Math 156: Calculus II **Name (Print):** ______________________

Spring 2023

Practice Problems for Final Exam

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1. Solve the differential equation $x^2 \frac{dy}{dx} = (x+1)y$.

2. Solve the differential equation $\frac{dy}{dx} = e^y x^2$.

3. Solve the initial-value problem $\frac{dy}{dx} = y^2(x+1)$, with $y(0) = 2$.

4. Solve the initial-value problem $\frac{dy}{dx} = e^{y-x}$, with $y(0) = 0$.

5. Find the area of the region bounded by the polar curve $r = e^{-\theta/4}$ from $\theta = \pi/2$ to $\theta = \pi$.

6. Find the area of the region enclosed by the polar curve $e = 3 + 2\cos(\theta)$.

7. Find the area of the region enclosed by one loop of the curve $r = 4\cos(3\theta)$.

For problems 8 to 14, determine whether each integral is convergent or divergent and evaluate those that are convergent.

8. $\displaystyle\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}\,dx$

9. $\displaystyle\int_{-\infty}^\infty xe^{-x^2}\,dx$

10. $\displaystyle\int_1^\infty \frac{\ln(x)}{x}\,dx$

11. $\displaystyle\int_0^1 \frac{1}{x}\,dx$

12. $\displaystyle\int_{-2}^3 \frac{1}{x^4}\,dx$

13. $\displaystyle\int_0^9 \frac{1}{\sqrt[3]{x-1}}\,dx$

14. $\displaystyle\int_0^{\pi/2} \sec^2(x)\,dx$

Evaluate each integral.

15. $\displaystyle\int_1^2 \frac{(x+1)^2}{x}\,dx$

16. $\displaystyle\int \frac{e^{\sin(x)}}{\sec(x)}\,dx$

17. $\displaystyle\int \frac{1}{2x^2+3x+1}\,dx$

18. $\displaystyle\int_0^{\pi/2} \sin^3(x)\cos^2(x)\,dx$

19. $\displaystyle\int \frac{\sin(\ln(x))}{x}\,dx$

20. $\displaystyle\int_1^2 \frac{\sqrt{x^2-1}}{x}\,dx$

21. $\displaystyle\int \frac{x-1}{x^2+2x}\,dx$

22. $\displaystyle\int \frac{1}{x\sqrt{x^2+1}}\,dx$

23. $\displaystyle\int \frac{x^2}{(4-x^2)^{3/2}}\,dx$

24. $\displaystyle\int \frac{\cos(x)}{1-\sin(x)}\,dx$

25. $\displaystyle\int_1^4 \sqrt{x}\ln(x)\,dx$

26. $\displaystyle\int_{-1}^1 \frac{e^{\tan^{-1}(x)}}{1+x^2}\,dx$

27. $\displaystyle\int \frac{1}{x^3\sqrt{x^2-1}}\,dx$

28. $\displaystyle\int x\sec(x)\tan(x)\,dx$

29. $\displaystyle\int_0^{\pi} x\cos^2(x)\,dx$

30. $\displaystyle\int e^{x+e^x}\,dx$

31. $\displaystyle\int \tan^{-1}(\sqrt{x})\,dx$

32. $\displaystyle\int \frac{1}{1+e^x}\,dx$

33. $\displaystyle\int \frac{e^{2x}}{1+e^x}\,dx$

34. $\displaystyle\int \frac{1}{x\sqrt{4x+1}}\,dx$

35. $\displaystyle\int \frac{1}{x\sqrt{4x^2+1}}\,dx$

36. $\displaystyle\int \frac{1}{x+x\sqrt{x}}\,dx$

37. $\displaystyle\int \sqrt{x}e^{\sqrt{x}}\,dx$

38. $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x}}\,dx$

39. $\displaystyle\int \frac{1}{x\ln(x)-x}\,dx$

40. $\displaystyle\int \frac{\sqrt{x}}{1+x^3}\,dx$

For problems 41 through 49, write an integral that gives the arc length of each given curve. Compute the integral if possible.

41. the curve $y = \frac{x^3}{3} + \frac{1}{4x}$, where $1 \le x \le 2$

42. the curve $y = \ln(\sec(x))$, where $0 \le x \le \pi/4$

43. the curve $y = x - \ln(x)$, where $1 \le x \le 4$

44. the curve parametrized by $x = 1 + 3t^2$, $y = 4 + 2t^3$, where $0 \le t \le 1$

45. the curve parametrized by $x = t + e^{-t}$, $y = t - e^{-t}$, where $0 \le t \le 2$

46. the curve parametrized by $x = t\sin(t)$, $y = t\cos(t)$, where $0 \le t \le 1$

47. the polar curve $r = 2\cos(\theta)$, where $0 \le \theta \le \pi$

48. the polar curve $r = \sin(6\sin(\theta))$, where $0 \le \theta \le \pi$

49. the polar curve $r = \theta^2$, where $0 \le \theta \le 2\pi$

50. Find the eccentricity of the conic $r = \dfrac{2}{3 + 3\sin(\theta)}$ and identify the type of conic.

51. Find the eccentricity of the conic $r = \dfrac{3}{4 - 8\cos(\theta)}$ and identify the type of conic.

52. Find the eccentricity of the conic $r = \dfrac{4}{5 - 4\sin(\theta)}$ and identify the type of conic.

For each curve given in problems 53 through 60, write one integral that gives the surface area resulting from rotating the curve about the x-axis. Then, write another integral that gives the surface area resulting from rotating the curve about the y-axis. Compute all integrals that are possible to solve.

53. the curve $y = x^3$, where $0 \le x \le 2$

54. the curve $x = y + y^3$, where $0 \le y \le 1$

55. the curve $x = \frac{1}{3}(y^2 + 2)^{3/2}$, where $1 \le y \le 2$

56. the curve $y = xe^x$, where $0 \le x \le 5$

57. the curve $y = \frac{1}{x}$, where $1 \le x \le 2$

58. the curve parametrized by $x = t\sin(t)$, $y = t\cos(t)$, where $0 \le t \le \pi/2$

59. the curve parametrized by $x = t^3$, $y = t^2$, where $0 \le t \le 1$

60. the curve parametrized by $x = t + e^t$, $y = e^{-t}$, where $0 \le t \le 1$

61. Find the area enclosed by the x-axis and the parametric curve $x = t^3 + 1$, $y = 2t - t^2$.

62. Find the area enclosed by the y-axis and the parametric curve $x = t^2 - 2t$, $y = \sqrt{t}$.

For each curve given in problems 63 through 68, find all points where the curve has a vertical tangent line or a horizontal tangent line.

63. the parametric curve $x = t^3 - 3t$, $y = t^2 - 3$

64. the parametric curve $x = \cos(\theta)$, $y = \cos(3\theta)$

65. the parametric curve $x = 1 + \ln(t)$, $y = t^2 + 2$

66. the polar curve $r = 3\cos(\theta)$

67. the polar curve $r = 1 + \cos(\theta)$

68. the polar curve $r = e^{\theta}$

69. Find a power series representation for the function $f(x) = \dfrac{2}{3-x}$ and determine the interval of convergence.

70. Find a power series representation for the function $f(x) = \dfrac{x^2}{x^4+16}$ and determine the interval of convergence.

71. Find a power series representation for the function $f(x) = \dfrac{x}{(1+4x)^2}$ and determine the radius of convergence.

72. Find a power series representation for the function $f(x) = \tan^{-1}(x^4)$ and determine the radius of convergence.

73. Find a power series representation for the function $f(x) = \ln\left(\dfrac{1+x}{1-x}\right)$ and determine the radius of convergence.

For each series, determine convergence or divergence. For convergent alternating series, also determine absolute convergence or conditional convergence.

74. $\displaystyle\sum_{n=0}^{\infty} \frac{n^2-1}{n^3+1}$

75. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n^2-1}{n^3+1}$

76. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^2}$

77. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$

78. $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{(2n)!}$

79. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n^3}+\frac{1}{3^n}\right)$

80. $\displaystyle\sum_{n=0}^{\infty} \frac{3^n n^2}{n!}$

81. $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n-1}3^{n+1}}{n^n}$

82. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$

83. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \cos(1/n^2)$

84. $\displaystyle\sum_{n=1}^{\infty} \tan(1/n)$

85. $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{e^{n^2}}$

86. $\displaystyle\sum_{n=0}^{\infty} \frac{n\ln(n)}{(n+1)^3}$

87. $\displaystyle\sum_{n=0}^{\infty} \frac{5^n}{3^n+4^n}$

88. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n}{n+1}\right)^{n^2}$

89. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1+1/n}}$

90. $\displaystyle\sum_{n=1}^{\infty} \left(\sqrt[n]{2}-1\right)^n$

91. $\displaystyle\sum_{n=0}^{\infty} \frac{e^n+1}{ne^n+1}$

92. $\displaystyle\sum_{n=0}^{\infty} ne^{-n}$

93. $\displaystyle\sum_{n=0}^{\infty} \frac{n^{100}100^n}{n!}$

94. $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{(\ln(n))^n}$

For each power series given in problems 95 through 100, find the radius of convergence and interval of convergence.

95. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2n-1}$

96. $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^4 4^n}$

97. $\displaystyle\sum_{n=0}^{\infty} \frac{n}{2^n(n^2+1)} x^n$

98. $\displaystyle\sum_{n=2}^{\infty} \frac{(x+2)^n}{2^n \ln(n)}$

99. $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^n}$

100. $\displaystyle\sum_{n=0}^{\infty} \frac{(5x-4)^n}{n^3}$

101. Find the Taylor series for $f(x) = (1-x)^{-2}$ centered at $a = 0$, and find the radius of convergence.

102. Find the Taylor series for $f(x) = 2^x$ centered at $a = 0$ and find the radius of convergence.

103. Find the Taylor series for $f(x) = \ln(x)$ centered at $a = 2$, and find the radius of convergence.

104. Find the Taylor series for $f(x) = e^{2x}$ centered at $a = 3$, and find the radius of convergence.