Homework 1, due Thursday, January 18

Please turn in well-written solutions for the following problems:

(1) Let A, B, C, and D be sets. Suppose that $A \setminus B \subseteq C \cap D$, and suppose that $x \in A$. Prove that if $x \notin D$, then $x \in B$.

(2) Use induction to prove that
$$\forall n \in \mathbb{Z}^+,\ 1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

(3) Use induction to prove that
$$\forall n \in \mathbb{Z}^+,\ 1^3 + 2^3 + 3^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

(4) Let f be a function with domain D, and suppose that $S \subseteq D$. We define $f(S) = \{f(x) \mid x \in S\}$.
   (a) Let A and B be subsets of D. Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$.

   (b) Give an example of a function f and sets A and B in the domain of f such that $f(A) \cap f(B) \not\subseteq f(A \cap B)$.

(5) Suppose that $f : A \to B$ and $g : B \to C$, and let $h = g \circ f$.
   (a) Prove that if f is onto and g is onto, then h is onto.

   (b) Prove that if h is onto, then g is onto.

   (c) Suppose that h is 1-1. Is it true that g is 1-1? Prove or give a counterexample.

   (d) Suppose that h is 1-1. Is it true that f is 1-1? Prove or give a counterexample.

(6) Suppose that A is an infinite set, and let $P(A)$ denote the power set of A. Prove that there is no onto function $f : A \to P(A)$. (Hint: modify the diagonalization argument.)