HOMEWORK 2, DUE WEDNESDAY, SEPTEMBER 11

Please turn in well-written solutions for the following:

(1) Suppose that A is an infinite set, and let $P(A)$ denote the power set of A. Prove that there is no onto function $f : A \to P(A)$. (Hint: modify the diagonalization argument.)

(2) The following page has a list of theorems. Prove the ones asked for below. In each proof, you may use the axioms of the real numbers given in class, and for each theorem on the list, you may use the theorems appearing before it on the list without proof.

   (i) Prove Theorem (5).

   (ii) Prove Theorem (10). (Hint: Assume that $ab = 0$ and that $a \neq 0$. Then it suffices to show that $b = 0$.)

   (iii) Prove Theorem (18).

   (iv) Prove Theorem (20).

   (v) Prove Theorem (25).

   (vi) Prove Theorem (27).

Theorems of Real Numbers

Let a, b, c, and d be real numbers.

(1) If $a+b=a+c$, then $b=c$.
(2) Given a and b, there is a unique element x such that $a+x=b$. We denote this x by $b-a$, and we define $-a$ to be the element $0-a$.
(3) $b-a=b+(-a)$.
(4) $-(-a)=a$.
(5) $a\cdot 0=0$.
(6) If $ab=ac$ and $a\neq 0$, then $b=c$.
(7) Given a and b with $b\neq 0$, there is a unique element x such that $ax=b$. We denote this x by $\dfrac{b}{a}$ or b/a, and we define a^{-1} to be the element $\dfrac{1}{a}$.
(8) If $a\neq 0$, then $\frac{b}{a}=ba^{-1}$.
(9) If $a\neq 0$, then $(a^{-1})^{-1}=a$.
(10) If $ab=0$, then $a=0$ or $b=0$.
(11) $(-a)b=-(ab)$.
(12) If $b\neq 0$ and $d\neq 0$, then $\dfrac{a}{b}+\dfrac{c}{d}=\dfrac{ad+bc}{bd}$.
(13) If $b\neq 0$ and $d\neq 0$, then $\left(\dfrac{a}{b}\right)\left(\dfrac{c}{d}\right)=\dfrac{ac}{bd}$.
(14) If $b\neq 0$, $c\neq 0$ and $d\neq 0$, then $\left(\dfrac{a}{b}\right)/\left(\dfrac{c}{d}\right)=\dfrac{ad}{bc}$.
(15) $-0=0$.
(16) $1^{-1}=1$.
(17) $-(a+b)=-a-b$.
(18) $(a-b)+(b-c)=a-c$.
(19) If $a\neq 0$ and $b\neq 0$, then $(ab)^{-1}=a^{-1}b^{-1}$.
(20) If $a<b$ and $b<c$, then $a<c$.
(21) If $a<b$, then $a+c<b+c$.
(22) If $a<b$ and $c>0$, then $ac<bc$.
(23) If $a\neq 0$, then $a^2>0$.
(24) If $a<b$, then $-b<-a$.
(25) $0<1$.
(26) If $a\leq b$ and $b\leq c$, then $a\leq c$.
(27) If $a\leq b$ and $b\leq a$, then $a=b$.
(28) If $ab<0$ then either $a<0$ and $b>0$ or $b<0$ and $a>0$.
(29) If $a<b$ and $c<d$, then $a+c<b+d$.
(30) If $0<a<b$, then $0<b^{-1}<a^{-1}$.