Homework 4, due Monday, March 8

Please turn in solutions for the following problems:

(1) Compute each of the following contour integrals:

(a) $\displaystyle\int_C e^z\,dz$, where C is the line segment from 0 to $1+i$

(b) $\displaystyle\int_C \frac{1}{z+4}\,dz$, where C is the circle of radius 1 centered at -4 traversed counterclockwise

(c) $\displaystyle\int_C z\,dz$, where C is the left semicircle from i to $-i$

(d) $\displaystyle\int_C z\overline{z}\,dz$, where C is the line segment from $-1+i$ to $1+5i$

(e) $\displaystyle\int_C z^3-6z^2+4\,dz$, where C is any curve joining $-1+i$ to 1

(2) Suppose $f(z)$ is the principal branch of z^i. That is, $f(z)=e^{i\,\mathrm{Log}(z)}$, for $-\pi<\mathrm{Arg}(z)<\pi$. Let C be the upper semicircle parametrized by $z(t)=e^{it}$, for $0\le t\le\pi$. Compute the integral $\int_C f(z)\,dz$.

(3) Let C be the positively oriented boundary of the square with corners at $2+2i$, $2-2i$, $-2+2i$, and $-2-2i$. Evaluate each integral:

(a) $\displaystyle\int_C \frac{z}{2z+1}\,dz$

(b) $\displaystyle\int_C \frac{\cos(z)}{z(z^2+8)}\,dz$

(4) Let C be the circle $|z-i|=2$, positively oriented. Evaluate each integral:

(a) $\displaystyle\int_C \frac{1}{z^2+4}\,dz$

(b) $\displaystyle\int_C \frac{1}{(z^2+4)^2}\,dz$

In addition, I suggest that you work these problems from the Brown/Churchill textbook (but do not turn in):

- Page 121, problems 2, 4
- Page 135, problems 1, 2, 4, 6
- Page 149, problems 1, 2
- Pages 160-161, problems 1, 2
- Pages 170-171, problems 1, 3, 4

Optional Proofs (not for credit):

- Let C be a contour with arc length L, and let f be a bounded function that is continuous on a domain containing C. Prove that
$$\left|\int_C f(z)\,dz\right| \leq L \cdot \max_{z\in C} |f(z)|\,.$$
- Show that if C is a positively oriented simple closed contour, then the area A of the region enclosed by C can be written as
$$A = \frac{1}{2i}\int_C \overline{z}\,dz.$$
- Show that if f is analytic inside and on a simple closed contour C, and z_0 is not on C, then
$$\int_C \frac{f'(z)}{z-z_0}\,dz = \int_C \frac{f(z)}{(z-z_0)^2}\,dz.$$