HOMEWORK 5, DUE FRIDAY, MARCH 24

Please turn in solutions for the following problems:

(1) Compute each of the following contour integrals:

    (a) $\int_C e^z \, dz$, where C is the line segment from 0 to $1 + i$

    (b) $\int_C \frac{1}{z+4} \, dz$, where C is the circle of radius 1 centered at -4 traversed counterclockwise

    (c) $\int_C z \, dz$, where C is the left semicircle from i to $-i$

    (d) $\int_C z\bar{z} \, dz$, where C is the line segment from $-1 + i$ to $1 + 5i$

    (e) $\int_C z^3 - 6z^2 + 4 \, dz$, where C is any curve joining $-1 + i$ to 1

(2) Suppose $f(z)$ is the principal branch of z^i. That is, $f(z) = e^{i \, \text{Log}(z)}$, for $-\pi < \text{Arg}(z) < \pi$. Let C be the upper semicircle parametrized by $z(t) = e^{it}$, for $0 \leq t \leq \pi$. Compute the integral $\int_C f(z) \, dz$.

(3) Let C be the positively oriented boundary of the square with corners at $2 + 2i$, $2 - 2i$, $-2 + 2i$, and $-2 - 2i$. Evaluate each integral:

    (a) $\int_C \frac{z}{2z + 1} \, dz$

    (b) $\int_C \frac{\cos(z)}{z(z^2 + 8)} \, dz$

(4) Let C be the circle $|z - i| = 2$, positively oriented. Evaluate each integral:

    (a) $\int_C \frac{1}{z^2 + 4} \, dz$

    (b) $\int_C \frac{1}{(z^2 + 4)^2} \, dz$

In addition, I suggest that you work these problems from the Brown/Churchill textbook (but do not turn in):

- Page 121, problems 2, 4
- Page 135, problems 1, 2, 4, 6
- Page 149, problems 1, 2
- Pages 160-161, problems 1, 2
- Pages 170-171, problems 1, 3, 4