{\sin(x)}$ at the point $(0, 1)$.

40. Let $f(x) = xe^x$. Find $f''(x)$.

41. Consider the curve $x - 2y = \ln(xy)$. Find $\frac{dy}{dx}$.

42. Consider the curve $(1+x)^2y^2 = e^x$. Find the equation of the tangent line to this curve at the point $(0, -1)$.

43. Let $f(x) = \int_1^{\ln(x)} e^t \, dt$. Find $f'(x)$.

44. The radius of a spherical bubble is increasing at a rate of 2 mm/min. At what rate is the volume increasing at the moment when the radius is 40 mm? You may use the fact that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.

45. Two people start from the same point. One walks east at 3 mph and the other walks south at 2 mph. How fast is the distance between the people changing after a half hour?

46. Use a linear approximation to the function $f(x) = \ln(x)$ at $a = 1$ to estimate the number $\ln(0.95)$.

47. Use a linear approximation to the function $f(x) = e^x$ at $a = 0$ to estimate the number $e^{-1/2}$.

48. Consider the function $f(x) = x^2 e^x$. Find the intervals on which f is increasing or decreasing, find the local maximum and minimum values of f, find the intervals of concavity of f, and find the inflection points of f.

49. Consider the function $f(x) = \dfrac{x^2 - 3x - 10}{x^2 - 4}$. Find the intervals on which f is increasing or decreasing, find the local maximum and minimum values of f, find the intervals of concavity of f, find the inflection points of f, find any vertical and horizontal asymptotes of f, and sketch the graph of f.

50. A box with a square base and an open top must have a volume of 1000 cm^3. Find the dimensions of the box that minimize the amount of material used.

51. A cylindrical box must have a volume of 1000 cm^3. The side of the box is to be made of a material that costs \$0.10 per cm^2 and the circular bases of the box are to be made of a material that costs \$0.50 per cm^2. Find the dimensions of the box that minimize the cost of the box. You may use the facts that the volume of a cylinder of radius r and height h is $\pi r^2 h$ and that the surface area of the side of the cylinder is $2\pi r h$.

52. Compute $\displaystyle\int_0^1 e^{2x}\,dx$.

53. Compute $\displaystyle\int_2^4 4 - \frac{6}{x^2}\,dx$.

54. Compute $\displaystyle\int_0^\pi \sqrt{x} - \cos(3x)\,dx$.

55. Compute $\displaystyle\int_0^1 \frac{4}{x^2+1}\,dx$.

56. Compute $\displaystyle\int_0^1 10^x\,dx$.

57. Compute $\displaystyle\int_0^1 \sinh(x)\,dx$.

58. Find $\displaystyle\int \frac{1}{x\ln(x)\ln(\ln(x))}\,dx$.

59. Find $\displaystyle\int \frac{x^2 - 2x + 1}{x}\,dx$.

60. Find $\displaystyle\int 1 + x^2 + \frac{1}{1+x^2}\,dx$.

61. Find $\displaystyle\int \frac{(\tan^{-1}(2x))^2}{4x^2+1}\,dx$.

62. Find $\displaystyle\int xe^{-x^2}\,dx$.

63. Find $\displaystyle\int \cot(x)\,dx$.

64. Find $\displaystyle\int \frac{1+x}{1+x^2}\,dx$.

65. Find $\displaystyle\int \sinh^2(x)\cosh(x)\,dx$.

66. Find the area of the region bounded by $y = \dfrac{1}{x}$, $y = \dfrac{1}{x^2}$, and $x = 2$.

67. Find the area of the region bounded by $y = \sin(x)$ and $y = e^x$ from $x = 0$ to $x = \pi/2$.

68. Find the volume of the solid whose base is the region bounded by $y = 2^x$ and $y = 1 + 2x - x^2$ and whose cross-sections perpendicular to the x-axis are squares.

69. Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = \cos^{-1}(x)$ about the y-axis.

70. Find the volume of the solid obtained by rotation the region between $y = \ln(x)$ and the x-axis from $x = 1$ to $x = 3$ about the x-axis.

71. Find the work done in moving a box from $x = 0$ to $x = 1$ meters if the force acting on it is $F(x) = 3x^2$ N.

72. Find the mass of a one-dimensional rod that is 3 feet long (starting at $x = 0$) and has a density function of $\rho(x) = e^{x/2}$ lb/ft.

73. Find the average value of the function $f(x) = \dfrac{\ln(x)}{x}$ on the interval $[1, e]$.

74. Find the average value of the function $f(x) = \sec^2(x)e^{\tan(x)}$ on the interval $[0, \pi/4]$.

75. Let $f(x) = \dfrac{x\sqrt{x}}{e^{\sin(x)}}$. Use logarithmic differentiation to find $f'(x)$.

76. Find the equation of the tangent line to the curve $y = \dfrac{(x+1)^6 \tan^{-1}(x)}{\cos^{-1}(x)}$ at the point $(0, 0)$. (Hint: Use logarithmic differentiation.)

77. Let $f(x) = \ln(x)^{\left(\frac{1}{x^2}\right)}$. Use logarithmic differentiation to find $f'(x)$.

78. Let $f(x) = \sin\left(\frac{x}{2}\right)^{\sin(x)}$. Use logarithmic differentiation to find the derivative, and use it to find the equation of the tangent line to the curve $y = f(x)$ at the point $(\pi, 1)$.

79. Let $f(x) = x^7 + 2x^5$. Find $(f^{-1})'(3)$.

80. You have a bowl that is a perfect half-sphere of radius 4 in. Water begins to enter the bowl at a rate of 1 in^3/min. You want to find the rate at which the water level is rising when the water level is 2 in. To do so, you need to do the following steps:

    (a) First, you want a formula for the volume of water in the bowl when the water level is at a height h. But you notice that this volume happens to be the volume of the solid obtained by rotating the region bounded by the y-axis, the line $y = -4 + h$, and the curve $y = -\sqrt{16 - x^2}$ about the y-axis. Find this volume, in terms of h.

    (b) Set up a related rates problem using your volume formula from part (a) to compute the solution.