Sketch the step response for \( H(s) = \frac{20}{s+2} \)

\[
pole = -2
\]
\[
\tau = -\frac{1}{\rho} = \frac{1}{2}
\]

Final Value (steady-state value) ⇒ use Final Value Theorem

\[
Y_{\text{ss}} = \lim_{s \to 0} s \cdot \frac{20}{s+2} = \lim_{s \to 0} \frac{20}{s} = \frac{20}{2} = 10
\]

\[
\text{from F.V.T. } H(s) \xrightarrow{\text{Step input}}
\]

Sketch the step response for \( H(s) = \frac{0.5}{s+0.5} \)

\[
pole = -0.5
\]
\[
\tau = 2
\]

\[
Y_{\text{ss}} = \lim_{s \to 0} H(s) = 1
\]

Slow exponential rise than the previous system because \( \tau \) is larger
Plot the step response of the following systems

\[ H(s) = \frac{10}{s^2 + 15s + 50} = \frac{10}{(s+5)(s+10)} \]

poles = -5, -10 \( \Rightarrow 5 > 1 \) (overdamped)

dominant time constant is the slower \( \tau \) (underdamped)

\( \tau = \frac{1}{5} \)

Find \( \zeta \) and \( \omega_n \)

\[ \frac{k}{s^2 + 2\zeta \omega_n s + \omega_n^2} \]

\( \text{general form} \)

\[ \Rightarrow \quad \omega_n = \sqrt{50} = 7.07 \]

\[ 2\zeta \omega_n = 15 \]

\[ \zeta = \frac{15}{2 \sqrt{50}} = 1.06 \]

Again \( \Rightarrow 5 > 1 \) \( \Rightarrow \) real, distinct poles \( \Rightarrow \) overdamped

Settling time

\( 5\% \rightarrow 3\tau = \frac{3}{5} \text{ second} \) \( \Rightarrow \text{Really only applies to underdamped systems.} \)

\( 2\% \rightarrow 4\tau = \frac{4}{5} \text{ second} \)

Percent overshoot \( \Rightarrow \) No overshoot \( \Rightarrow \) Critically Damped

Steady-state value

\[ y_{ss} = \lim_{s \to 0} H(s) = \frac{1}{5} \]

\[ \frac{1}{5} \]

\[ This \ part \ due \ to \ the \ fact \ that \ there \ are \ two \ poles \]

A first-order system has a response that is perfectly exponential.
Step Response for $H(s) = \frac{10}{s^2 + 20s + 100} = \frac{10}{(s + 10)(s + 10)}$

poles = -10, -10 → Real, Repeated Poles
i.e. $\zeta = 1$ Critically Damped
$\omega_n = \sqrt{100} = 10$
$\tau = \frac{10}{10}$
$Y_{ss} = \lim_{s \to 0} H(s) = \frac{10}{100} = \frac{1}{10}$
No overshoot because the system is critically damped

Settling time

2% t = 3\tau = \frac{3}{10} \text{ second} \quad \text{Really only applies to Underdamped Systems}
5% t = 4\tau = \frac{4}{10} = \frac{2}{5} \text{ second}
Step response for \[ H(s) = \frac{10}{s^2 + 10s + 100} \]

Polles \[ p = \frac{-10 \pm \sqrt{100 - 400}}{2} = \frac{-10 \pm 10\sqrt{3}}{2} = -5 \pm j5\sqrt{3} = -5 \pm j8.66 \]

\[ \Rightarrow \text{Complex Poles} \Rightarrow \text{Under Damped} \]

\[ \begin{align*}
    k &= \frac{10}{s^2 + 2\beta \omega_n s + \omega_n^2} = \frac{10}{s^2 + 10s + 100} \\
    \Rightarrow \omega_n &= \sqrt{100} = 10 \\
    2\beta \omega_n &= 10 \\
    \beta &= \frac{1}{2} = 0.5 \Rightarrow \text{Under damped} \\
    \eta &= \frac{1}{3\omega_n} = \frac{1}{5} = 0.2 \\
    \text{Settling Time, } t_s \\
    2\% &\Rightarrow t_s \approx 3\eta = \frac{3}{5} \text{ second} \\
    5\% &\Rightarrow t_s \approx 4\eta = \frac{4}{5} \text{ second} \\
    \text{Percent Overshoot} \\
    \text{P.O. } \approx 100\% \times e^{-\frac{\pi}{\sqrt{1-0.25}}} = 100\% \times e^{-\frac{\pi}{2} / \sqrt{1-0.25}} = 26.76\% \\
    \text{Final Value} \\
    Y_{ss} = \lim_{s \to 0} H(s) = \frac{1}{10} \\
    \text{Peak value} = \frac{\text{P.O. } \times Y_{ss} + Y_{ss}}{100\%} = \frac{26.76 \times 0.1 + 0.1}{100\%} = 0.1268 \\
    T = \frac{2\pi}{\omega_d} \\
    \omega_d = 8.66 \\
    T = \frac{2\pi}{8.66} = 0.7255
\]
Use MATLAB to generate the step response of the following systems. Find the rise time and percent overshoot (if applicable).

a. \( H(s) = \frac{20}{s + 2} \)

b. \( H(s) = \frac{10}{s^2 + 10s + 100} \)

% find the step response of the following two systems

% first-order system
num = [20];
den = [1 2];
tt = 0:0.01:2;
figure;
subplot(2,1,1);
step(num,den,tt);
title('Step Response of First-Order System')

% second-order system
num = [10];
den = [1 10 100];
tt = 0:0.01:2;
subplot(2,1,2);
step(num,den,tt);
title('Step Response of Second-Order System')
axis([0 2 0 0.12]);

The rise time and the percent overshoot can be found from the following plots.
Alternatively, we can use more sophisticated MATLAB code to find out the rise times and percent overshoot for us. Type “help function_name” to learn more about MATLAB functions.

```matlab
% first-order system
num = [20];
den = [1 2];
tt = 0:0.01:2;
figure;
subplot(2,1,1);
step(num,den,tt);
[y] = step(num,den,tt);
title('Step Response of First-Order System')
[I1,J1] = find(y >= 0.1*10);
[I2,J2] = find(y >= 0.9*10);
tr = tt(I2(1))-tt(I1(1))

% second-order system
num = [10];
den = [1 10 100];
tt = 0:0.01:2;
subplot(2,1,2);
step(num,den,tt);
[y] = step(num,den,tt);
title('Step Response of Second-Order System')
axis([0 2 0 0.12]);
[I,J] = find(y >= 0.1);
tr = tt(I(1))
y_max = max(y);
y_ss = y(end);
percent_overshoot = ((y_max - y_ss)/y_ss)*100
```

![Step Response of First-Order System](image1)

![Step Response of Second-Order System](image2)
From the following step response plots, determine the transfer function. Also, determine the rise times.

First-Order System  \[ H(s) = \frac{k}{s + p} \]

\[ \tau = 0.63 \times Y_{ss} = 0.63 \times 2 = 1.26 \]
\[ \tau = \frac{1}{\gamma} = \frac{1}{0.63} = 1.6 \]
\[ p = -\frac{1}{\tau} = -5 \]
\[ Y_{ss} = \lim_{s \to 0} H(s) = \frac{k}{\tau} = 2 \]
\[ k = 2 \cdot p = 2 \cdot (-5) = -10 \]
\[ \therefore H(s) = \frac{10}{s + 5} \]

No overshoot

Second-Order System

\[ \tau_r = 0.18 \]

Alternating Method

Use Percent Overshoot to Find \( s \)

Calculate \( \omega_n = \frac{\omega_d}{\sqrt{1 - s^2}} \)

Then use Final Value Theorem for \( k \)

First-Order System - Rise time, \( \tau_r \rightarrow \) Never exceeds final value

\[ H(s) = \frac{10}{s + 5} \]

\[ \therefore \] use \( \tau_r \) for 10% to 90% of final value

\[ \tau_r \approx 0.52 - 0.02 = 0.5 \]

Second-Order System

\[ T = 0.91 - 0.32 = 0.65 = \frac{2\pi}{\omega_d} \]
\[ \therefore \omega_d = \frac{2\pi}{0.65} = 9.67 \]
\[ Y_{ss} = 0.05 \]
\[ H(s) = \frac{k}{s^2 + 2\omega_n s + \omega_n^2} = \frac{k}{(s + \omega_n^2)^2 + \omega_n^2} \]
\[ P.O. \approx 0.07 - 0.05 = 100\% \approx 40\% \]
\[ P.O. \approx 100\% \approx -\frac{3\pi/\sqrt{1-\omega_n^2}}{} \]

Actual (what was plotted) \( H(s) = \frac{10}{s^2 + 5s + 100} \)

Close to analytic expression