Please turn in solutions for the following problems:

(1) Find each limit or explain why it does not exist:

(a) $\displaystyle\lim_{z \to -2i} \frac{z^3 - 8i}{z + 2i}$

(c) $\displaystyle\lim_{z \to \infty} \frac{4z^6 - 7z^3}{(z^2 - 4)^3}$

(b) $\displaystyle\lim_{z \to 8+i} \frac{1}{1 - \text{Im}(z)}$

(d) $\displaystyle\lim_{z \to \infty} \frac{|z|}{z}$

(2) Use the rules for differentiation to find the derivative of each function.

(a) $f(z) = e^{z^3 - z}$

(b) $f(z) = \cos^3(z^2)$

(c) $f(z) = \dfrac{z + 1}{z + i}$, where $z \neq i$

(d) $f(z) = (\text{Log}(z))^3$, where z is not on the negative real axis

(3) Let $g(z) = \bar{z}$. Write in the form $g(x+iy) = u(x+iy)+iv(x+iy)$. Check the Cauchy-Riemann equations to determine if this function is differentiable.

(4) Suppose that f is an entire function such that $f(z) = u(z) + iv(z)$, where $u(x + iy) = 2x^2 + 2x + 1 - 2y^2$. Determine what v must be.

(5) Let w be a function from $\mathbb{R}$ to $\mathbb{C}$ defined by $w(t) = t^2 - e^{it}$.

(a) Compute the derivative $w'(t)$.

(b) Compute the integral $\displaystyle\int_0^3 w(t)\, dt$.

In addition, I suggest that you work these problems from the Brown/Churchill textbook (but do not turn in):

- Pages 55-56, problems 3, 10
- Page 62, problem 1
- Page 71, problems 2, 4
- Pages 77, problems 1, 2
- Page 81, problem 1
- Page 104, problem 9
- Page 121, problems 2, 4