Prob 6.2

Given:
- plate cam
- reciprocating flat face follower
- displacement diagram: 1. Rise $L_1 = 2\text{ in}$
  $\beta_1 = 180^\circ$
  Simple harmonic motion
  2. Fall $L_2 = 2\text{ in}$
  $\beta_2 = 180^\circ$
  Simple harmonic motion
- Prime circle radius $R_o = 2\text{ in}$
- ccw rotation
- follower stem offset $e = 0.75\text{ in}$ (in direction that reduces bending stress during rise.)

Find:
1) Displacement Diagram
2) Cam profile
3) Required Follower Face width
4) Will there be undercuts

Sol'n: 1) Displacement Diagram

\[ Y = \frac{L_1}{2} \left( 1 - \cos \frac{\pi \theta}{\beta_1} \right) \]
\[ = \frac{2}{2} \left( 1 - \cos \frac{\pi \theta}{\pi} \right) \]
\[ = 1 - \cos \theta \]

2. For $\pi \leq \theta \leq 2\pi$

Full return simple harmonic motion use Eq. 6.15a
\[ Y = \frac{L_2}{2} \left( 1 + \cos \frac{\pi (\theta - \beta_2)}{\beta_2} \right) \]
\[ = \frac{2}{2} \left( 1 + \cos \frac{\pi (\theta - \pi)}{\pi} \right) \]
\[ = 1 + \cos (\theta - \pi) \]

\[ Y = L_2 = 2 \text{ in} \]
\[ \beta_2 = \pi \text{ rad} \]
2) Cam profile

<table>
<thead>
<tr>
<th>Θ</th>
<th>0</th>
<th>30</th>
<th>60</th>
<th>90</th>
<th>120</th>
<th>150</th>
<th>180</th>
<th>210</th>
<th>240</th>
<th>270</th>
<th>300</th>
<th>330</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>0</td>
<td>0.134</td>
<td>0.5</td>
<td>1.0</td>
<td>1.5</td>
<td>1.867</td>
<td>2.0</td>
<td>1.867</td>
<td>1.5</td>
<td>1.0</td>
<td>0.5</td>
<td>0.134</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Scale 1:2

max to left

max to right

3) Required Follower Face Width

- How far does the face need to extend in each direction?
- Measure on profile diagram or use Eq. 6.29:

\[
\text{Face width} \geq y'_{\text{max}} - y'_{\text{min}}
\]

\[
\geq 1 - (-1)
\]

Face width \(> 2\)

- Must extend linch to left and linch to right of center.

\[y'_{\text{max}} = 1 \text{ at } \theta = 90^\circ\]

\[y'_{\text{min}} = -1 \text{ at } \theta = 270^\circ\]

4) Will there be undercuts?

- Examine profile figure or use Eq. 6.28.
- In figure, undercuts would look like

\[y'_{\text{honey for profile}} \text{ or } y'_{\text{honey for profile}}\]

To get to here
- using Eq. 6.28

\[ R_o > \rho_{\min} - y - y'_{\min} \]

- use \( \rho_{\min} = 0 \) when checking for undercutting but it should really be higher than zero to avoid high contact stresses.

\[ \rho_{\min} > 0 \quad \rho_{\min} = 0 \quad \rho_{\min} < 0 \]

\[ y = 1 - \cos \theta \]
\[ y' = \sin \theta \]
\[ y'' = \cos \theta \leq y''_{\min} = -1 \quad (\text{when } \theta = \pi (180^\circ)) \]
\[ y = 1 - \cos 180^\circ = 1 - (-1) = 2 \]

\[ R_o > 0 - 2 - (-1) \]
\[ R_o > -1 \quad \text{to not have undercutting. Since } R_o = 2, \text{ undercutting will not occur.} \]
Problem 6.4

Given:
- plate cam - cw rotation
- oscillating roller follower
- y motion 1. Full rise - cycloidal
   \[ \beta_1 = 150^\circ, \quad L_1 = 30^\circ \]
   \[ \frac{5}{6}\pi \text{ rad} \]
2. Dwell
   \[ \beta_2 = 30^\circ \]
   \[ \frac{\pi}{6} \text{ rad} \]
3. Full return - cycloidal
   \[ \beta_3 = 120^\circ, \quad L_3 = 30^\circ \]
   \[ \frac{2}{3}\pi \text{ rad} \]
4. Dwell
   \[ \beta_4 = 60^\circ \]
   \[ \frac{\pi}{3} \text{ rad} \]
- prime circle radius \( R_0 = 60 \text{ mm} \)
- roller radius \( R_r = 10 \text{ mm} \)
- follower length \( 100 \text{ mm} \)
- follower pivot point \( 125 \text{ mm} \) to left of cam rotation axis

Find:
- \( \phi_{max} \) - maximum pressure angle
- will there be undercutting?

Sol'n:
First construct displacement diagram
For $0 \leq \theta \leq 150^\circ$

have full-rise cycloidal motion

$$y = L_1 \left( \frac{\theta_1}{\beta_1} - \frac{1}{2\pi} \sin \frac{2\pi}{3} \frac{\theta_1}{\beta_1} \right)$$  \hspace{1cm} (Eq. 6.13a)

Be careful!

Use radians for $\theta$ & $\beta$ in these equations!

$$y = 30 \left( \frac{\theta_1}{\beta_1} - \frac{1}{2\pi} \sin \frac{2\pi}{3} \frac{\theta_1}{\beta_1} \right)$$

For $150^\circ \leq \theta \leq 180^\circ$

$$y = 30^\circ$$

For $180^\circ \leq \theta \leq 300^\circ$

have full-return cycloidal motion

$$y = L_3 \left( 1 - \frac{\theta_3}{\beta_3} + \frac{1}{2\pi} \sin \frac{2\pi}{3} \frac{\theta_3}{\beta_3} \right)$$  \hspace{1cm} (Eq. 6.16a)

$$= 30 \left( 1 - \frac{\theta_3}{\beta_3} + \frac{1}{2\pi} \sin \frac{2\pi}{3} \frac{\theta_3}{\beta_3} \right)$$

For $300^\circ \leq \theta \leq 360^\circ$

$$y = 0^\circ$$

Scale 1:3

If this was a radial reciprocating follower, we could use the nomograph (Fig. 6.28) to find the $\phi_{max}$: assume $L = 30$ mm

$$\frac{R_i}{L} = \frac{60}{30} = 2$$

$\beta_1 = 150^\circ \hspace{1cm} \beta_3 = 120^\circ$

from nomograph $\phi_{max} = 22^\circ$
If this were the same radial reciprocating follower.

From Fig. 6.31(a) - for $\beta_1 = 150^\circ$ - full rise cycloidal

$$\frac{P_{\text{min}} + R_n}{R_o} = 1$$

$$\frac{P_{\text{min}} + 10}{60} = 1$$  \hspace{1cm} P_{\text{min}} = 50 \text{ mm} \leq \text{very generous}$$

From Fig. 6.31(b) - for $\beta_3 = 120^\circ$ - full return cycloidal

$$\frac{P_{\text{min}} + 10}{60} = 0.9$$  \hspace{1cm} P_{\text{min}} = 4.4 \text{ mm}$$

$$P_{\text{min}} > 0 \text{ - no undercuts on full return motion.}$$

Homework 6.1 & 6.3
Example 6.2

**Given:** Plate cam, reciprocating follower

\[ \omega = 150 \text{ rpm} = \frac{150 \frac{\text{rev}}{\text{min}} \times 2\pi \text{ rad}}{60 \frac{\text{sec}}{\text{min}}} = 5\pi \text{ rad/s} \]

**Motion:**
1. Half rise from dwell to constant velocity
2. Constant velocity rise \( \dot{y} = 25 \text{ in/s} \)
   \( \gamma' = \frac{\dot{y}}{\omega} = \frac{25}{5\pi} = \frac{5}{\pi} \text{ in.} \)
   for \( L_2 = 1.25 \text{ in.} \)

\[ \frac{L_2}{\beta_2} = \gamma' \quad \beta_2 = \frac{L_2}{\gamma'} = \frac{1.25}{(5/\pi)} = \frac{\pi}{4} \text{ rad} = 45^\circ \]

3. Half rise to top
4. Full return
5. Dwell for 0.10 sec.

\[ \frac{\beta_5}{t_5} = \omega \quad \beta_5 = \omega t_5 = 5\pi \times 0.1 = \frac{\pi}{2} \text{ rad} = 90^\circ \]

**Total lift** \( L = 3.0 \text{ in.} \)

**Find:** Displacement diagram full specification: \( L, L_1, L_5, \beta_1, \beta_5 \)
- type of eqn. for each motion.

**Sol'n:** Plot required motions as solid lines (next page)
Plot anticipated transition motions as dashed lines.
Identify knowns & unknowns

<table>
<thead>
<tr>
<th>Motion</th>
<th>( \beta )</th>
<th>( L )</th>
<th>Type of Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{\pi}{2} ) ( (45^\circ) )</td>
<td>1.25</td>
<td>Constant velocity rise</td>
</tr>
<tr>
<td>3</td>
<td>?</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>4</td>
<td>?</td>
<td>3.0</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{\pi}{2} ) ( (90^\circ) )</td>
<td>0</td>
<td>Dwell ( (y = 0) )</td>
</tr>
</tbody>
</table>

**Note that** \( L_1 + 1.25 + L_3 = 3.0 \) \( \text{ (1) } \)
\[ L_1 + L_3 = 1.75 \]

**Note that** \( \beta_1 + \frac{\pi}{4} + \beta_3 + \frac{\pi}{2} = 2\pi \) \( \beta_1 + \beta_3 + \beta_4 = \frac{5\pi}{4} \) \( \text{ (2) } \)
For $0 \leq \theta \leq \beta$
- Need to choose a type of half-rise motion.

<table>
<thead>
<tr>
<th></th>
<th>Desired</th>
<th>Harmonic (Fig. 6.2a)</th>
<th>Cycloidal (Fig. 6.22a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_A$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$y_A'$</td>
<td>0</td>
<td>$\geq 0$</td>
<td>0</td>
</tr>
<tr>
<td>$y_A''$</td>
<td>0</td>
<td>0</td>
<td>$\geq 0$</td>
</tr>
<tr>
<td>$y_B$</td>
<td>$L_1$</td>
<td>$L$</td>
<td>$L$</td>
</tr>
<tr>
<td>$y_B'$</td>
<td>$\frac{5}{11}$</td>
<td>$&gt; 0$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$y_B''$</td>
<td>0</td>
<td>0</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_B'''$</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- Acceleration requirement is more important than jerk requirement, therefore choose Cycloidal.
- Actual motion drawn in solid line above.
For $\beta_1 + \beta_2 \leq \Theta \leq \beta_1 + \beta_2 + \beta_3$ or $0 \leq \Theta \leq \beta_3$

- Need to choose a type of half-rise motion

<table>
<thead>
<tr>
<th>Desired</th>
<th>Harmonic (Fig. 6.20b)</th>
<th>Cycloidal (Fig. 6.22b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{3c}$</td>
<td>$L_1 + L_2$</td>
<td>$0$ - but can add $L_1 + L_2$</td>
</tr>
<tr>
<td>$y_{3c}$</td>
<td>$5/\Pi$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$y_3''c$</td>
<td>$0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_3'''c$</td>
<td>$0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_3d$</td>
<td>$L_1 + L_2 + L_3$</td>
<td>$&gt; 0$</td>
</tr>
<tr>
<td>$y_3'd$</td>
<td>$0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_3''d$</td>
<td>$0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_3'''d$</td>
<td>Open Choice</td>
<td>$0$</td>
</tr>
</tbody>
</table>

For $0 \leq \Theta \leq \beta_4$

- Need to choose a type of full return motion

<table>
<thead>
<tr>
<th>Desired</th>
<th>Harmonic (Fig. 6.17)</th>
<th>Cycloidal (Fig. 6.18)</th>
<th>8th Order Poly (Fig. 6.19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_4d$</td>
<td>$L_4$</td>
<td>$L$</td>
<td>$L$</td>
</tr>
<tr>
<td>$y_4'd$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$y_4''d$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_4'''d$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_4'd$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$y_4''d$</td>
<td>$0$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_4'''d$</td>
<td>$0$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
<tr>
<td>$y_4''d$</td>
<td>$0$</td>
<td>$&gt; 0$</td>
<td>$&lt; 0$</td>
</tr>
</tbody>
</table>

- Meeting the acceleration goals is more important than jerk, therefore choose [8th Order Polynomial]

Also note that, at $B_1$, $y_{18} = \frac{5}{\Pi}$ to match constant velocity rise motion

$$y_{18} = \frac{L_1}{\beta_1} \left(1 - \cos \frac{\Pi \Theta}{\beta_1}\right) = \frac{L_1}{\beta_1} \left(1 - \cos \frac{\Pi (1-0)}{\beta_1}\right) = \frac{L_1}{\beta_1} = \frac{5}{\Pi}$$ (3)

Similarly, at $C$, $y_{3c} = \frac{5}{\Pi}$ to match end of constant velocity rise motion

$$y_{3c} = \frac{\Pi L_3}{2 \beta_3} \cos \frac{\Pi \Theta_3}{2 \beta_3} = \frac{\Pi L_3}{2 \beta_3} \cos \frac{\Pi (0)}{2 \beta_3} = \frac{\Pi L_3}{2 \beta_3} = \frac{5}{\Pi}$$ (4)
Also note that at D:

\[ E_{0,7}\frac{15.8049}{\beta_4^2} = -\frac{\pi^2 L_3}{4\beta_3^2} \]

\[ E_{0,7}\frac{15.8049}{\beta_4^2} = -\frac{\pi^2 L_3}{4\beta_3^2} \]

\[ \therefore \frac{15.8049}{\beta_4^2} = +\frac{\pi^2 L_3}{4\beta_3^2} \] (5)

We have 5 equations to solve for 5 unknowns.

1. Separate \( L_3 \) in Eq. 4: \( L_3 = \frac{10}{\pi^2} \beta_3 \)

2. Substitute into Eq. 5: \( \frac{15.8049}{\beta_4^2} = \frac{\pi^2}{4} \left( \frac{10}{\pi^2} \beta_3^2 \right) \)

\[ \beta_3 = \frac{10}{4 \times 15.8049} \beta_4^2 = 0.158179 \beta_4^2 \] (6)

3. Substitute \( L_3 \) from Eq. 3 \& \( L_3 \) from Eq. 4 into Eq. 1

\[ \frac{5\beta_1}{2\pi} + \frac{10\beta_3}{\pi^2} = 1.75 \]

\[ \beta_1 = \left( 1.75 - \frac{10}{\pi^2} \beta_3 \right) \left( \frac{2\pi}{5} \right) = 0.7\pi - \frac{4}{\pi} \beta_3 \] (7)

4. Substitute \( \beta_3 \) from (6) into (7)

\[ \beta_1 = 0.7\pi - \frac{4}{\pi} \left( 0.158179 \beta_4^2 \right) = 0.7\pi - 0.20140 \beta_4^2 \] (8)

5. Substitute \( \beta_1 \) from (8) and \( \beta_3 \) from (6) into (2)

\[ 0.7\pi - 0.20140 \beta_4^2 + 0.158179 \beta_4^2 + \beta_4 = \frac{5\pi}{4} \]

\[ 0.043221 \beta_4^2 - \beta_4 + 1.72786 = 0 \]

\[ \beta_4 = \frac{1 \pm \sqrt{(-1)^2 - 4(0.043221)(1.72786)}}{2(0.043221)} = 1.88074, 21.2567 \]

more than 2\( \pi \).
Sub. \( \beta_4 \) into Eq. 8
\[
\beta_1 = 2.19913 - 0.20140 (1.88074)^2 = 1.48674
\]
Sub. \( \beta_4 \) into Eq. 6
\[
\beta_3 = 0.15818 (1.88074)^2 = 0.55951
\]
Sub. \( \beta_1 \) into Eq. (3)
\[
L_1 = 0.79577 (1.48674) = 1.1831
\]
Sub. \( \beta_3 \) into Eq. (4)
\[
L_3 = 1.01321 (0.55951) = 0.5669
\]

Summarizing

<table>
<thead>
<tr>
<th>Half-rise Cycloidal</th>
<th>( L_1 = 1.1831 ) in</th>
<th>( \beta_1 = 1.48674 ) rad = 85.184°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant Velocity Rise</td>
<td>( L_2 = 1.2500 ) in</td>
<td>( \beta_2 = 0.78540 ) rad = 45.000°</td>
</tr>
<tr>
<td>Half-rise Harmonic</td>
<td>( L_3 = 0.5669 ) in</td>
<td>( \beta_3 = 0.55951 ) rad = 32.058°</td>
</tr>
<tr>
<td>Full return 8th Order Poly.</td>
<td>( L_4 = 3.0000 ) in</td>
<td>( \beta_4 = 1.88074 ) rad = 107.758°</td>
</tr>
<tr>
<td>Dwell</td>
<td>( L_5 = 0.0000 ) in</td>
<td>( \beta_5 = 1.57080 ) rad = 90.000°</td>
</tr>
</tbody>
</table>

Homework 6.7 & 6.9