Name:
You are allowed one sheet of notes.

1. A child weighing 30 kg jumps laterally (sideways) onto a trampoline from a platform that is at the same height as the trampoline. If the effective stiffness of the trampoline is $1 \mathrm{kN} / \mathrm{m}$ in the vertical direction, compute the amplitude of the child's vibration.

- Assume that the vertical velocity of the child is zero when it contacts the trampoline. Be aware that the equilibrium position of the trampoline changes when the weight of the child is added.
- Use the equation $A=\operatorname{sqrt}\left(w_{n}^{2} x_{0}^{2}+v_{0}^{2}\right) / w_{n}$ to calculate $A$, even though some insight into the problem would allow you to avoid using this equation.


2. A horizontal spring-mass system has a mass of 200 g and spring stiffness of $4.44 \mathrm{~N} / \mathrm{m}$. If the mass is given an initial velocity of $0.1 \mathrm{~m} / \mathrm{s}$ to the right from a starting point 0.05 m
 to the left of the equilibrium point, give:

- the equation of motion of the mass $(x(t))$
- the position of the mass at $t=0.1 \mathrm{~s}, t=0.5 \mathrm{~s}$ and $t=1 \mathrm{~s}$
- a graph of $x(t)$ from $t=0 \mathrm{~s}$ to $t=1.5 \mathrm{~s}$, clearly showing the amplitude (as a numeric value), correct period, and initial conditions.

3. Buildings must be designed to withstand lateral vibrations initiated from wind gusts, earthquakes and roof-mounted air handling equipment. The figure shows how the vibrations for buildings are modeled. The floors are simply masses and the support columns act as cantilever beams to provide lateral stiffness.

(a)

(b)

(c)

(d)

Figure P3. Modeling the lateral vibrations of a building
A one story military building has a roof mass of 10000 kg and support columns that have a combined lateral stiffness equivalent to 98700 $\mathrm{N} / \mathrm{m}$. From experience, the maximum amplitude of acceleration of the roof is $0.4935 \mathrm{~m} / \mathrm{s}^{2}$, which occurs whenever the large roof-mounted air conditioning unit starts up.

The military wants to install lasers on the roof for a security system. What is the maximum movement the lasers will experience, if their mass is negligible compared to the mass of the roof?
4. The front of a Porsche Carrera GT3 weighs 680 kg . The combined stiffness of the front springs is $70000 \mathrm{~N} / \mathrm{m}$. How stiff should each of the two front shocks be to make the suspension critically damped?

If the vehicle hits a bump setting up the initial conditions of $x_{0}=30 \mathrm{~mm}$ and $v_{0}=0$, what is the displacement after $0.5,1.0$ and 2.0 seconds?
5. A restaurant's swinging kitchen door, which swings both out of and into the kitchen, has a closer with a spring ( $\mathrm{k}=71 \mathrm{~N}-\mathrm{m} / \mathrm{rad}$ ) and damper ( $\mathrm{c}=7.5 \mathrm{~N}-\mathrm{m}-\mathrm{s} / \mathrm{rad}$ ). The mass moment of inertia of the door with respect to the hinge is $20 \mathrm{~kg}-\mathrm{m}^{2}$.

If the door is held open and then let go from an angle of $50^{\circ}$, how long will the door continue to swing? How many times will it swing before it is closed? Assume that the door is considered closed when the amplitude has decreased to $2.5^{\circ}$. Use the small angle approximation that $\sin (\theta) \approx \theta$.


Image from tuscon.olx.com
6. A large half-pipe is being designed for a new skateboard park. Derive the system equation and natural frequency for the motion of skateboarders using the half-pipe.

- Model only the side to side motion as shown in the bottom picture.
- Assume that the half-pipe cross-section is circular.
- Assume the center of gravity of the skateboarder stays directly above the skateboard.
- Assume that energy loss due to friction or damping is negligible.
- Assume that $I_{p}$ for the skateboard wheels is negligible.
- Only consider motions close to the bottom of the half-pipe (i.e., assume that $\sin \theta \approx \theta$ )


Will there be a difference between bigger (heavier) skateboarders and smaller (lighter) skateboarders in terms of how quickly they cycle back and forth?

7. A good spiderman and an evil spiderman roam the streets of New York. Plot the motion for both the evil spiderman $\left(\theta_{1}(t)\right)$ and the good spiderman $\left(\theta_{2}(t)\right)$ if both spidermen swing from webs of equal length attached to the same point on scaffolding at the top of the middle building and jump at the same time. Note that:

- The height of the scaffolding on the middle building is 200 m above the ground.
- The evil spiderman weighs 68 kg .

- The good spiderman weighs 77 kg .
- The initial height of the evil spiderman is 103.58 m .
- The initial height of the good spiderman is 125 m .
- The lengths of the webs are 150 m .
- Use the small angle approximation in your calculations and assume no damping: $\theta+\frac{g}{2} \theta=0$
- Plot the approximate motion $\left(\theta_{1}(t)\right.$ and $\left.\theta_{2}(t)\right)$ for one period, clearly showing:
- the amplitudes (as numeric values),
- the initial conditions, and
- the time at the middle and end of the first period.
- Also answer the additional questions:
- What are their exact positions $\left(\theta_{1}\right.$ and $\left.\theta_{2}\right)$ after 2.0 s ?
- Who will reach the Bank first?
- When will the good spiderman catch up to the evil spiderman? $(t=$ ? $)$
- Where will the good spiderman catch up to the evil spiderman? $\left(\theta_{1}=\theta_{2}=\right.$ ?)

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Question 6 cont'd.
8. The amplitude of a vibration decreases to one fifth of the original amplitude over five cycles. What is the damping ratio?

