

## FINAL EXAM MATERIAL AND EXPECTATIONS

For the final exam, you should be able to do the following things:

### Chapter 12.

- Know the basics of how equations in three variables describe 3D shapes and surfaces
- Represent a 3D vector in component form, in terms of the standard basis vectors, and in terms of magnitude and direction angles
- Add, subtract, and scalar multiply 3D vectors
- Take dot product of two vectors
- Use dot product to check if vectors are perpendicular
- Compute the angle between two vectors
- Compute the scalar and vector projections from one vector onto another
- Take cross product of two 3D vectors
- Use cross product to produce a vector perpendicular to two given vectors
- Use cross product to check if vectors are parallel
- Use cross product to compute torque
- Given a point and a direction vector, write parametric equations for line through the point with the direction
- Given two points, write parametric equations for the segment connecting them
- Given the equation of a plane, find the normal vector to the plane
- Given a normal vector, write the equation of a plane with that normal vector
- Use normal vectors to compute the angle between planes or check if planes are parallel/perpendicular

### Chapter 13.

- Given a vector function, identify the component functions and find the domain
- Compute limits of vector functions
- Compute the derivative of a vector function
- Compute the definite integral of a vector function
- Compute the unit tangent vector,  $\mathbf{T}(t)$ , of a vector function
- Compute the arc length of a curve drawn out by a vector function
- Compute the arc length function of a vector function
- Compute the curvature of a vector function
- Compute the unit normal vector  $\mathbf{N}(t)$  of a vector function
- Compute the binormal vector  $\mathbf{B}(t)$  of a vector function
- Compute the normal plane to a vector function at  $t$
- Compute the osculating plane to a vector function at  $t$
- If  $\mathbf{r}(t)$  gives the position of an object at time  $t$ , find the velocity and acceleration of the object
- Given the acceleration of an object and the initial position and velocity, find the vector function that gives the position of the object
- Use the acceleration of gravity to find velocity and position of projectiles and answer questions about time

**Chapter 14.**

- Know basics about graphs of functions of several variables
- Compute domain of a function of several variables
- Compute level curves/level surfaces
- Compute limits of functions of two variables
- Show that a limit does not exist by checking along different lines or curves
- Compute partial derivatives and second partial derivatives, including mixed partial derivatives, of functions of several variables
- Know the different notations for partial derivatives ( $f_x$  vs  $\frac{\partial f}{\partial x}$ , etc)
- Compute the tangent plane to a surface  $z = f(x, y)$
- Use the linearization of a function to estimate a nearby function value
- Use the chain rule to compute partial derivatives of compositions
- Compute the gradient vector of a function of several variables
- Compute directional derivatives
- Know the geometric properties of the gradient
- Find local extrema of a function of two variables by finding critical points and using the second derivative test
- Use Lagrange multipliers to maximize or minimize a function of two variables subject to some constraint

**Chapter 15.**

- Compute an iterated integral
- Use Fubini to reverse the order of integration correctly for both 2 variable and 3 variable integrals
- Convert an integral over  $xy$ -coordinates to an integral in polar coordinates
- Given a density function over a region or solid, use an integral to compute total mass/charge and the center of mass/charge
- Write an integral giving the surface area of a surface  $z = f(x, y)$  over a region
- Convert an integral over  $xyz$ -coordinates to an integral in cylindrical coordinates
- Convert an integral over  $xyz$ -coordinates to an integral in spherical coordinates
- Given a transformation of  $uv$ -space into  $xy$ -space, compute the Jacobian of the transformation
- Use the Jacobian to write and solve an integral using a transformation

**Chapter 16.**

- Given a vector field, identify its component functions
- Compute line integrals of scalar functions over curves ( $ds$ ,  $dx$ , or  $dy$ )
- Compute line integrals of vector fields over curves
- Know what a conservative vector field is
- Determine whether a vector field is conservative and find a potential function  $f$  for any conservative vector field
- Understand how the fundamental theorem of line integrals applies to integrals of conservative vector fields over closed curves
- Use Green's theorem to write a line integral as a double integral, and vice-versa, and use this to compute such integrals