Sketch the following discrete-time signals.

a) \( x[n] = u[n-5] - u[n-6] \)

Plot the two portions of the signal separately, and then subtract:

\[
\begin{align*}
&u[n-5] & & u[n-6] \\
&0 & & 0 & & \cdots & & 0 & & 0 & & \cdots \\
&1 & & 2 & & 3 & & 4 & & 5 & & 6 & & 7 & & n
\end{align*}
\]

b) \( x[n] = 10u[-n+2] - 5u[n-2] \)

\( 10u[-n+2] \) turns on for \(-n+2 \geq 0 \Rightarrow n \leq 2\)

\[
\begin{align*}
&10u[-n+2] & & 5u[n-2] \\
&\cdots & & \cdots \\
&1 & & 2 & & 3 & & 4 & & 5 & & 6 & & n
\end{align*}
\]

Therefore, subtracting:

\[
\begin{align*}
&\cdots & & \cdots \\
&5 & & 5 & & 5 & & \cdots \\
&1 & & 2 & & 3 & & 4 & & 5 & & 6 & & n
\end{align*}
\]
c) \( x[n] = 4 \delta[n+5] + (n+5)u[n+3] - nu[n] \)

Plot each individual portion of this signal separately.

\[ \begin{align*}
4\delta[n+5] & \quad \text{Shifted impulse function} \\
(n+5)u[n+3] & \quad \text{Shifted ramp function that starts at } n = -3 \\
-nu[n] & \\
\end{align*} \]

Add all together.

\[ x[n] \]

---

d) \( x[n] = (0.1)^n \left(u[n] - u[n-5]\right) \)

A decreasing exponential that is only turned on for samples 0 through 4.

\[ \begin{align*}
x[0] &= (0.1)^0 = 1 \\
x[1] &= (0.1)^1 = 0.1 \\
x[2] &= (0.1)^2 = 0.01 \\
x[3] &= (0.1)^3 = 0.001 \\
x[4] &= (0.1)^4 = 0.0001 \\
\end{align*} \]
A discrete-time signal, $x[n]$, is shown below. Sketch the following signals:

\[ x[n] \]

\[ \begin{array}{c}
\text{-5} & \text{-4} & \text{-3} & \text{-2} & \text{-1} & 1 & 2 & 3 & 4 & 5 & n
\end{array} \]

\[ \begin{array}{c}
\text{-2}
\text{-1}
\text{1}
\end{array} \]

a) $y[n] = x[n-3]$

- delayed by 3 samples

\[ x[n-3] \]

\[ \begin{array}{c}
\text{-5} & \text{-4} & \text{-3} & \text{-2} & \text{-1} & 1 & 2 & 3 & 4 & 5 & 6 & n
\end{array} \]

\[ \begin{array}{c}
\text{-2}
\text{-1}
\text{1}
\end{array} \]

b) $y[n] = x[3-n]$

- There is both time reversal and time shifting

: Let $v[n] = x[-an]$

Then $y[n] = v[n+b] = x[-a(n+b)] = x[an + ab]$

Matching terms

\[ a = -1 \]
\[ b = 3 \]

$y[n] = v[n-3] = x[-n+3]$

- delayed version of $v[n]$

\[ \begin{array}{c}
\\text{...}
\text{-2}
\text{-1}
\text{1}
\text{2}
\text{3}
\text{4}
\text{5}
\text{6}
\end{array} \]

\[ \begin{array}{c}
\text{-2}
\text{-1}
\text{1}
\text{2}
\text{3}
\text{4}
\end{array} \]
c) \[ y[n] = x[3n] \]

This is time scaling, also known as subsampling in the discrete-time domain. This subsampling is valid because 3 is an integer, and we must ensure that 3n is an integer (the argument of x must be an integer). Therefore, we are subsampling at a rate of 3, meaning that the output, \( y[n] \), only looks at every third value of \( x[n] \).

Simply put, we can plug in values of \( n \) (integer values) into \( x \) to get the resulting values of \( y[n] \).

For example,

Let \( n = 0 \)
\[
\begin{align*}
y[0] &= x[3 \cdot 0] = x[0] = 1
\end{align*}
\]

Let \( n = 1 \)
\[
\begin{align*}
\end{align*}
\]

Let \( n = 2 \)
\[
\begin{align*}
\end{align*}
\]

Also, let \( n = -1 \)
\[
\begin{align*}
y[-1] &= x[3 \cdot (-1)] = x[-3] = -2
\end{align*}
\]

\[ y[n] \]

Again, we are simply taking every third sample value of \( x \) to determine the values of \( y \).
d) \( y[n] = x[3n+1] \)

This transformation includes both time scaling and time shifting.

Let \( u[n] = x[n+b] \)

Then \( y[n] = u[an] = x[an+b] \)

Matching terms

\[
\begin{align*}
  a &= 3 \\
  b &= 1 \\
  u[n] &= x[n+1]
\end{align*}
\]

Now, we subsample at a rate of 3

For example, plugging in values of \( n \), we get

Let \( n = 0 \)

\[
y[0] = u[3(0)] = x[3(0)+1] = u[1] = x[1] = 1
\]

Let \( n = 1 \)

\[
\]

Let \( n = -1 \)

\[
y[-1] = u[3(-1)] = x[3(-1)+1] = u[0] = x[-2] = -1
\]

(or we could simply look at the plots and do this by inspection)

\[
y[n] = u[3n] = x[3n+1]
\]
e) \( y[n] = x[n] \cdot u[3-n] \)

This is the multiplication of two signals. First, let us find out what \( u[3-n] \) is:

\[
u[3-n] = \begin{cases} 
1 & 3-n \geq 0 \Rightarrow n \leq 3 \\
0 & 3-n < 0 
\end{cases}
\]

\[\Rightarrow u[3-n] \text{ is given by the following plot} \]

\[
\begin{array}{c}
\ldots \\
\vdots \\
3 \\
\vdots \\
n \\
\end{array}
\]

We also notice from the plot of \( x[n] \) that all the values of \( x[n] \) for \( n > 3 \) are zero.

\[\Rightarrow y[n] = x[n] \cdot u[3-n] = x[n] \]

\[
\begin{array}{c}
y[2] \\
y[1] \\
y[0] \\
y[-1] \\
y[-2] \\
\vdots \\
-2 \\
\end{array}
\]

\[
\begin{array}{c}
-1 \\
0 \\
1 \\
2 \\
3 \\
4 \\
5 \\
n \\
\end{array}
\]
f) \[ y[n] = x[n-2] \delta[n-2] \]

This is the multiplication of two signals—one is a delayed version of \( x[n] \) and the other is a unit pulse function.

(This is also the shifting property for the discrete-time domain)

\[
\begin{array}{c}
x[n-2] \\
\hline
-3 & -2 & 1 & 2 & 3 & 4 & 5 \\
\end{array}
\]

\[
\begin{array}{c}
\delta[n-2] \\
\hline
2 & \ldots & \ldots & \ldots & \ldots & n \\
\end{array}
\]

\[
\begin{array}{c}
y[n] \\
\hline
2 & \ldots & \ldots & \ldots & \ldots & n \\
\end{array}
\]

Alternatively, plug in the sample value of the only non-zero value of the unit pulse function.

Let \( n = 2 \)

\[ y[2] = x[2-2] \delta[2-2] = x[0] (1) = 1 \]

All other values of \( n \) produce a 0 valued output from \( \delta \) and, thus, \( y[n] \)
9) \( y[n] = x[(n-1)^2] \)

The simplest way to determine this output is to plug in values of \( n \).

Let \( n = 0 \)
\[
y[0] = x[(0-1)^2] = x[1] = 1
\]

Let \( n = 1 \)
\[
y[1] = x[(1-1)^2] = x[0] = 1
\]

Let \( n = 2 \)
\[
\]

Let \( n = 3 \)
\[
\]

Also for negative values of \( n \)

Let \( n = -1 \)
\[
y[-1] = x[(-1-1)^2] = x[4] = 0
\]

Let \( n = -2 \)
\[
y[-2] = x[(-2-1)^2] = x[9] = 0
\]

Therefore, \( y[n] \) can be sketched as

\[
y[n]
\]

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & n \\
\end{array}
\]
The following continuous-time signal is to be discretized. What is the minimum sampling frequency that must be used in order to avoid aliasing?

\[ x(t) = 1 + 5 \cos (2\pi(10) t) + 10 \cos (2\pi(100) t) \]

Solution

\[ x(t) \] contains frequency components at 0 Hz, 10 Hz, and 100 Hz.

\[ \therefore \text{Nyquist Rate} \Rightarrow f_{\text{ns}} = 2 \max(f) = 2(100 \text{ Hz}) = 200 \text{ Hz} \]

\[ x(t) \] must be sampled at a frequency \( > 200 \text{ Hz} \)

(Good practice \( \Rightarrow f_s \geq 20 f_{\text{ns}} = 4 \text{ kHz} \))
Determine if the following system properties are valid.

\[ y(t) = x(-t) \]  Causal?

Let \( y(-t) = x(1) \) ⇒ Input precedes output

\[ \text{Not Causal} \]

\[ y(t) = (t+5)x(t) \]  Memoryless?

Output only depends on "t" and current state of \( x(t) \)

\[ \text{Memoryless} \]

\[ y(t) = x(5) \]  Memoryless?

⇒ Always depends on a particular value of \( x(t) \) ⇒ \( t = 5 \)
⇒ Could be looking to past, present, or future

\[ \text{Has Memory} \]

\[ y(t) = 2x(t) \]  Stable (BIBO)?

If \( |x(t)| \leq B_1 \) ⇒ Some boundary \( B_1 \)
then \( |y(t)| \leq B_2 \)
where \( B_2 = 2B_1 \)

\[ y(t) \text{ will always be } \leq B_2 = 2B_1 \text{ for all input bounded by } B_1 \]

\[ \text{Stable} \]
Determine if the following system properties are valid:

\[ y(t) = x(t) + a \quad \text{Linear?} \]

**Homogeneity Test**

\[ S \{ K x(t) \} = K x(t) + a \]

\[ K y(t) = K (x(t) + a) = K x(t) + K a \]

\[ S \{ K x(t) \} \neq K y(t) \]

**Nonlinear**

\[ y(t) = t x(2t) \]

**Homogeneity Test**

\[ S \{ K x(t) \} = K t x(2t) \]

\[ K y(t) = K t x(2t) \]

\[ S \{ K x(t) \} \neq K y(t) \quad \Rightarrow \text{Passes Homogeneity Test} \]

**Additivity Test**

\[ S \{ x_1(t) + x_2(t) \} = t \left( x_1(2t) + x_2(2t) \right) = t x_1(2t) + t x_2(2t) \]

Let \( y_1(t) = t x_1(2t) \)

\( y_2(t) = t x_2(2t) \)

\( y(t) = y_1(t) + y_2(t) = t x_1(2t) + t x_2(2t) \)

\[ S \{ x_1(t) + x_2(t) \} = y_1(t) + y_2(t) \]

\[ \Rightarrow \text{Passes Additivity Test} \]

**Linear**
Determine if the following system properties apply.

\[ y(t) = \int_0^T x(t - \tau) \, d\tau \quad \text{Time Invariant?} \]

\[ y(t) = \int_0^T x(t - \tau) \, d\tau = \int_0^T x(t - td - \tau) \, d\tau \]

\( \Rightarrow \) They are equal

Time Invariant

\[ y(t) = x(2t) \quad \text{Time Invariant?} \]

\[ y(t) = x(2t - td) = x(2(t - td)) \]

\( \Rightarrow \) They are not equal!

Time Varying
Determine the following properties of the given discrete-time system:

1. Causality
2. Memory
3. Stability
4. Linearity
5. Time Invariance
6. LTIS

Let \( y[n] = \left( \frac{n+0.5}{n-0.5} \right)^2 x[n] \)

1. Causal: only depends on present value of \( n \rightarrow x[n] \)
2. Memoryless: only depends on present value of \( n \rightarrow x[n] \)
3. Stability

Let all inputs \( |x[n]| < M \)

\[
\lim_{n \to \infty} y[n] = \lim_{n \to \infty} \left( \frac{n+0.5}{n-0.5} \right)^2 (M) \rightarrow M
\]

Maximum for \( y[n] \) (for \( n>0 \))

\[
y[1] = \left( \frac{1.5}{0.5} \right)^2 M = 9M
\]

\[
|y[n]| \leq 9M = R \rightarrow \text{Bounded} \rightarrow \text{Stable}
\]

4. Linearity

Additivity Test

Let \( x_1[n] \rightarrow y_1[n] = \left( \frac{n+0.5}{n-0.5} \right)^2 x_1[n] \)

Let \( x_2[n] \rightarrow y_2[n] = \left( \frac{n+0.5}{n-0.5} \right)^2 x_2[n] \)

\[
x_1[n] + x_2[n] \rightarrow \left( \frac{n+0.5}{n-0.5} \right)^2 \left[ x_1[n] + x_2[n] \right] = y_1[n] + y_2[n] \rightarrow \text{Additive}
\]

Homogeneity Test

Let \( x[n] \rightarrow y[n] = \left( \frac{n+0.5}{n-0.5} \right)^2 x[n] \)

Let \( a x[n] \rightarrow a \left( \frac{n+0.5}{n-0.5} \right)^2 x[n] = a y[n] \rightarrow \text{Homogeneous}
\]

\[\therefore \text{Linear}\]
5. **Time Invariance**

![Diagram](image)

Apply \( n \rightarrow n - n_d \) to all \( n \).

They do not agree

Time Varying

6. **LTI?**

*Not LTI. It is time varying.*