In-class Example for Chapter 3: Velocity Analysis

1) Robot Arm

What is the velocity of point D with respect to point O₁ if:

- Given:
  - $\theta_2 = 45°$
  - $\theta_3 = 0°$
  - $\theta_4 = 0°$
  - $\omega_2 = +10°/sec$
  - $\omega_3 = +10°/sec$
  - $\omega_4 = -20°/sec$

- Find: $\vec{V}_D$

Solution: First we do a position analysis - only need to solve for position differences on same link

- $\overline{R}_{BA} = 2\ m \angle 45°$
- $\overline{R}_{CB} = 1\ m \angle 0°$
- $\overline{R}_{DC} = 0.15\ m \angle 0°$

Then we do the velocity calculations

- $\overline{V}_{AO} = 0$
- $\overline{V}_B = \overline{V}_{BA} + \overline{V}_A$

- $\overline{V}_C = \overline{V}_B + \overline{V}_{CB}$

- $\overline{V}_D = \overline{V}_C + \overline{V}_{DC}$

- $\overline{V}_D = \overline{V}_A + \overline{V}_{BA} + \overline{V}_{CB} + \overline{V}_{DC}$

- $\overline{V}_D = -0.247\ \hat{i} + 0.369\ \hat{j}\ m/s$
- $\overline{V}_D = 0.444\ m/s \angle 123.8°$

- Don't mix position & velocity vector diagrams
Problem 3.6

Given:

\[ \begin{align*}
B & \rightarrow 350 \text{ miles/hr} \\
200 \text{ miles} & \rightarrow 390 \text{ miles/hr} \\
60° & \rightarrow 45° \\
\end{align*} \]

Find:

a) \( \min R_{AB} \) \( \triangleq \) \( \text{At time } t = t_1 \) or \( t_c = t_c \)

b) \( t_{c1} (\text{clock time at } \min R_{BA}) \)

Sol'n:

\[ \begin{align*}
\vec{V}_A &= \vec{V}_B + \vec{V}_{AB} \\
\vec{V}_{AB} &= \vec{V}_A - \vec{V}_B = 390 \, \cos 45° \hat{i} + 390 \, \sin 45° \hat{j} - 350 \hat{i} \\
&= -74.2 \hat{i} + 275.8 \hat{j} \text{ miles/hr} \\
\end{align*} \]

\[ \begin{align*}
\vec{R}_{AB} &= \vec{R}^0_{AB} + \vec{V}_{AB} \, t \\
&= 200 \cos (-60°) \hat{i} + 200 \sin (-60°) \hat{j} - 74.2 t \hat{i} + 275.8 t \hat{j} \\
&= (100 - 74.2 t) \hat{i} + (-173.2 + 275.8 t) \hat{j} \text{ miles} \\
\end{align*} \]

\[ \begin{align*}
(R_{AB})^2 &= (100 - 74.2 t)^2 + (-173.2 + 275.8 t)^2 \\
&= 10,000 - 14840 t + 5505.6 t^2 + 29998.2 - 95537.1 t + 76065.6 t^2 \\
&= 39998.2 - 110377.1 t + 81571.2 t^2 \\
\end{align*} \]

\[ \begin{align*}
\frac{d(R_{AB})^2}{dt} &= -110377.1 + 163142.4 t = 0 \text{ when } t = t_1, \\
\Rightarrow t_1 &= 0.6766 \text{ hr} = 41 \text{ minutes} \\
\Rightarrow t_c &= 6:41 \text{ pm} \\
\end{align*} \]

\[ \begin{align*}
\min R_{AB} &= \sqrt{(100 - 74.2 (0.6766))^2 + (-173.2 + 275.8 (0.6766))^2} \\
&= 51.6 \text{ miles} \\
\end{align*} \]

Homework: 3.3, 3.5 & 3.8
Problem 3.14

Given

\[ \omega_2 = 60 \text{ rad/s ccw} \]
\[ R_{A02} = R_{BA} = 6 \text{ in} \]
\[ R_{A04} = R_{B04} = 10 \text{ in} \]
\[ R_{CA} = 8 \text{ in} \]

Find a) \( \vec{v}_C \) b) \( \vec{w}_2 \) \( \vec{w}_4 \)

First need to do position analysis

\[
\vec{R}_B = \vec{R}_{BA} + \vec{R}_A = \vec{R}_{B04} + \vec{R}_{A04}
\]

\[
\vec{R}_A - \vec{R}_{A04} = \vec{R}_{B04} - \vec{R}_{BA}
\]

\[
\vec{C} = \vec{A} + \vec{B}
\]

\[
\theta_A = \theta_c + \cos^{-1} \frac{c^2 + A^2 - 18^2}{2AC}
\]

\[
\theta_{R_{BA}} = \theta_c + \cos^{-1} \frac{c^2 + R_{BA}^2 - R_{BA}^2}{2R_{BA}C}
\]

\[
= 149^\circ \pm \cos^{-1} \frac{11.66^2 + 10^2 - 6^2}{2(10)(11.66)}
\]

\[
= 149^\circ \pm 31^\circ = 118^\circ \text{ or } 180^\circ
\]

\[
\theta_B = \theta_c + \cos^{-1} \frac{c^2 + B^2 - A^2}{2BC}
\]

\[
\theta_{R_{BA}} + 180 = \theta_c + \cos^{-1} \frac{c^2 + R_{BA}^2 - R_{BA}^2}{2R_{BA}C}
\]

\[
= 149.0^\circ \pm \cos^{-1} \frac{11.66^2 + 6^2 - 10^2}{2(6)(11.66)}
\]

\[
= 149^\circ \pm 59^\circ = 90^\circ \text{ or } 208^\circ
\]

\[
\theta_{R_{BA}} = 270^\circ \text{ or } 28^\circ
\]
Next do the velocity analysis

\[ \vec{V}_A = \vec{\omega}_z \times \vec{R}_A \text{ must be in rad.} \]

\[ V_A = \omega_z R_A = (60 \times 6) = 360 \text{ in/s} \]

\[ \Theta V_A = \Theta R_A + 90^\circ = 90^\circ + 90^\circ = 180^\circ \]

\[ \vec{V}_B = \vec{V}_{BA} + \vec{V}_A = \vec{V}_{BO_4} + \vec{V}_{0_4} \]

**Case 3**

\[ \vec{V}_A = \vec{V}_{BO_4} - \vec{V}_{BA} \]

\[ B = C \frac{\sin(\Theta_C - \Theta_A)}{\sin(\Theta_B - \Theta_A)} \]

\[ \vec{V}_{BA} = V_A \frac{\sin(\Theta_{V_A} - \Theta_{V_{BA}})}{\sin(\Theta_{V_{BA}} + 180 - \Theta_{V_{BO_4}})} \]

\[ = 360 \frac{\sin(180 - 208)}{\sin(118 + 180 - 208)} \]

\[ = -169.0 \text{ in/s} \]

\[ \therefore \vec{V}_{BA} = 169.0 \text{ in/s} < 298^\circ \]

\[ A = C \frac{\sin(\Theta_C - \Theta_B)}{\sin(\Theta_A - \Theta_B)} \]

\[ \vec{V}_{BO_4} = V_A \frac{\sin(\Theta_{V_A} - (\Theta_{V_{BA}} + 180))}{\sin(\Theta_{V_{BO_4}} - (\Theta_{V_{BA}} + 180))} \]
\[ V_{BD} = 360 \frac{\sin (180^\circ - (118 + 180))}{\sin (208^\circ - (118 + 180))} \]
\[ = 317.86 \text{ in/s} \]

\[ \vec{V}_B = \vec{V}_{BA} + \vec{V}_A = 169.0 \cos 298^\circ \uparrow + 169.0 \sin 298^\circ \uparrow - 360 \uparrow \]
\[ = -280.7 \uparrow - 149.2 \uparrow \text{ in/s} \]

\[ |\omega_2| = \frac{V_{BA}}{R_{BA}} = \frac{169}{6} = 28.2 \text{ rad/s} \]

\[ \omega_2 = -28.2 \text{ rad/s} \]

\[ |\omega_3| = \frac{V_{BD}}{R_{BD}} = \frac{317.86}{15} = 21.2 \text{ rad/s} \]

\[ \omega_3 = +31.8 \text{ rad/s} \]

\[ \vec{V}_C = \vec{V}_{CA} + \vec{V}_A \]

\[ \vec{V}_{CA} = \omega_2 \times \vec{R}_{CA} \]

\[ V_{CM} = \omega_z R_{CM} = -28.2 \times 8 = -225.6 \text{ in/s} \]

\[ \Theta_{VC} = \Theta_{RCA} + 90 \]

\[ = 208 + 90 = 298 \]

\[ \vec{V}_{CA} = 225.6 \angle 118^\circ \]

Homework: 3.11 & 3.15

For 3.15, define a point C at D4.
Apparent Velocity

\[ \vec{V}_{p_j} = \vec{V}_{p_i} + \vec{V}_{p_ji} \]

The equation involves 1 point in 2 C.S. as opposed to 2 points in 1 C.S. as we usually do.

Example 3.4 from text

**Given:**

\[ \begin{align*}
V_A &= 300 \text{ km/h} \\
V_B &= 2000 \text{ km/h}
\end{align*} \]

**Find:** \( V_{B3/2} \)

**Sol’n:**

We are interested in point B
- rocket is at \( B_3 \)
- pilot is looking at reference point \( B_2 \) (looking straight right, in \( x_2 \) direction)

Applying the equation:

\[ \vec{V}_{B3} = \vec{V}_{B2} + \vec{V}_{B3/2} \]

\[ \vec{V}_{B3/2} = \vec{V}_{B3} - \vec{V}_{B2} \]

\[ \omega_2 = \frac{V_{R0}}{R_{A01}} = \frac{300}{5} = 60 \text{ rad \ h}^{-1} \text{ ccw} \]

\[ \vec{V}_{B2} = \vec{V}_A + \vec{V}_{B2A} = \vec{V}_A + \omega_2 \times \vec{R}_{B2A} = 300 \hat{j} + 60(30) \hat{i} = 2100 \hat{i} \text{ km/h} \]

\[ \vec{V}_{B3/2} = 2000 \hat{i} - 2100 \hat{j} \]

\[ \vec{V}_{B3/2} = -100 \hat{j} \text{ km/h} \]

Contrast this with the velocity difference:

\[ \vec{V}_{BA} = \vec{V}_B - \vec{V}_A \]

\[ = 2000 \hat{i} - 300 \hat{j} \]

\[ \vec{V}_{BA} = 1700 \hat{i} \text{ km/h} \]