Homework 3, due Tuesday, February 22

Please turn in solutions for the following problems:

(1) Find each limit or explain why it does not exist:

(a) $\lim_{z\to -2i} \frac{z^3-8i}{z+2i}$

(b) $\lim_{z\to 8+i} \frac{1}{1-\text{Im}(z)}$

(c) $\lim_{z\to\infty} \frac{4z^6-7z^3}{(z^2-4)^3}$

(d) $\lim_{z\to\infty} \frac{|z|}{z}$

(2) Use the rules for differentiation to find the derivative of each function.

(a) $f(z) = e^{z^3-z}$

(b) $f(z) = \cos^3(z^2)$

(c) $f(z) = \frac{z+1}{z+i}$, where $z \neq i$

(d) $f(z) = (\text{Log}(z))^3$, where z is not on the negative real axis

(3) Let $g(z) = \overline{z}$. Write in the form $g(x+iy) = u(x+iy)+iv(x+iy)$. Check the Cauchy-Riemann equations to determine if this function is differentiable.

(4) Suppose that f is an entire function such that $f(z) = u(z) + iv(z)$, where $u(x+iy) = 2x^2+2x+1-2y^2$. Determine what v must be.

(5) Recall that we defined $\tan^{-1}(z) = \frac{i}{2}\log\left(\frac{i+z}{i-z}\right)$. Let $f(z) = \tan^{-1}(z)$, where we use the principal branch Log of the logarithm to define this as a single-valued function. Write in the form $f(x+iy) = u(x+iy)+iv(x+iy)$. (That is, find u and v.) Then, check the Cauchy-Riemann equations to determine if this function is differentiable. If so, find the derivative, $f'(z)$.

(Note: This problem is extremely heavy on calculations, so I'll make this problem worth optional bonus points, but it should be rewarding and fun if you make it to the end!)

In addition, I suggest that you work these problems from the Brown/Churchill textbook (but do not turn in):

- Pages 55-56, problems 3, 10
- Page 62, problem 1
- Page 71, problems 2, 4
- Pages 77, problems 1, 2
- Page 81, problem 1