Math 156: Calculus II **Name (Print):** ______________________

Fall 2017

Practice Problems for Final Exam

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1. Compute $\lim_{x\to 1} \frac{x^3-2x^2+1}{x^3-1}$.

2. Compute $\lim_{x\to 0} \frac{e^{2x}-1}{\sin(x)}$.

3. Compute $\lim_{x\to \infty} \frac{\ln(x)}{\sqrt{x}}$.

4. Compute $\lim_{x\to 0^+} \frac{\ln(x)}{\sqrt{x}}$.

5. Compute $\lim_{x\to -1} \frac{x^9+1}{x^5+1}$.

6. Compute $\lim_{x\to 0} \frac{\sqrt{1+2x}-\sqrt{1-4x}}{x}$.

7. Compute $\lim_{x\to 0} \frac{e^x-1-x}{x^2}$.

8. Compute $\lim_{x\to 0} \frac{\ln(1+x)}{\sin(x)+e^x-1}$.

9. Compute $\lim_{x\to \infty} x^3e^{-x^2}$.

10. Compute $\lim_{x\to 0^+} x^{\sqrt{x}}$.

11. Compute $\lim_{x\to \infty} x^{1/x}$.

12. Compute $\lim_{x\to 0^+} (1-x)^{1/x}$.

Evaluate each integral.

13. $\displaystyle\int_1^2 \frac{(x+1)^2}{x}\,dx$

14. $\displaystyle\int \frac{e^{\sin(x)}}{\sec(x)}\,dx$

15. $\displaystyle\int \frac{1}{2x^2+3x+1}\,dx$

16. $\displaystyle\int_0^{\pi/2} \sin^3(x)\cos^2(x)\,dx$

17. $\displaystyle\int \frac{\sin(\ln(x))}{x}\,dx$

18. $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$

19. $\displaystyle\int \frac{x-1}{x^2+2x}\,dx$

20. $\displaystyle\int \frac{1}{x\sqrt{x^2+1}}\,dx$

21. $\displaystyle\int \frac{x^2}{(4-x^2)^{3/2}}\,dx$

22. $\displaystyle\int \frac{\cos(x)}{1-\sin(x)}\,dx$

23. $\displaystyle\int_1^4 \sqrt{x}\ln(x)\,dx$

24. $\displaystyle\int_{-1}^1 \frac{e^{\tan^{-1}(x)}}{1+x^2}\,dx$

25. $\displaystyle\int \frac{1}{x^3\sqrt{x^2-1}}\,dx$

26. $\displaystyle\int x\sec(x)\tan(x)\,dx$

27. $\displaystyle\int_0^{\pi} x\cos^2(x)\,dx$

28. $\displaystyle\int e^{x+e^x}\,dx$

29. $\displaystyle\int \tan^{-1}(\sqrt{x})\,dx$

30. $\displaystyle\int \frac{1}{1+e^x}\,dx$

31. $\displaystyle\int \frac{e^{2x}}{1+e^x}\,dx$

32. $\displaystyle\int \frac{1}{x\sqrt{4x+1}}\,dx$

33. $\displaystyle\int \frac{1}{x\sqrt{4x^2+1}}\,dx$

34. $\displaystyle\int \frac{1}{x+x\sqrt{x}}\,dx$

35. $\displaystyle\int \sqrt{x}e^{\sqrt{x}}\,dx$

36. $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$

37. $\displaystyle\int \frac{1}{x\ln(x)-x}\,dx$

38. $\displaystyle\int \frac{\sqrt{x}}{1+x^3}\,dx$

Determine whether each integral is convergent or divergent and evaluate those that are convergent.

39. $\displaystyle\int_0^{\infty} \frac{x^2}{\sqrt{1+x^3}}\, dx$

40. $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2}\, dx$

41. $\displaystyle\int_1^{\infty} \frac{\ln(x)}{x}\, dx$

42. $\displaystyle\int_0^{1} \frac{1}{x}\, dx$

43. $\displaystyle\int_{-2}^{3} \frac{1}{x^4}\, dx$

44. $\displaystyle\int_0^{9} \frac{1}{\sqrt[3]{x-1}}\, dx$

45. $\displaystyle\int_0^{\pi/2} \sec^2(x)\, dx$

46. Find the centroid of the region bounded by the curves $y = x^2$ and $x = y^2$.

47. Find the centroid of the region bounded by the curves $y = e^x$, $y = 0$, $x = 0$, and $x = 1$.

48. Find the centroid of the region bounded by the curves $y = \sin(x)$, $y = \cos(x)$, $x = 0$, and $x = \pi/4$.

49. Find the centroid of the region bounded by the curves $x + y = 2$ and $x = y^2$.

50. Find the area of the region bounded by the polar curve $r = e^{-\theta/4}$ from $\theta = \pi/2$ to $\theta = \pi$.

51. Find the area of the region enclosed by the polar curve $e = 3 + 2\cos(\theta)$.

52. Find the area of the region enclosed by one loop of the curve $r = 4\cos(3\theta)$.

For problems 53 through 61, write an integral that gives the arc length of each given curve. Compute the integral if possible.

53. the curve $y = \frac{x^3}{3} + \frac{1}{4x}$, where $1 \leq x \leq 2$

54. the curve $y = \ln(\sec(x))$, where $0 \leq x \leq \pi/4$

55. the curve $y = x - \ln(x)$, where $1 \leq x \leq 4$

56. the curve parametrized by $x = 1 + 3t^2$, $y = 4 + 2t^3$, where $0 \leq t \leq 1$

57. the curve parametrized by $x = t + e^{-t}$, $y = t - e^{-t}$, where $0 \leq t \leq 2$

58. the curve parametrized by $x = t\sin(t)$, $y = t\cos(t)$, where $0 \leq t \leq 1$

59. the polar curve $r = 2\cos(\theta)$, where $0 \leq \theta \leq \pi$

60. the polar curve $r = \sin(6\sin(\theta))$, where $0 \leq \theta \leq \pi$

61. the polar curve $r = \theta^2$, where $0 \leq \theta \leq 2\pi$

62. Find the eccentricity of the conic $r = \frac{2}{3 + 3\sin(\theta)}$ and identify the type of conic.

63. Find the eccentricity of the conic $r = \frac{3}{4 - 8\cos(\theta)}$ and identify the type of conic.

64. Find the eccentricity of the conic $r = \frac{4}{5 - 4\sin(\theta)}$ and identify the type of conic.

For each curve given in problems 65 through 72, write one integral that gives the surface area resulting from rotating the curve about the x-axis. Then, write another integral that gives the surface area resulting from rotating the curve about the y-axis. Compute all integrals that are possible to solve.

65. the curve $y = x^3$, where $0 \leq x \leq 2$

66. the curve $x = y + y^3$, where $0 \leq y \leq 1$

67. the curve $x = \frac{1}{3}(y^2 + 2)^{3/2}$, where $1 \leq y \leq 2$

68. the curve $y = xe^x$, where $0 \leq x \leq 5$

69. the curve $y = \frac{1}{x}$, where $1 \leq x \leq 2$

70. the curve parametrized by $x = t\sin(t)$, $y = t\cos(t)$, where $0 \leq t \leq \pi/2$

71. the curve parametrized by $x = t^3$, $y = t^2$, where $0 \leq t \leq 1$

72. the curve parametrized by $x = t + e^t$, $y = e^{-t}$, where $0 \leq t \leq 1$

73. Find the area enclosed by the x-axis and the parametric curve $x = t^3 + 1$, $y = 2t - t^2$.

74. Find the area enclosed by the y-axis and the parametric curve $x = t^2 - 2t$, $y = \sqrt{t}$.

For each curve given in problems 75 through 80, find all points where the curve has a vertical tangent line or a horizontal tangent line.

75. the parametric curve $x = t^3 - 3t$, $y = t^2 - 3$

76. the parametric curve $x = \cos(\theta)$, $y = \cos(3\theta)$

77. the parametric curve $x = 1 + \ln(t)$, $y = t^2 + 2$

78. the polar curve $r = 3\cos(\theta)$

79. the polar curve $r = 1 + \cos(\theta)$

80. the polar curve $r = e^{\theta}$

81. Find a power series representation for the function $f(x) = \dfrac{2}{3-x}$ and determine the interval of convergence.

82. Find a power series representation for the function $f(x) = \dfrac{x^2}{x^4+16}$ and determine the interval of convergence.

83. Find a power series representation for the function $f(x) = \dfrac{x}{(1+4x)^2}$ and determine the radius of convergence.

84. Find a power series representation for the function $f(x) = \tan^{-1}(x^4)$ and determine the radius of convergence.

85. Find a power series representation for the function $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$ and determine the radius of convergence.

For each series, determine convergence or divergence. For convergent alternating series, also determine absolute convergence or conditional convergence.

86. $\displaystyle\sum_{n=0}^{\infty} \frac{n^2-1}{n^3+1}$

87. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n^2-1}{n^3+1}$

88. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

89. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$

90. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!}$

91. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3} + \frac{1}{3^n}\right)$

92. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n n^2}{n!}$

93. $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n-1}3^{n+1}}{n^n}$

94. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$

95. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos(1/n^2)$

96. $\displaystyle\sum_{n=1}^{\infty} \tan(1/n)$

97. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{e^{n^2}}$

98. $\displaystyle\sum_{n=0}^{\infty} \frac{n\ln(n)}{(n+1)^3}$

99. $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{3^n+4^n}$

100. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

101. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

102. $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)^n$

103. $\displaystyle\sum_{n=0}^{\infty} \frac{e^n+1}{ne^n+1}$

104. $\displaystyle\sum_{n=0}^{\infty} ne^{-n}$

105. $\displaystyle\sum_{n=0}^{\infty} \frac{n^{100}100^n}{n!}$

106. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(\ln(n))^n}$

For each power series given in problems 107 through 112, find the radius of convergence and interval of convergence.

107. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2n-1}$

108. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

109. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{2^n(n^2+1)} x^n$

110. $\displaystyle\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln(n)}$

111. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^n}$

112. $\displaystyle\sum_{n=0}^{\infty} \frac{(5x-4)^n}{n^3}$

113. Find the Taylor series for $f(x) = (1-x)^{-2}$ centered at $a = 0$, and find the radius of convergence.

114. Find the Taylor series for $f(x) = 2^x$ centered at $a = 0$ and find the radius of convergence.

115. Find the Taylor series for $f(x) = \ln(x)$ centered at $a = 2$, and find the radius of convergence.

116. Find the Taylor series for $f(x) = e^{2x}$ centered at $a = 3$, and find the radius of convergence.