Spur Gear Design
Idealized Spur Gears

The speed ratio is given by:

$$\frac{\omega_3}{\omega_2} = \frac{R_2}{R_3}$$
Tooth pitch

- However, in order for the gears to mesh, they must have the same tooth pitch
Tooth pitch

- “Circular pitch” is arc-length per tooth:

\[ p = \]

- In metric use “module”:

\[ m = \]

- With inch units use “diametral pitch”:

\[ P = \]
Standard (AGMA) Gears

- Gears come with standard pitches and pressure angles

### TABLE 7.1 Standard Tooth Sizes

<table>
<thead>
<tr>
<th>Standard Diametral Pitches</th>
<th>U.S. Customary, teeth/in</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coarse</td>
<td>1, 1.125, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16</td>
</tr>
<tr>
<td>Fine</td>
<td>20, 24, 32, 40, 48, 64, 80, 96, 120, 150, 200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Standard Modules</th>
<th>SI, mm/tooth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Preferred</td>
<td>1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40, 50</td>
</tr>
<tr>
<td>Next choice</td>
<td>1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36, 45</td>
</tr>
</tbody>
</table>

### TABLE 7.2 Standard Tooth Systems for Spur Gears

<table>
<thead>
<tr>
<th>System</th>
<th>Pressure Angle, $\phi$ (deg)</th>
<th>Addendum, $a$</th>
<th>Dedendum, $d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full depth</td>
<td>20°</td>
<td>$1/P$ or $1m$</td>
<td>$1.25/P$ or $1.25m$</td>
</tr>
<tr>
<td>Full depth</td>
<td>22.5°</td>
<td>$1/P$ or $1m$</td>
<td>$1.25/P$ or $1.25m$</td>
</tr>
<tr>
<td>Full depth</td>
<td>25°</td>
<td>$1/P$ or $1m$</td>
<td>$1.25/P$ or $1.25m$</td>
</tr>
<tr>
<td>Stub teeth</td>
<td>20°</td>
<td>$0.8/P$ or $0.8m$</td>
<td>$1/P$ or $1m$</td>
</tr>
</tbody>
</table>
Pressure Angle

- As meshed gears rotate, the teeth slide against each other and the contact point changes continuously, but the contact point always lies on the pressure line.
- The direction of the force is always in the direction of the pressure line (i.e., normal to surface at contact point).
The Involute Gear Tooth Profile

• In order to maintain contact on the line of action with the force always at the same pressure angle, an involute profile must be used. It is generated as shown.
Addendum and Dedendum

- **Addendum** ($a$) is the distance from the pitch circle to the top of the teeth.
- **Dedendum** ($d$) is the distance to the bottom.
Finished Spur Gears

Clearance: $c = d - a$
Tooth thickness at pitch circle: $t = p / 2$
Racks and Internal Gears

- Rack

- Internal Gear
Interference and Undercutting

- If the teeth are too large for the pitch diameter, there will be interference on the flank of the driving teeth during approach.
- There should only be contact between points A and B (where the base circle meets the pressure line). Points C and D (where the addendum circles meet the pressure line) should be between A and B.
- If teeth are too large, the gear designer must:
  - provide undercutting – leave space at base of teeth → very weak teeth
  - increase number of teeth → smaller teeth are weaker
  - increase pitch circle radius → bigger, faster, noisier gears
  - increase pressure angle → more friction
  - make gear teeth stubbier → non-standard gears
Interference and Undercutting

To avoid interference:

\[ CP \leq AP \]
\[ PD \leq PB \]

\[ AP = R_2 \sin \Phi \]
\[ PB = R_3 \sin \Phi \]

\[ CP = \sqrt{(R_3 + a)^2 - (R_3 \cos \phi)^2} - R_3 \sin \phi \]
\[ PD = \sqrt{(R_2 + a)^2 - (R_2 \cos \phi)^2} - R_2 \sin \phi \]
Contact Ratio

- Average number of teeth in contact at once.

\[ CD = CP + PD \]
\[ p_b = p \cos \Phi \]
Length & Angle of Approach & Recess

Angle of approach: \( \alpha_2 = \frac{CP}{r_2} \)

Angle of recess: \( \beta_2 = \frac{PD}{r_2} \)

Length of path of:
- approach: \( CP \)
- recess: \( PD \)
Increasing the center distance