Math 124, 4.4 Exponential and Logarithmic Equations

Strategy for solving exponential and logarithmic equations:

- Rewrite each side of the equation as a single exponential or logarithm.
- Eliminate the exponential or logarithm using one of the rules on the right.

$$\begin{bmatrix} a^{u} = a^{v} \Longrightarrow u = v \\ \log_{a}(u) = \log_{a}(v) \Longrightarrow u = v \end{bmatrix}$$

$$\begin{bmatrix} a^{u} = b^{v} \Longrightarrow \log(a^{u}) = \log(b^{v}) \\ a^{u} = b \Longrightarrow \log(a^{u}) = \log(b) \end{bmatrix}$$

$$\log_{a}(x) = y \Longleftrightarrow x = a^{y}$$

• Solve the resulting equation.

Laws of Logarithms:

$$\log_a(x) + \log_a(y) = \log_a(x \cdot y)$$

$$\log_a(x) + \log_a(y) = \log_a(x \cdot y)$$

$$\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$$

$$r \cdot \log_a(x) = \log_a(x^r)$$

Solve each of the following equations. Remember to check your answers.

1.)
$$2^{3x+5} = 2^{5x-1}$$

4.)
$$\ln(x+3) = \ln(4x-1)$$

2.)
$$5^{x-2} = 625$$

5.)
$$\log_{11}(x) + \log_{11}(x-5) = \log_{11}(x+27)$$

3.)
$$9^x = 27$$

6.)
$$\log(x) + \log(x+1) = \log 12$$

7.)
$$e^{5x} = 12$$

11.)
$$\log_5(2x+3) = 2$$

8.)
$$3 \cdot 7^x = 81$$

12.)
$$\log_2(x) + \log_2(x+2) = 3$$

9.)
$$2^{x+5} = 3^{2x-4}$$

13.)
$$\ln(x) + \ln(2 - x) = 0$$

10.)
$$e^x = 2^{x+1}$$

14.)
$$2\ln(x+3) = \ln 49$$