Math 456 - Complex Variables, Spring 2023
Course Syllabus

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Office: Learning Resource Center 323K
Office hours: Mon: 11-12, Tues: 9-10 & 6pm-7pm (online), Wed: 11-12, Thurs: 2-3, Fri: 11-12 The Tuesday evening online office hour will be accessible through Google Meet with the meeting code TechMathLeary. Other office hours will be in person, and I may be available by appointment.

Class Room/Time: INN-B 304, MWF 9:00-9:50 am
Course website: community.wvu.edu/∼bal0018/math456S23.html (QR-code below)

Homework assignments and course announcements will be posted on the website. Please be sure to check the website regularly, and to regularly check the email address you have on record. You are responsible for any information posted on the course website.

Textbook: There is no required textbook for the class. An optional supplementary textbook that you may wish to use is *Complex Variables and Applications*, 8th edition, by Brown and Churchill. I can also give other recommendations for complex analysis books upon request, both at the level of this course and more advanced.

Catalog Data: MATH 456. Complex Variables. (3-0) Credits 3. Complex numbers, functions of a complex variable; analytic functions; the logarithm and related functions; power series; Laurent series and residues; conformal mapping and applications.
Prerequisite: MATH 251

Course Objective: Upon completion of the course the student should have a sufficient understanding of the techniques of complex analysis to apply them to problems in engineering and the physical sciences.

Learning Outcomes: Upon successful completion of the course, the student will be able to:

1. Represent complex numbers algebraically and geometrically.
2. Define and analyze limits and continuity of complex functions.
3. Define and find the derivative of a complex function.
4. Apply the concept and consequences of analyticity and the Cauchy-Riemann equations.
5. Analyze sequences and series of complex functions and types of convergence.
6. Evaluate complex contour integrals directly and by the fundamental theorem, apply the Cauchy-Goursat theorem and the Cauchy integral formula.
7. Represent functions as Laurent series, classify singularities and poles, find residues, and evaluate complex integrals using the residue theorem.

Course material: After spending the entire calculus sequence studying functions of real variables, we now investigate what happens if we instead look at functions of complex variables. This course will start with some simple arithmetic and algebra with this new system of numbers, before investigating how differential and integral calculus work in this world, which will yield some surprising and beautiful properties.

Topics:

1. Complex Numbers (Chapter 1) - 4 days
   (a) Algebra in $\mathbb{C}$
   (b) Geometry in the complex plane
2. Elementary Functions (Chapter 3) - 2 days
   (a) Exponential function
   (b) Branches of Logarithmic Functions
   (c) Trigonometric Functions
   (d) Hyperbolic Functions
3. Analytic Functions (Chapter 2) - 4 days
   (a) Limits and Continuity
   (b) Differentiation
   (c) Cauchy-Riemann Equations
   (d) Analytic Functions
4. Integrals (Chapter 4) - 4 days
   (a) Line integrals
   (b) Cauchy-Goursat Theorem
   (c) Cauchy Integral Formula
   (d) Morera's Theorem
   (e) Liouville's Theorem
   (f) Fundamental Theorem of Algebra
5. Series (Chapter 5) - 4 days
   (a) Taylor series
   (b) Laurent series
   (c) Convergence of series
   (d) Integration and differentiation of series
6. Residues and Poles (Chapters 6 and 7) - 4 days
   (a) Residue Theorem
   (b) Computing residues at poles
   (c) Evaluation of improper real integrals
   (d) Integrals involving sine and cosine
7. Mapping by Elementary Functions (Chapter 8) - 3 days
   (a) Linear functions
   (b) Linear fractional transformations
   (c) Complex Exponential Function and other Special Functions
8. Conformal Mapping (if time permits) (Chapter 9)

Grading: Your final grade will be based on homework, three exams during the semester, and a final exam. Your final course score will be computed via the following grading scheme:

- 20% Homework + 20% Exam 1 + 20% Exam 2 + 20% Exam 3 + 20% Final Exam

Letter Grade Cutoffs: A: 90%, B: 80%, C: 70%, D: 60%, F: below 60%

Homework: Homework problems will be assigned weekly. I will write out homework assignments and post them on the course website, though additional optional problems from the supplementary textbook may be suggested. The homework is the most important part of this class. To be more precise, not knowing how to do the homework is the most important part of this class. At this level, mathematics is learned by trying to solve problems and failing over and over again. Therefore, you should be prepared to spend many hours each week in frustration, finding flaws in your arguments, or simply stuck not knowing what to do next! To that end, you are expected to write out your own solutions for the homework problems. Any verbatim copying of solutions, either from another student or from a textbook or online resource, is prohibited and is considered plagiarism.

Exams: There will be three exams during the semester, tentatively scheduled for Friday, February 3; Wednesday, March 1; and Wednesday, April 5. These will be 50 minute exams taken during the regular lecture time. The final exam time has been set by the university, and will be on Wednesday, May 3 from 8:00-9:50 am. Make-up exams will only be given to students with excused absences, and such make-up exams must be scheduled within 24 hours of the missed exam.

Getting Help: Always remember: asking for help when you need it is not a sign of weakness, but a sign of strength! Please feel free to virtually attend my office hours or email me if you have questions about the course material. If you are unable to make it to my regularly scheduled office hours, I am willing to make an appointment to meet at another time if possible. Additionally, you can get help in the Math Tutoring Lab in LRC 323 from 8 AM to 4:30 PM. Free tutoring is also available through Student Support Services, located in Benedum 130, and the Student Success Center, located in the library on the second floor of LRC. Finally, I would also encourage the formation of study groups, to learn from each other and help each other learn.

Class policies:

- Graphing or programmable calculators will never be allowed during any exams. Scientific calculators will be considered on an exam by exam basis. You may use any calculator to help you do the homework if you wish, but you should keep in mind that you may be required to solve similar problems without a calculator on the quizzes and exams.
- While class attendance does not directly factor into your grade computation, attendance of each lecture is highly recommended. Regular attendance will tend to lead to better understanding of the course material, which tends to lead to better performance on exams.
- If you believe a problem on a homework assignment or midterm exam has been graded incorrectly, you must notify the instructor of your complaint within 7 days of the date the exam is handed back. If you are unable to retrieve your graded material at the time it is handed back, it is your responsibility to make arrangements with the instructor to retrieve the material at another time.

Institutional Policies: Students are responsible for reviewing policies on inclusivity, academic integrity, incompletes, sale of course materials, sexual misconduct, adverse weather, as well as student evaluation of instruction, days of special concern/religious holiday statements, and the updated COVID-19 statement. For these detailed policies of West Virginia University, please review: https://tlcommons.wvu.edu/syllabus-policies-and-statements.