Block Diagrams

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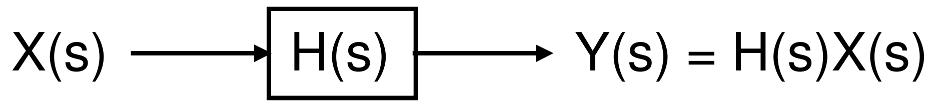
EE 327

Block Diagrams

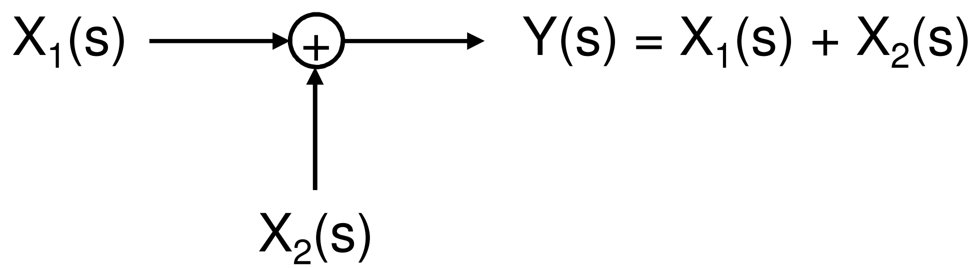
- Symbolic representation of complex signals
  - Easier to understand the relationships between subsystems using block digrams than by looking at the equations representing them, or even schematics
  - Often easier to obtain a transfer function for the overall system by first drawing block diagrams

Basic Elements

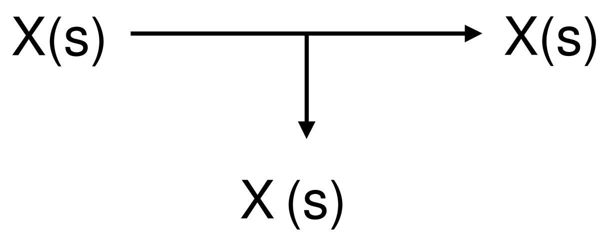
1. Block



2. Adder



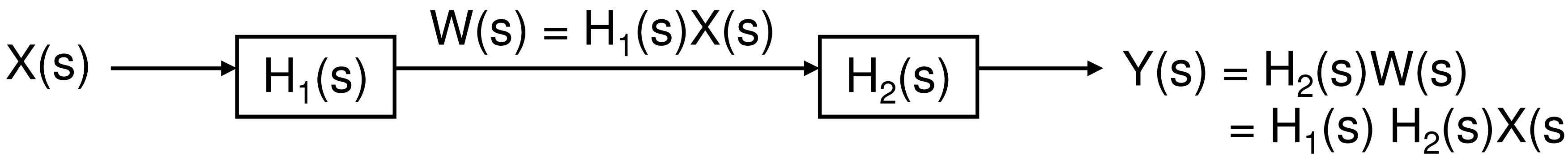
3. Node



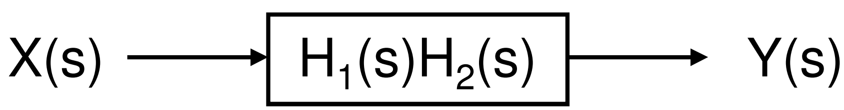
Basic Connections

- Series / Cascade
- Parallel
- Feedback

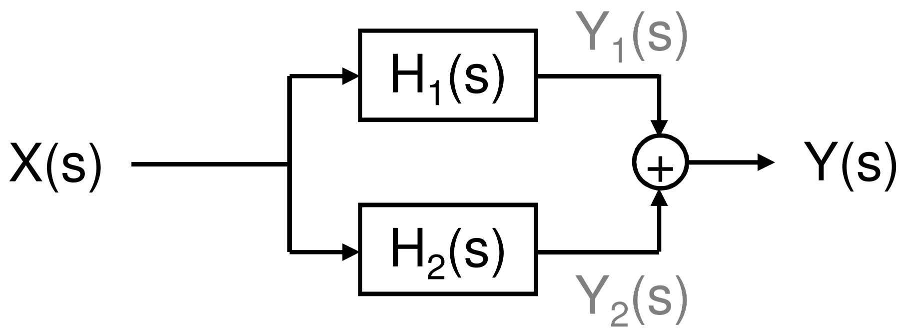
Series / Cascade Connections



Equivalent to a single block of



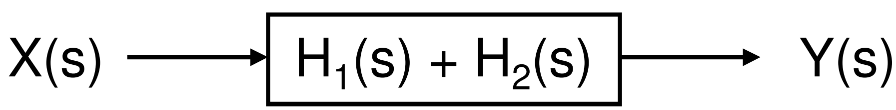
Parallel Connections



$Y_1(s) = H_1(s)X(s)$
$Y_2(s) = H_2(s)X(s)$

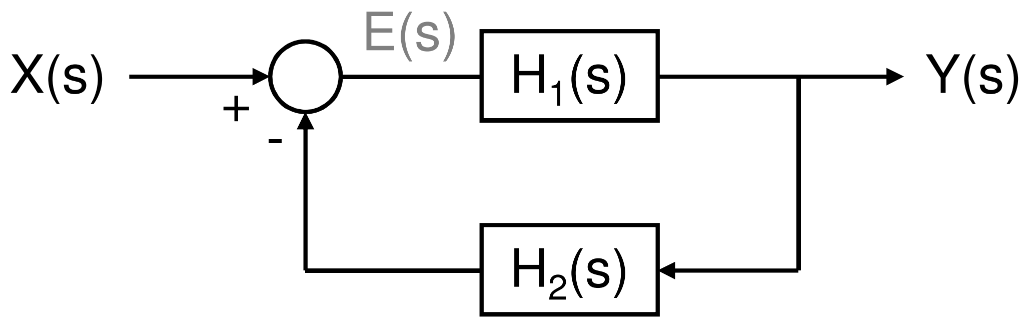
$$Y(s) = Y_1(s) + Y_2(s)$$
$$= [H_1(s) + H_2(s)]X(s)$$

Equivalent to a single block of



Feedback Connection

(Negative Feedback)



$$Y(s) = H_1(s)E(s)$$
$$E(s) = X(s) - H_2(s)Y(s)$$

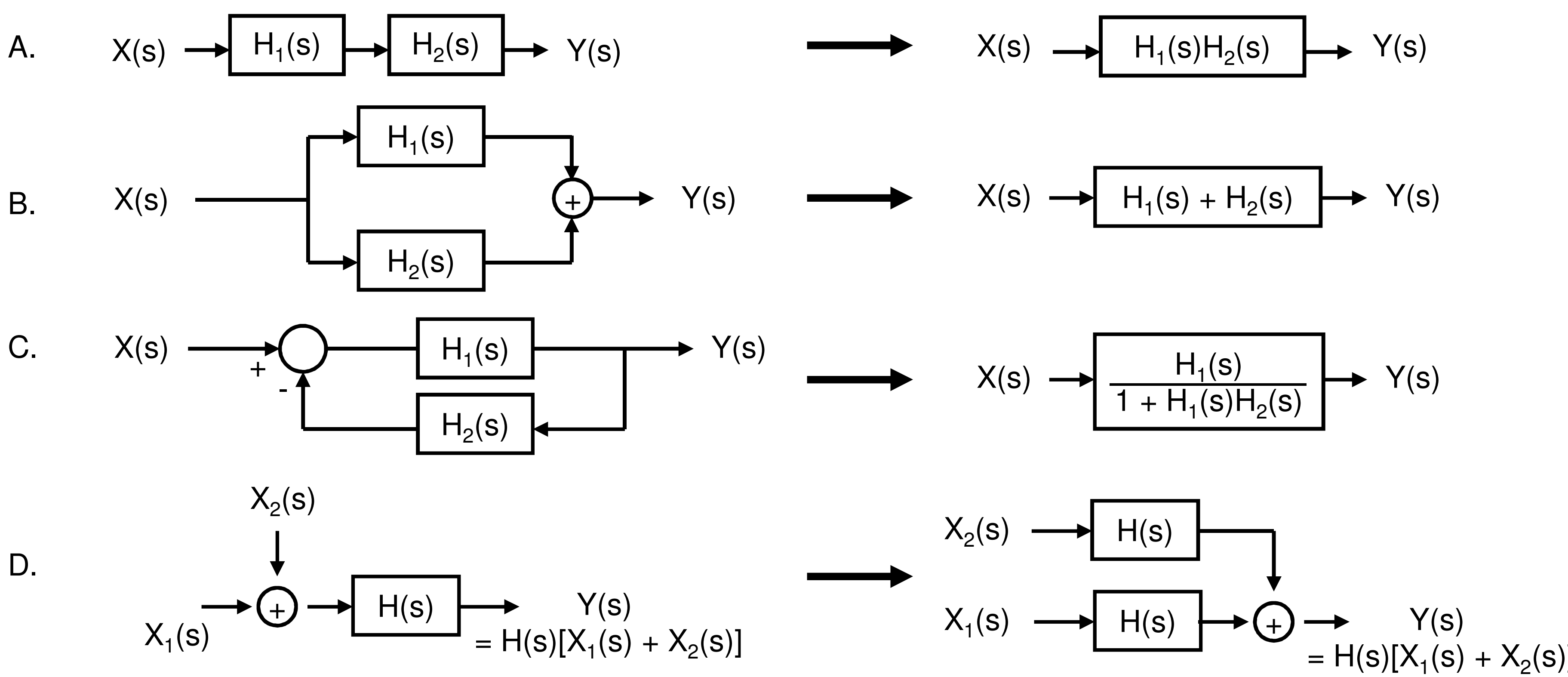
$$Y(s) = H_1(s)[X(s) - H_2(s)Y(s)]$$
$$= H_1(s)X(s) - H_1(s)H_2(s)Y(s)$$

$$Y(s)[1 + H_1(s)H_2(s)] = H_1(s)X(s)$$

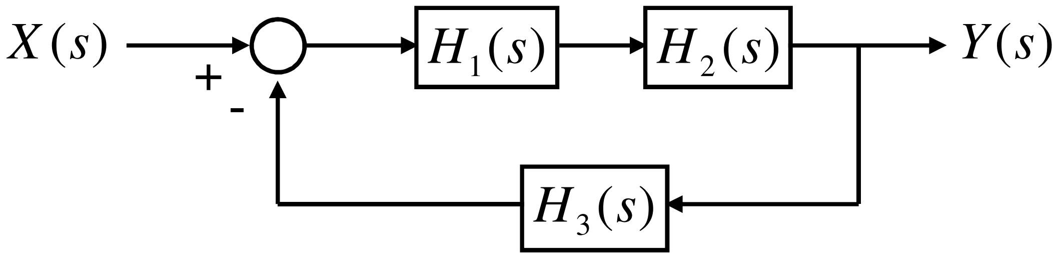
$$\frac{Y(s)}{X(s)} = \frac{H_1(s)}{1 + H_1(s)H_2(s)}$$ (Equivalent Block)

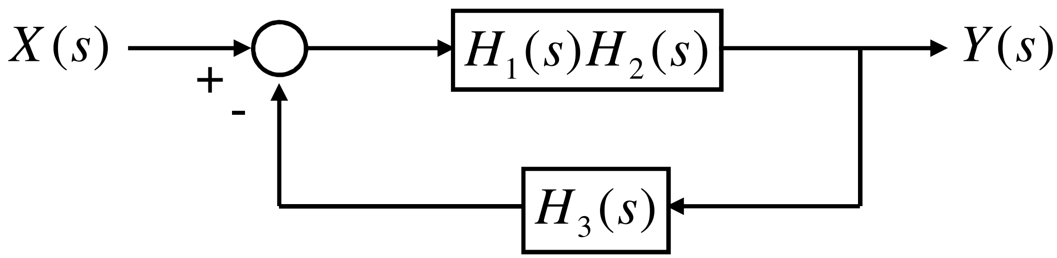
Complex System Modeling

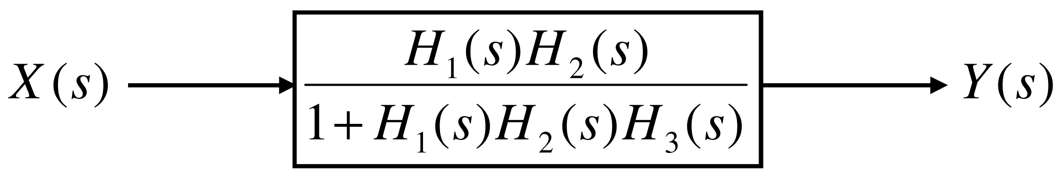
1. Obtain a transfer function for each subsystem
2. Determine equations that show the interactions of each subsystem
3. Draw a block diagram (Hint. Start from input and work to output or vice versa)
4. Perform block-diagram reduction



Block-Diagram Reduction Example 1





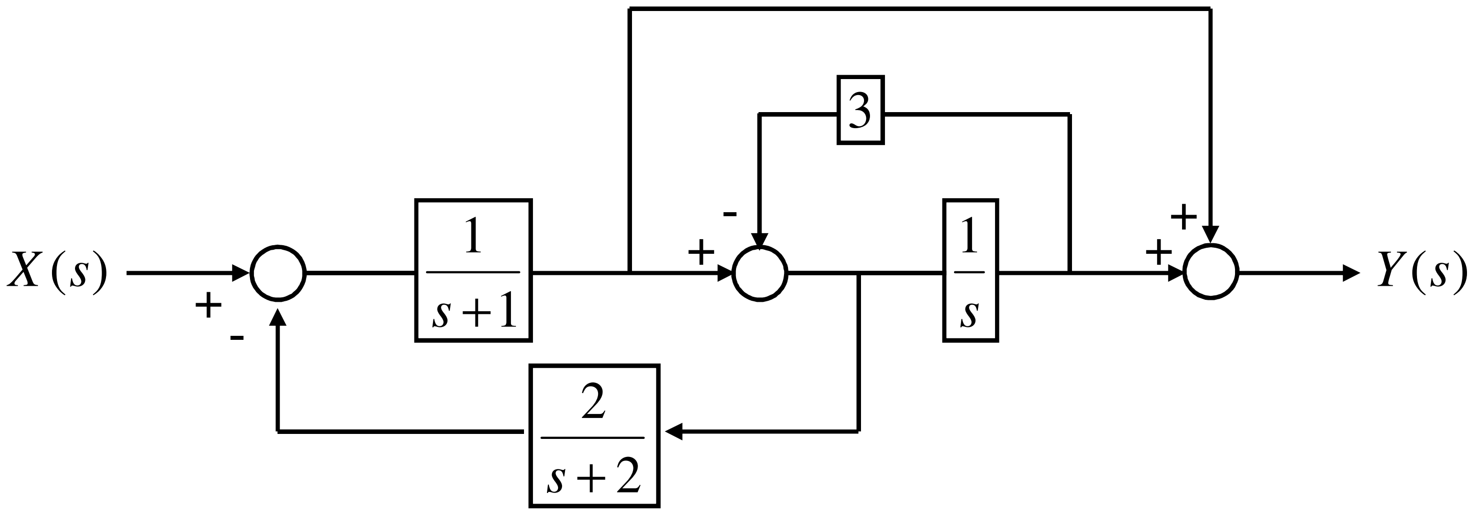


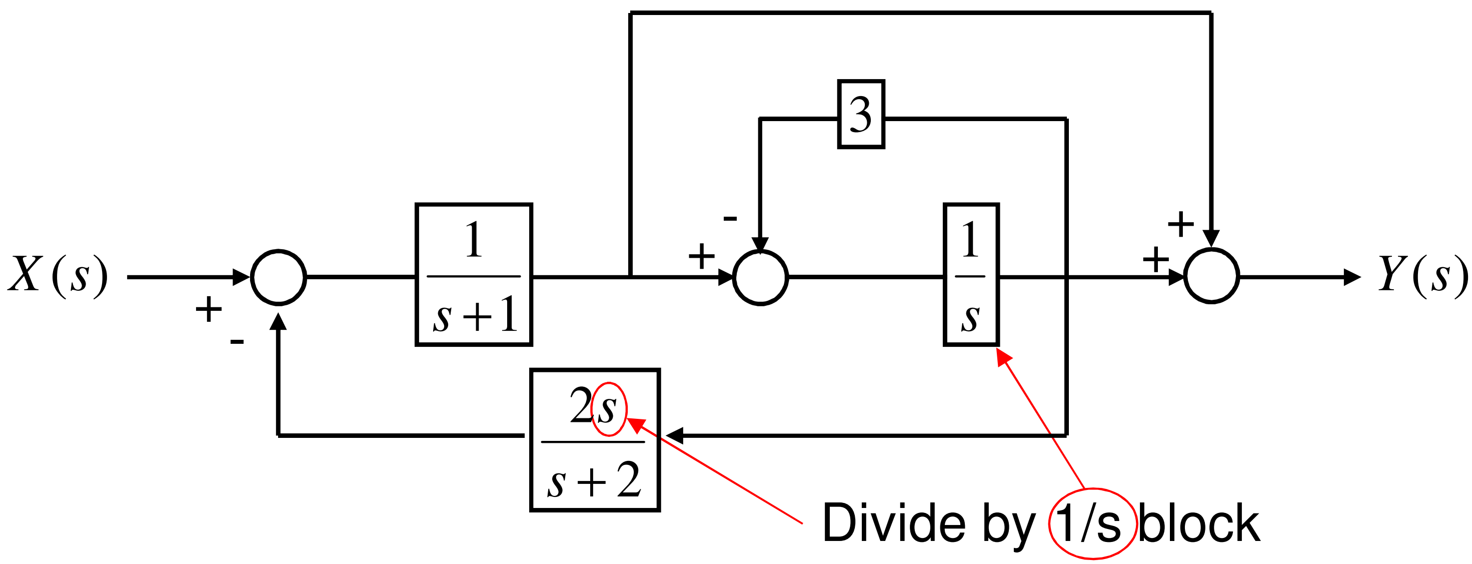
If $H_1(s) = 10,\ H_2(s) = \dfrac{1}{s+2},\ H_3(s) = \dfrac{s+1}{s+10}$

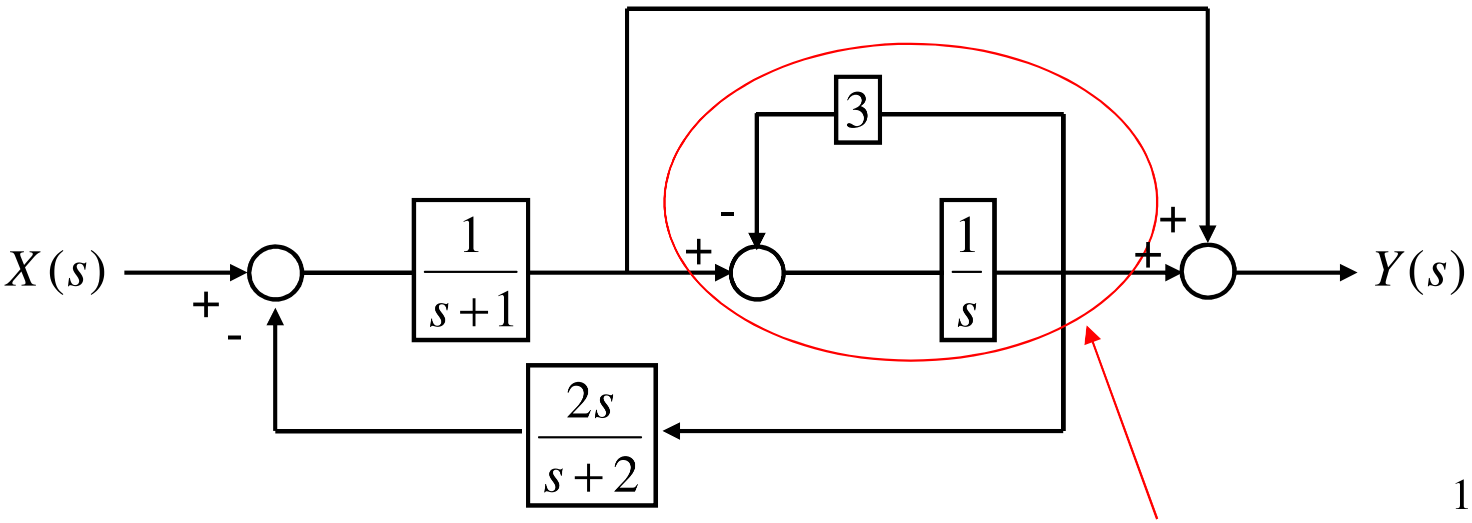
$$H(s) = \frac{H_1(s)H_2(s)}{1 + H_1(s)H_2(s)H_3(s)} = \frac{\dfrac{10}{s+2}}{1 + \dfrac{10(s+1)}{(s+2)(s+10)}} =$$

$$= \frac{10(s+10)}{s^2 + 22s + 30}$$

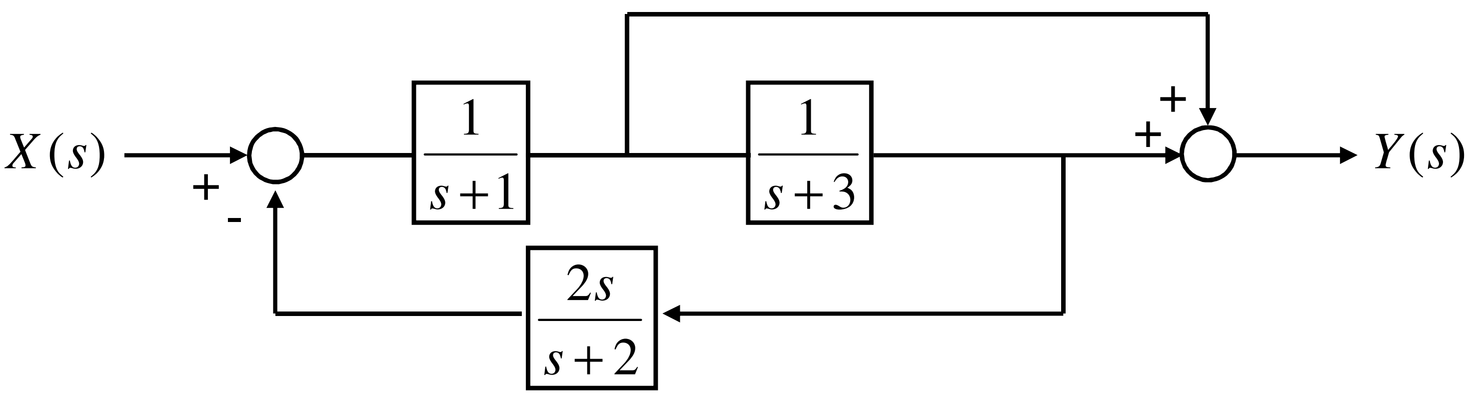
Block-Diagram Reduction Example 2

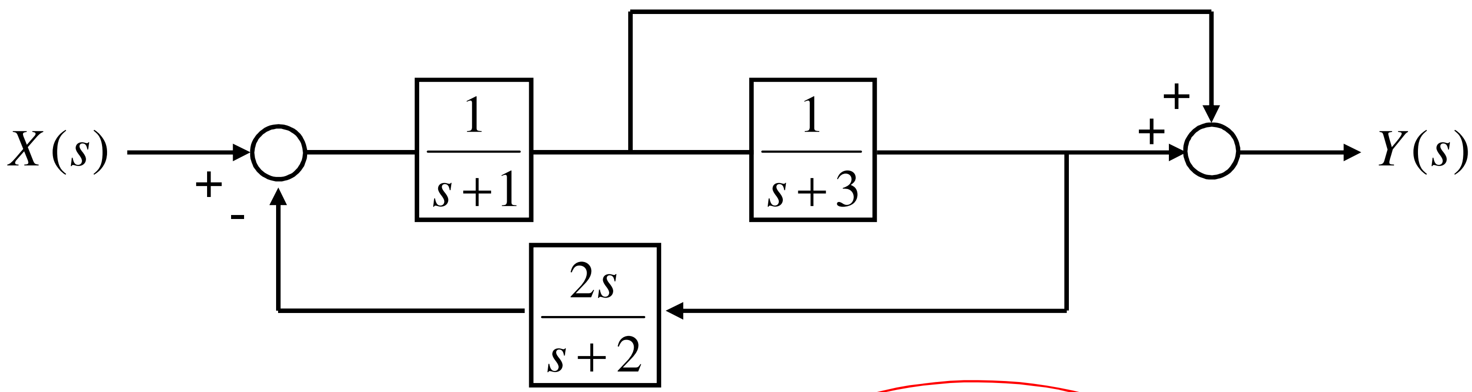


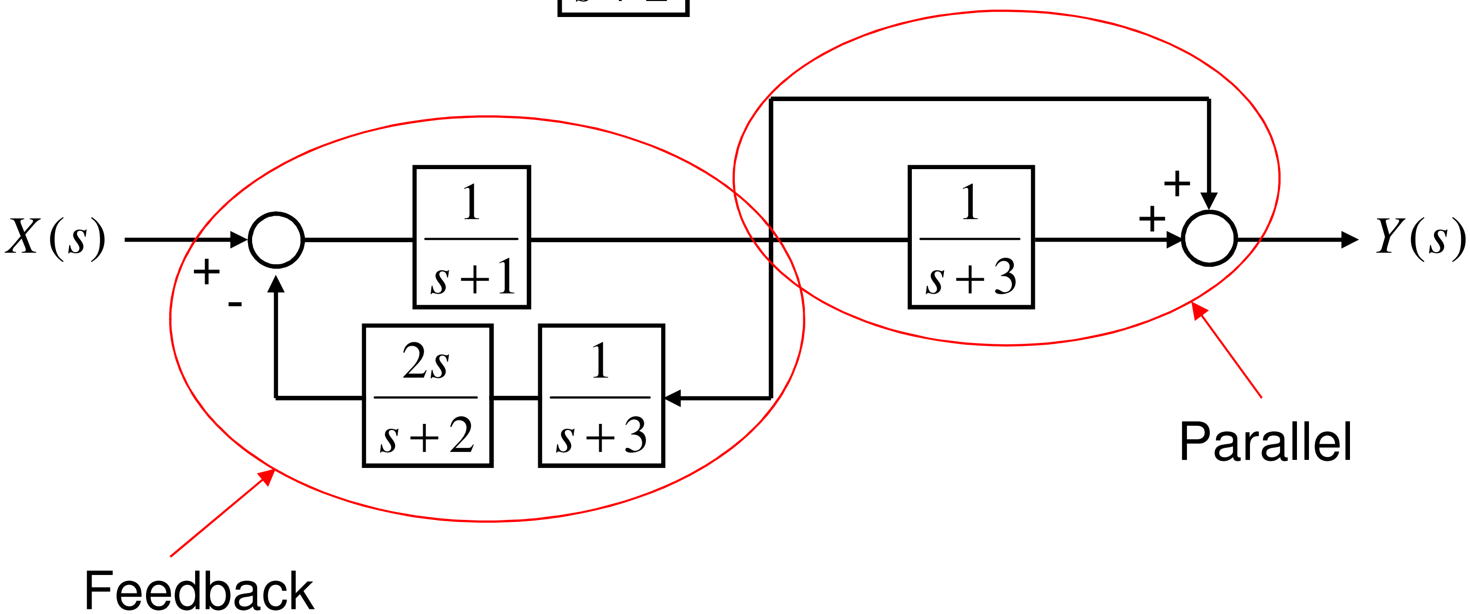


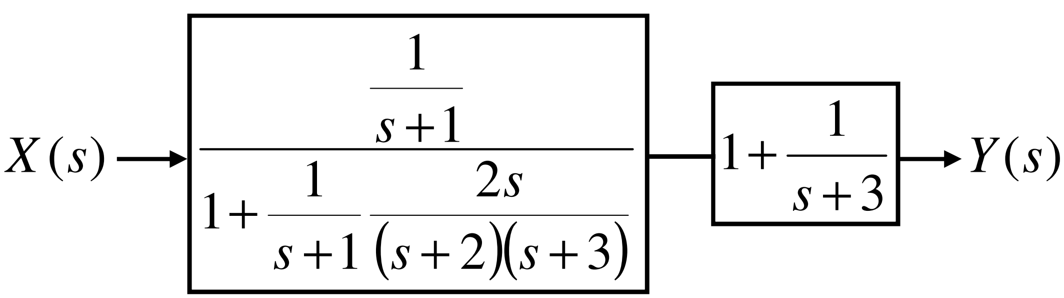


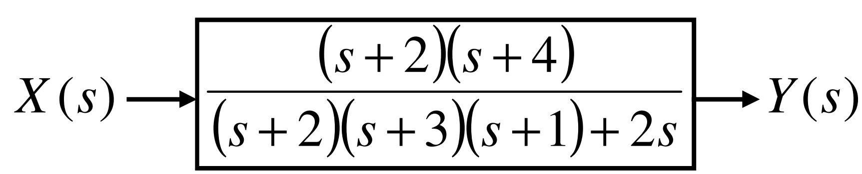
Feedback Block $\dfrac{\dfrac{1}{s}}{1+\left(\dfrac{1}{s}\right)(3)} = \dfrac{1}{s+3}$




X(s)
+
-
1/(s+1)
1/(s+3)
+
+
Y(s)
2s/(s+2)

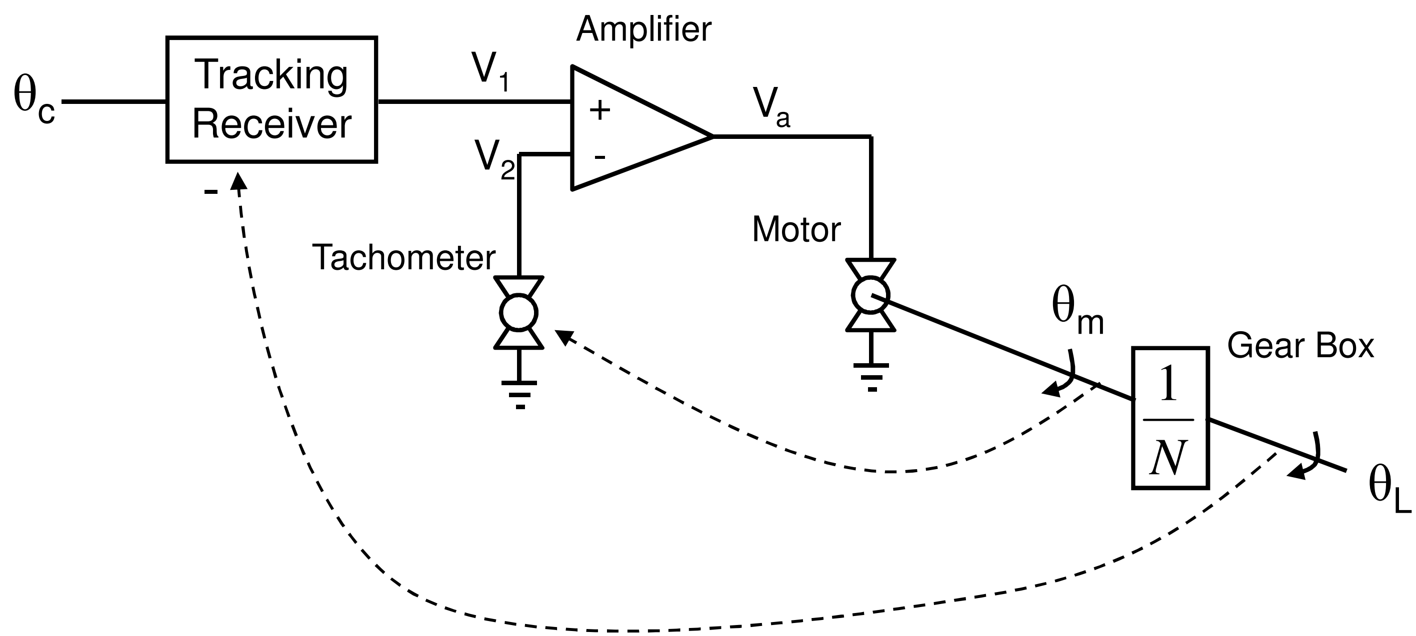

X(s)
+
-
1/(s+1)
2s/(s+2)
1/(s+3)
1/(s+3)
+
+
Y(s)
Feedback
Parallel


X(s)
$\dfrac{\frac{1}{s+1}}{1+\frac{1}{s+1}\frac{2s}{(s+2)(s+3)}}$
$1+\dfrac{1}{s+3}$
Y(s)

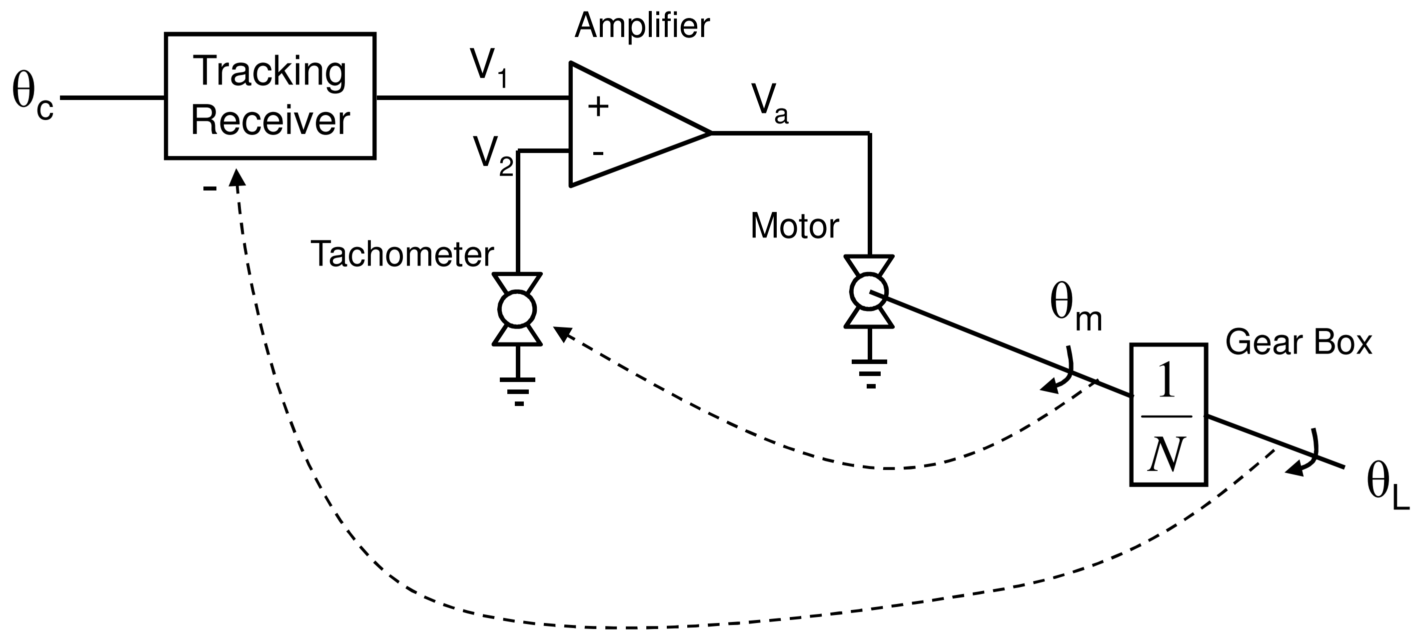

X(s)
$\dfrac{(s+2)(s+4)}{(s+2)(s+3)(s+1)+2s}$
Y(s)

Large System Example

Satellite Tracking System



- Trying to track a satellite with an antenna
- Input to the system is a control angle (i.e. the angle we want to point the antenna)
- Output is the actual angle of the antenna
- The overall system requires several components to complete its task



System Input
θ_c,
System Output
θ_L
System Signals
θ_c, θ_m, θ_L, V_1, V_2, V_a
System Constants
J, K_a, K_b, K_t, K_T

Subsystem Equations

1. Tracking Receiver
   Input → θ_c, θ_L
   Output → V_1

$$V_1 = H_1(s)[\theta_c - \theta_L]$$

$H_1(s)$ is a complicated transfer function

2. Amplifier
   Input → $V_1 - V_2$
   Output → V_a

$$V_a = K_a(V_1 - V_2)$$
$$H_2(s) = K_a$$

3. Motor
   Input → V_a
   Output → θ_m

$$J\ddot{\theta}_m + K_T K_b \dot{\theta}_m = K_T V_a$$

TF is $s^2 J\theta_m + sK_T K_b \theta_m = K_T V_a$

$$H_3(s) = \frac{K_T}{s^2 J + sK_T K_b}$$

4. Tachometer
   Input → θ_m
   Output → V_2

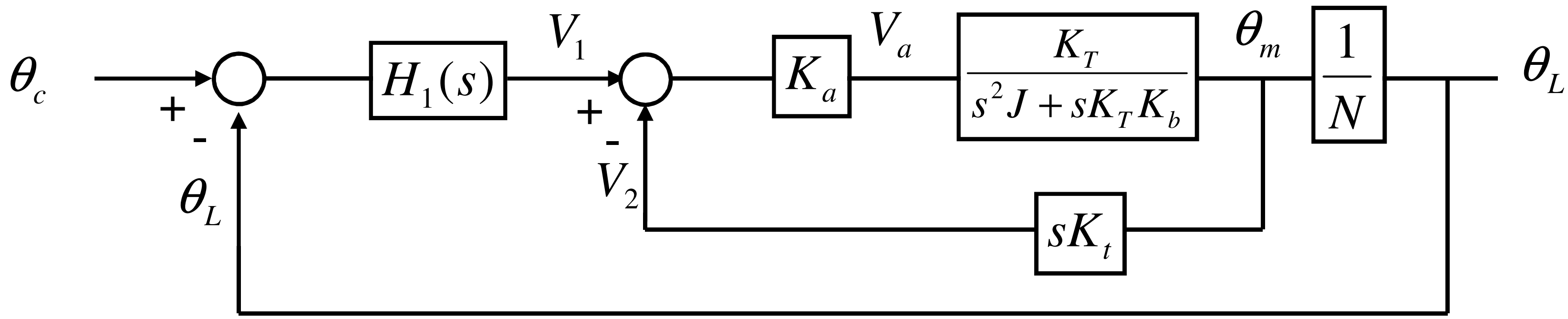
$$V_2 = K_t \dot{\theta}_m$$
$$H_4(s) = sK_t$$

5. Gear Box
   Input → θ_m
   Output → θ_L

$$\theta_L = \frac{1}{N}\theta_m$$
$$H_5(s) = \frac{1}{N}$$

Create a System Block Diagram

(Let us work from input to output)



Subsystem Equations

1. Tracking Receiver
   Input → θ_c, θ_L
   Output → V_1

   $$V_1 = H_1(s)[\theta_c - \theta_L]$$

   $H_1(s)$ is a complicated transfer function

2. Amplifier
   Input → $V_1 - V_2$
   Output → V_a

   $$H_2(s) = K_a$$

3. Amplifier
   Input → V_a
   Output → θ_m

   $$H_3(s) = \frac{K_T}{s^2 J + sK_T K_b}$$

4. Tachometer
   Input → θ_m
   Output → V_2

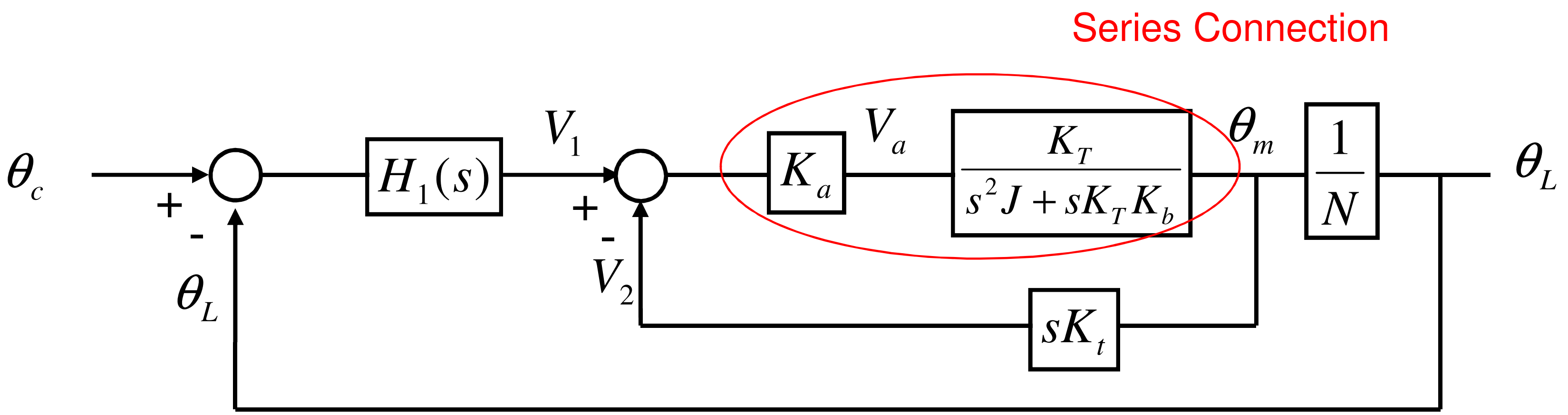
   $$H_4(s) = sK_t$$

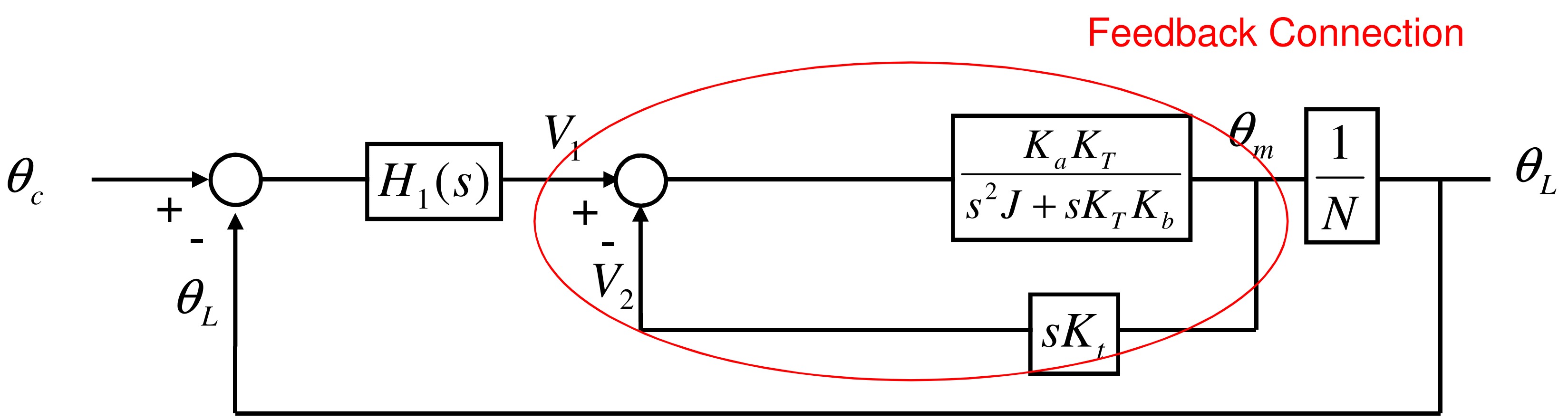
5. Gear Box
   Input → θ_m
   Output → θ_L

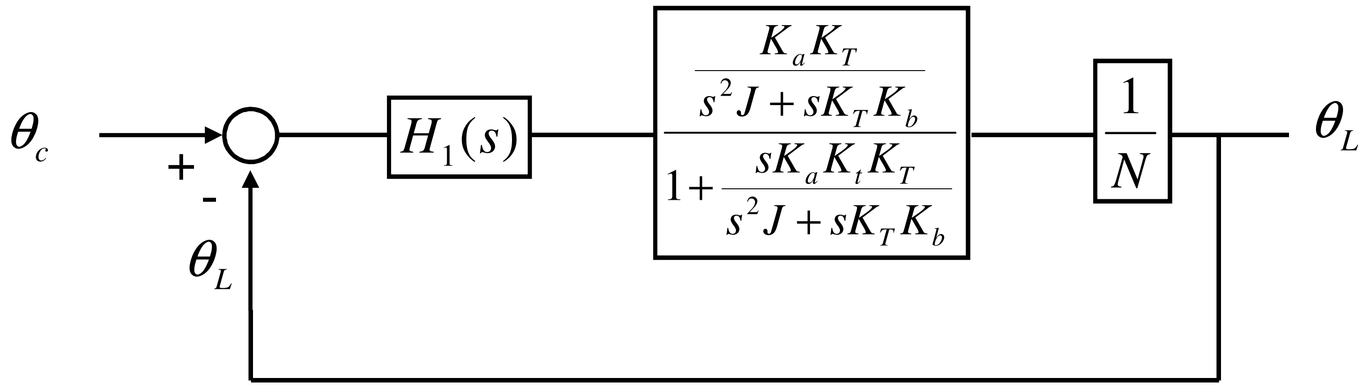
   $$H_5(s) = \frac{1}{N}$$

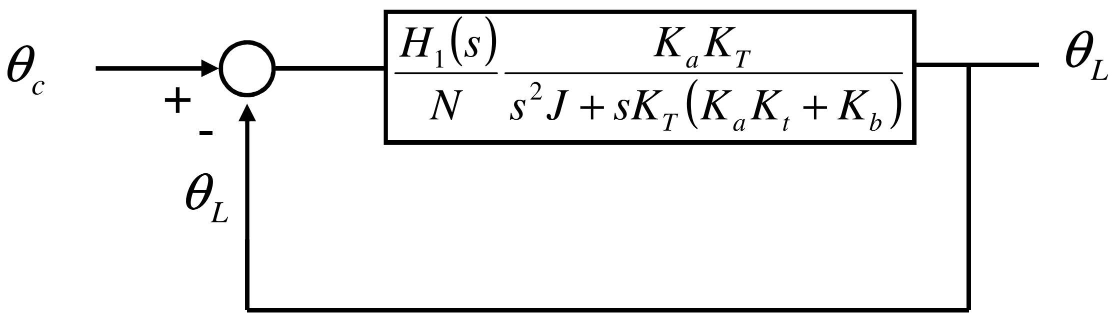
Connect like nodes

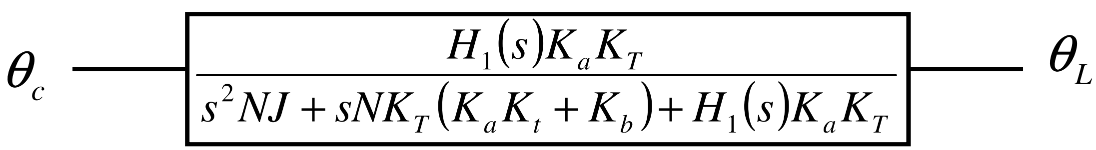
Perform Block-Diagram Reduction






θ_c
+
-
θ_L
$H_1(s)$
$\dfrac{\dfrac{K_a K_T}{s^2 J + s K_T K_b}}{1 + \dfrac{s K_a K_t K_T}{s^2 J + s K_T K_b}}$
$\dfrac{1}{N}$
θ_L


θ_c
+
-
θ_L
$\dfrac{H_1(s)}{N} \dfrac{K_a K_T}{s^2 J + s K_T (K_a K_t + K_b)}$
θ_L


θ_c
$\dfrac{H_1(s) K_a K_T}{s^2 N J + s N K_T (K_a K_t + K_b) + H_1(s) K_a K_T}$
θ_L