Homework 1, due Friday, January 29

Please turn in solutions for the following problems:

(1) Let $z = 1 + 2i$, and $w = 2 - i$. Compute each of the following:

(a) $z + 3w$
(b) $w^2 - \overline{z}$
(c) $\dfrac{5z}{2w}$
(d) $z\overline{w}$
(e) $\overline{zw}$
(f) $2z - iw$
(g) $z^2 - 4i\overline{z} + 3 - 2i$

(2) For each of the following complex numbers, give the polar or exponential form, using the principal argument.
(a) $1 + i\sqrt{3}$
(b) $-2 - 2i$
(c) $\left(\dfrac{1+i}{\sqrt{2}}\right)^4$

(3) Write the complex number $2e^{i\pi/4}$ in the form $a + bi$.

(4) Find all solutions of the equation $(z + 1)^4 = 1 - i$.

(5) Sketch the set of points in the complex plan determined by each of the following conditions:
(a) $|z| = 3$
(b) $|z - 2| = |z - i|$
(c) $\mathrm{Re}[(1 - i)\overline{z}] = 0$

(6) Write the equation of the circle of radius 2 centered at $4 + i$.

In addition, I suggest that you work these problems from the Brown/Churchill textbook (but do not turn in):

- Page 5, problems 1, 2, 4
- Page 8, problems 1, 2
- Page 12, problem 5
- Pages 14-15, problems 1, 2, 7
- Pages 22-23, problems 1, 2, 5
- Pages 29-30, problems 2, 6

Optional Proofs (not for credit):

- Prove that $|z| = 1$ if and only if $\overline{z} = 1/z$.
- Prove that $|z + w|^2 - |z - w|^2 = 4\,\mathrm{Re}(z\overline{w})$.
- Suppose that a is a real number. Prove that $|z|^2 + a^2 = |z + a|^2 - 2\,\mathrm{Re}(Az)$.