

ex.  $X(z) = \frac{z^3 + 1}{z^3 - z^2 - z - 2}$

Find the inverse Z transform

roots = 2,  $-0.5 \pm j 0.866$  $\deg(\text{num}) = \deg(\text{den})$ 

$$\frac{X(z)}{z} = \frac{z^3 + 1}{(z)(z^2 - z - 2)} = \frac{k_0}{z} + \frac{k_1}{z + 0.5 + j 0.866} + \frac{k_1^*}{z + 0.5 - j 0.866} + \frac{k_2}{z - 2}$$

$$k_0 = \left. \frac{X(z)}{z} \right|_{z=0} = -\frac{1}{2}$$

$$k_1 = \left. \frac{X(z)}{z} (z + 0.5 + j 0.866) \right|_{z=-0.5 - j 0.866} = 0.429 + j 0.0825$$

$$k_1^* = 0.429 - j 0.0825$$

$$k_2 = \left. \frac{X(z)}{z} (z - 2) \right|_{z=2} = 0.643$$

$$X(z) = -0.5 + \frac{(0.429 + j 0.0825)z}{z + 0.5 + j 0.866} + \frac{(0.429 - j 0.0825)z}{z + 0.5 - j 0.866} + \frac{0.643z}{z - 2}$$

$$x[n] = -0.5 \delta[n] + (0.429 + j 0.0825)(-0.5 - j 0.866)^n u[n] + (0.429 - j 0.0825)(-0.5 + j 0.866)^n u[n] + 0.643(2)^n u[n]$$

combine complex terms

$$|-0.5 \pm j 0.866| = 1$$

$$|0.429 + j 0.0825| = 0.436$$

$$\angle(-0.5 \pm j 0.866) = \pi - \tan^{-1} \frac{0.866}{0.5} = \frac{4\pi}{3} \text{ rad} \quad \angle(0.429 + j 0.0825) = \tan^{-1} \frac{0.0825}{0.429} = 10.89^\circ$$

$$\therefore x[n] = -0.5 \delta[n] + 0.873 \cos\left(\frac{4\pi}{3} n + 10.89^\circ\right) + 0.643(2)^n, n \geq 0$$