Other Forms of Damping
Section 2.7
Forced Vibration with Coulomb Damping

• A single degree-of-freedom system with mass 100 g, spring stiffness of 10 N/m and a Coulomb damping coefficient of 0.05 is excited by a harmonic force of 0.5 N amplitude at 1 Hz.

• Approximate the displacement amplitude for the vibration of the mass.
### Equations of Motion for Vibration Measurement Devices

#### Non-Conservative Forces

<table>
<thead>
<tr>
<th>Type of Force</th>
<th>Source</th>
<th>Amount of Force</th>
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</thead>
<tbody>
<tr>
<td>Linear viscous damping</td>
<td>Slow fluid</td>
<td></td>
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<tr>
<td>Air damping</td>
<td>Fast fluid</td>
<td></td>
</tr>
<tr>
<td>Coulomb damping</td>
<td>Sliding friction</td>
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<tr>
<td>Solid/structural/hysteretic damping</td>
<td>Internal damping</td>
<td></td>
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</tbody>
</table>
Coulomb Damping

\[ m\ddot{x} + \mu N \, \text{sgn}(\dot{x}) + kx = F_0 \sin \omega t \]

- Too hard to solve with harmonic excitation term added!
- How about using an approximation? Use a value of \( c_{eq} \) in

\[ m\ddot{x} + c_{eq} \dot{x} + kx = F_0 \sin \omega t \]

that dissipates the same amount of energy per cycle.
- Considering only the steady-state response, if \( F_0 >> \mu N \),

\[ c_{eq} = \frac{4\mu N}{\pi \omega X} \quad \text{or} \quad \zeta_{eq} = \frac{2\mu N}{\pi n \omega_n \omega X} \]
Coulomb Damping

- Substituting into solution:

\[ X = \frac{F_0}{k} \sqrt{\left(1 - r^2\right)^2 + \left(\frac{4\mu N}{\pi kX}\right)^2} \]

- Solving for \( X \):

\[ X = \frac{F_0}{k} \sqrt{1 - \left(\frac{4\mu N}{\pi F_0}\right)^2} \]

\[ \theta = \tan^{-1} \left( \frac{4\mu N}{\pi kX (1 - r^2)} \right) = \tan^{-1} \frac{2\xi_{eq} r}{1 - r^2} \]