HOMEWORK 2, DUE WEDNESDAY, FEBRUARY 10

Please turn in solutions for the following problems:

(1) Sketch each set and determine if each set is open, closed, connected, or bounded:

(a) $|2z - 4 + 6i| \leq 4$
(b) $1 < |z| < 2$
(c) $\mathrm{Re}(z) > \mathrm{Im}(z)$
(d) $0 \leq \arg(z) \leq \pi/4$
(e) $(\mathrm{Re}(z))^2 + 1 = (\mathrm{Im}(z))^2$

(2) For each of the following functions, determine the largest possible domain of definition.

(a) $f(z) = \dfrac{1}{z^2+1}$
(b) $f(z) = \dfrac{z}{z - \overline{z}}$
(c) $f(z) = \dfrac{1}{1 - |z|^2}$
(d) $f(z) = \mathrm{Arg}\left(\dfrac{1}{z}\right)$

(3) Consider the function $f(z) = \mathrm{Arg}(z)$, and let S be the horizontal line $\mathrm{Im}(z) = 1$. Sketch the image set $f(S)$.

(4) Let $f(z) = e^z$. Compute each value. (Round to an appropriate number of decimal places if needed).

(a) $f(-1 + i\pi)$
(b) $f(\pi/2)$

(5) Consider the function $f(z) = e^z$, and let S be the vertical line $\mathrm{Re}(z) = 1$. Sketch the image set $f(S)$.

(6) Let $f(z) = \mathrm{Log}(z)$. Compute each value. (Write in terms of π if possible, but otherwise, round to an appropriate number of decimal places if needed).

(a) $f(\sqrt{3} - i)$
(b) $f(-4 + 4i)$

(7) Consider the function $f(z) = \mathrm{Log}(z)$, and let S be the right half-circle of radius 1 centered at 0. That is, $S = \{z \in \mathbb{C} \mid |z| = 1 \text{ and } \mathrm{Re}(z) \geq 0\}$. Sketch the image set $f(S)$.

In addition, I suggest that you work these problems from the Brown/Churchill textbook (but do not turn in):

- Page 33, problems 1, 2, 3
- Page 44, problems 3, 4, 7
- Page 92, problems 1, 8
- Page 97, problems 1, 2, 7