1. For the one-dimensional problem shown, calculate:
   a. The global stiffness matrix before the application of boundary conditions.
   b. The reduced stiffness matrix after the application of boundary conditions.

\[ k_1 = 10,000 \text{ N/mm} \]
\[ k_2 = 5,000 \text{ N/mm} \]
\[ k_3 = 10,000 \text{ N/mm} \]
\[ F = 500 \text{ N} \]
2. Give the correct order for the following FEA tasks, considering both how SolidWorks works and the current best practices.

<table>
<thead>
<tr>
<th>Step</th>
<th>Task</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>a. Add Boundary Conditions</td>
</tr>
<tr>
<td>2</td>
<td>b. Solve for displacements</td>
</tr>
<tr>
<td>3</td>
<td>c. Visualize the results</td>
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<tr>
<td>4</td>
<td>d. Solve for strains</td>
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<tr>
<td>5</td>
<td>e. Create the mesh</td>
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<tr>
<td>6</td>
<td>f. Create geometry</td>
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<tr>
<td>7</td>
<td>g. Anticipate physical behavior, possibly using check calculations</td>
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<tr>
<td>8</td>
<td>h. Determine whether FEA is warranted</td>
</tr>
<tr>
<td>9</td>
<td>i. Solve for stresses</td>
</tr>
</tbody>
</table>

3. In SolidWorks, explain what each of the following is for:
   a. Simulation Manager
   b. Fixture
   c. Results
   d. Probe
   e. Post-Processing toolbar
4. To what do DOF 1, DOF 2, … DOF 6 refer, when applying user-defined restraints in the Lab Assignments?

5. What is the difference between truss (or rod or bar) elements and beam elements?

6. What does the FEA software do when the yield stress is exceeded in a linear static analysis?

7. If an element has a quadratic shape function in $x$, will the strain also be a quadratic function of $x$?
8. Given the truss structure shown, calculate the stress and strain in truss element 1 if:

\[ A_1 = 0.0004 \, \text{m}^2 \]
\[ E_1 = 200 \times 10^9 \, \text{Pa} \]
\[ L_1 = 2 \, \text{m} \]

\[
\begin{bmatrix}
    u_1 \\
    v_1 \\
    u_2 \\
    v_2 \\
    u_3 \\
    v_3
\end{bmatrix} = \begin{bmatrix}
    0 \\
    0 \\
    0 \\
    0 \\
    -1 \times 10^{-3} \\
    -2 \times 10^{-3}
\end{bmatrix}
\]
9. Give the equivalent nodal forces and moments that represent the distributed force on the beam shown. Give your answer in the form of the global force vector \( \mathbf{R} \).

\[ q = 100 \text{ N/m} \]

\[
\begin{bmatrix}
F_{1x} \\
F_{1y} \\
M_1 \\
F_{2x} \\
F_{2y} \\
M_2 \\
F_{3x} \\
F_{3y} \\
M_3 \\
F_{4x} \\
F_{4y} \\
M_4 \\
F_{5x} \\
F_{5y} \\
M_5
\end{bmatrix}
= \mathbf{R}
\]
10. For the example on the right:
   (i) Solve for the two elemental stiffness matrices.
   (ii) Assemble the global stiffness matrix.
   (iii) Compute the global applied force vector \( \mathbf{R} \) considering only the gravitational force acting on the rod elements.
   (iv) After applying the appropriate restraint condition(s), solve for the nodal displacements.
   (v) Solve for the reaction force(s) at the restraint(s).
   (vi) Solve for the element strains.
   (vii) Solve for the element stresses.

\[
\begin{align*}
L_1 &= L_2 = 1 \text{ m} \\
A_1 &= 0.001 \text{ m}^2 \\
A_2 &= 0.004 \text{ m}^2 \\
E_1 &= E_2 = 70 \times 10^9 \text{ Pa} \\
\rho_1 &= \rho_2 = 2700 \text{ kg/m}^3
\end{align*}
\]
11. In Question 10, what is the displacement at the middle of element 1 (i.e., at 0.5 m from the top)?

12. Plot the displacement of both elements as a function of the distance from the top.

13. In Question 10, what is the strain at the middle of element 1 (i.e., at 0.5 m from the top)?

14. Plot the strain of both elements as a function of the distance from the top.
15. In Question 10, what is the stress at the middle of element 1 (i.e., at 0.5 m from the top)?

16. Plot the stress of both elements as a function of the distance from the top.

17. In the above questions, will the answers be the exact answers? If your answer is no, what aspect of the problem makes it so the FEA answer is not fully correct?

18. Consider the horizontal TRUSS element with cantilevered support conditions. Is the stiffness matrix singular (i.e., would you be able to solve for the displacements)?
19. For the beam elements shown (with shape functions given below), the nodal
displacements have been calculated in meters and radians as:

\[
D = \begin{bmatrix}
v_1 \\
\theta_1 \\
v_2 \\
\theta_2 \\
v_3 \\
\theta_3 \\
\end{bmatrix} = \begin{bmatrix}
0 \\
-0.0076 \\
-0.010 \\
0 \\
0 \\
0.0076 \\
\end{bmatrix}
\]

\[
\begin{align*}
D &= \begin{bmatrix}
v_1 \\
\theta_1 \\
v_2 \\
\theta_2 \\
v_3 \\
\theta_3 \\
\end{bmatrix} = \begin{bmatrix}
0 \\
-0.0076 \\
-0.010 \\
0 \\
0 \\
0.0076 \\
\end{bmatrix} \\
e) & Plot (sketch) the vertical displacement \( v(x) \) for the entire beam (both elements).
\]

b) Plot the angle of the beam \( \theta(x) \) for the entire beam.
c) Plot the moment \( M(x) \) carried by the beam for the entire beam.
d) Plot the stress \( \sigma(x) \) at the bottom of the beam.
e) Plot the stress \( \sigma(x) \) at the middle of the beam cross-section.

- In the plots, write actual values at node positions.
- Indicate the order of the polynomials for each plot.

Recall that for a beam element \( v(x) = Nd \), where:

\[
N = \begin{bmatrix}
1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \\
-\frac{2x^2}{L} + \frac{x^3}{L^2} \\
-\frac{3x^2}{L^2} - \frac{2x^3}{L^3} - \frac{x^2}{L^2} + \frac{x^3}{L^2}
\end{bmatrix}
\]

\[
\theta(x) = \frac{dv}{dx}
\]

\[
M = EI \frac{d^2v}{dx^2} = E/BD, \quad \text{where: } B = \begin{bmatrix}
-\frac{6}{L^2} - \frac{12x}{L^3} \\
-\frac{4}{L} + \frac{6x}{L^2} \\
-\frac{6}{L^2} - \frac{12x}{L^3} - \frac{2}{L} + \frac{6x}{L^2}
\end{bmatrix}
\]
Question 19 cont’d.
Question 19 cont’d.