The Nature of FEA Approximations

Demonstrated with a tapered bar element

Finite Element Approximations

• The previous lectures showed the procedure for how a FEA is performed, but it was technically not a FEA.

• A FEA technically requires subdividing a body into regions in order to use simple approximations to describe complicated displacement, strain, stress, etc. fields.

• There may be several reasons for subdividing a straight, axially loaded bar into several elements.
Varying cross-section

- A bar element with varying cross-section does not have constant strain, therefore $\delta = \frac{PL}{AE}$ cannot be used.
- We could develop a new equation for a bar with non-uniform cross-section, but instead, we approximate the solution with a set of constant cross-section bar elements.

Distributed Load

- A bar element with distributed loading does not have constant strain. ($\delta = \frac{PL}{AE}$ cannot be used.)
- We could develop a new equation for a bar with distributed loading, but in the Finite Element Method, we approximate the solution with a set of bar elements with loaded nodes in between.
Axially Loaded Members – General Problem

Displacement: \( u(x) \)

Strain: \( \varepsilon(x) = \frac{du}{dx} \)

Stress: \( \sigma = E \varepsilon \)

Force: \( P = A \sigma \)

Equilibrium: \( \frac{dP}{dx} + p(x) = 0 \quad \frac{d}{dx} \left( EA \frac{du}{dx} \right) + p(x) = 0 \)

Boundary Conditions: \( u(0) \) or \( \sigma(0) \) and \( u(L) \) or \( \sigma(L) \)

Slide from B.S. Altan, Michigan Technological U.