

The following continuous-time signal is to be discretized. What is the minimum sampling frequency that must be used in order to avoid aliasing?

$$x(t) = 1 + 5 \cos((2\pi)(10)t) + 10 \cos((2\pi)(100)t)$$

Solution

$x(t)$  contains frequency components at 0 Hz, 10 Hz, and 100 Hz

$$\therefore \text{Nyquist Rate} \Rightarrow f_{ns} = 2 f_{max} = 2(100 \text{ Hz}) = 200 \text{ Hz}$$

$x(t)$  must be sampled at a frequency  $> 200 \text{ Hz}$

(Good practice  $\Rightarrow f_s \geq 20 f_{ns} = 4 \text{ kHz}$ )

Given a system

$$\dot{y} + 5y = 2x$$

What is the minimum sampling frequency that would be reasonable on the output  $y(t)$ ?

Solution

$$\dot{y} + 5y = 2x$$

Find the transfer function

$$sY(s) + 5Y(s) = 2X(s)$$

$$H(s) = \frac{2}{s+5} = \frac{\frac{2}{5}}{\frac{s}{5} + 1}$$

Frequency Response

$$H(j\omega) = \frac{2}{5} \frac{1}{\frac{j\omega}{5} + 1}$$

corner frequency at  $5 \text{ rad/sec}$

All frequencies below  $5 \text{ rad/sec}$  are in the passband

All frequencies above  $5 \text{ rad/sec}$  are "attenuated"

$\therefore$  We are only concerned with frequencies  $\leq 5 \text{ rad/sec}$   
(in the pass band)

Nyquist rate  $\Rightarrow f_{ns} = 2f_{max}$

$$\omega_{ns} = 2\omega_{max} = (2)(5 \text{ rad/sec}) = 10 \text{ rad/sec}$$

$$\omega_s \geq 10 \text{ rad/sec}$$

(Good practice  $\rightarrow \omega_s \geq 20\omega_{ns} = 200 \text{ rad/sec}$ )

A sampling frequency of  $f_s = 1 \text{ MHz}$  is used with the following discrete-time filter

$$H(z) = \frac{z}{z + 0.8}$$

Determine the frequency response given a sampling frequency of  $f_s = 1 \text{ MHz}$ , and plot the corresponding magnitude response.

Solution

$$\text{Frequency response} \rightarrow H(e^{j\omega}) = \frac{e^{j\omega}}{e^{j\omega} + 0.8}$$

$$\text{Let } \omega = \pi \tau_s \text{ where } \tau_s = \frac{1}{f_s}$$

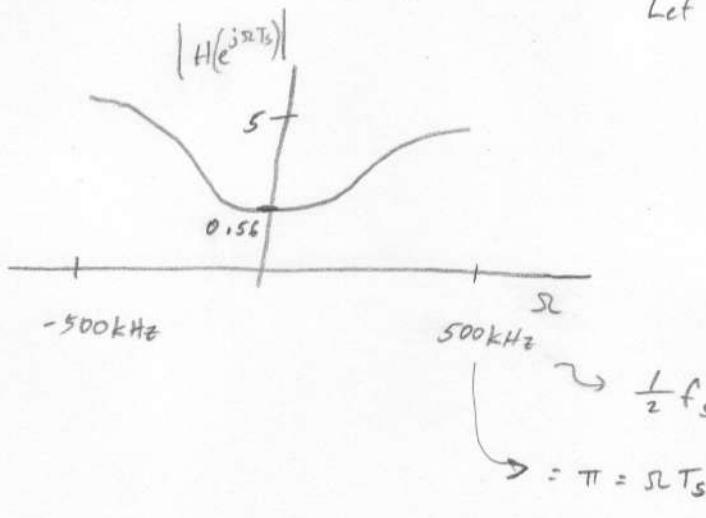
$$H(e^{j\omega})|_{\omega=\pi \tau_s} = (He^{j\pi \tau_s}) = \frac{e^{j\pi \tau_s}}{e^{j\pi \tau_s} + 0.8}$$

$$|H(e^{j\pi \tau_s})|^2 = \frac{e^{j\pi \tau_s}}{e^{j\pi \tau_s} + 0.8} \cdot \frac{e^{-j\pi \tau_s}}{e^{-j\pi \tau_s} + 0.8} = \frac{1}{1.64 + 1.6 \cos(\pi \tau_s)}$$

$$|H(e^{j\pi \tau_s})| = \frac{1}{\sqrt{1.64 + 1.6 \cos(\pi \tau_s)}}$$

$$\text{Let } \pi \tau_s = 0 \quad |H(e^{j0})| = 0.56$$

$$\text{Let } \pi \tau_s = \pm \pi \quad |H(e^{j\pi})| = 5$$



$$\Rightarrow \pi = \pi \tau_s = \frac{\pi}{2 f_s}$$

$$\therefore \pi = \pi(1 \text{ MHz}) = 2\pi(500 \text{ Krad/sec}) \\ \Rightarrow 500 \text{ kHz}$$