Operations on Continuous-Time Signals

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EE 327

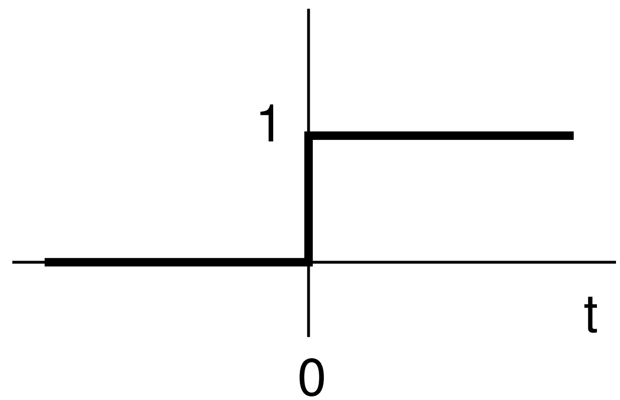
Continuous-Time Signals

- Continuous-Time Signals
  - Time is a continuous variable
  - The signal itself need not be continuous

- We will look at several common continuous-time signals and also operations that may be performed on them

Unit Step Function → u(t)

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



- Used to characterize systems
- We will use u(t) to illustrate the properties of continuous-time signals

Operations of CT Signals

1. Time Reversal $y(t) = x(-t)$
2. Time Shifting $y(t) = x(t-t_d)$
3. Amplitude Scaling $y(t) = Bx(t)$
4. Addition $y(t) = x_1(t) + x_2(t)$
5. Multiplication $y(t) = x_1(t)x_2(t)$
6. Time Scaling $y(t) = x(at)$

1. Time Reversal

- Flips the signal about the y axis
- y(t) = x(-t)

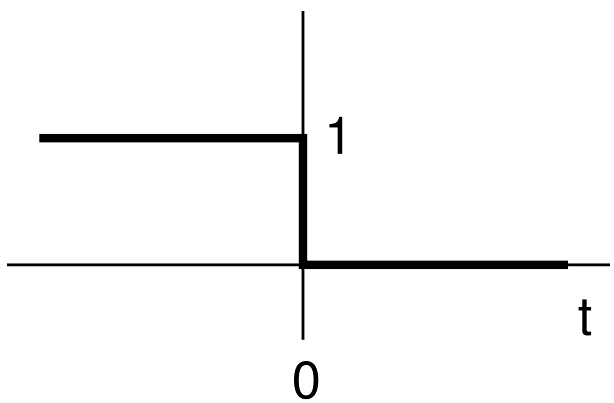
ex. Let x(t) = u(t), and perform time reversal

<u>Solution</u>: Find y(t) = u(-t)
Let "a" be the argument of the step function → u(a)

$$u(a) = \begin{cases} 1 & a \geq 0 \\ 0 & a < 0 \end{cases}$$

Let a = -t, and plug in this value of "a"

$$u(-t) = \begin{cases} 1 & t \leq 0 \\ 0 & t > 0 \end{cases}$$



2. Time Shifting / Delay

- $y(t) = x(t - t_d)$
- Shifts the signal left or right
- Shifts the origin of the signal to t_d

- Rule → Set $(t - t_d) = 0$ (set the argument equal to zero)

  → Then move the origin of $x(t)$ to t_d

- Effectively, $y(t)$ equals what $x(t)$ was t_d seconds ago

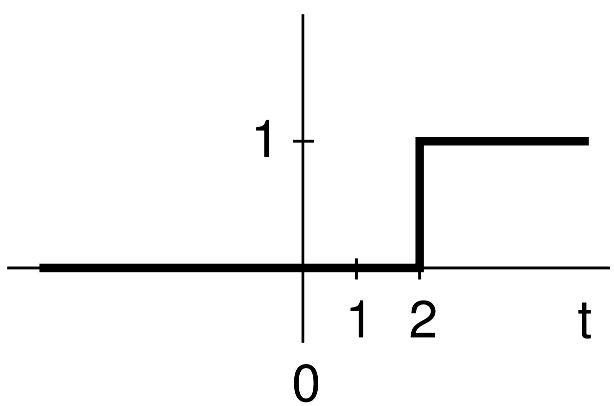
2. Time Shifting / Delay

ex. Sketch y(t) = u(t – 2)

Method 1

Let "a" be the argument of "u"

$$y(a)=\begin{cases}1 & a\geq 0\\ 0 & a<0\end{cases}=\begin{cases}1 & t-2\geq 0\\ 0 & t-2<0\end{cases}=\begin{cases}1 & t\geq 2\\ 0 & t<2\end{cases}$$



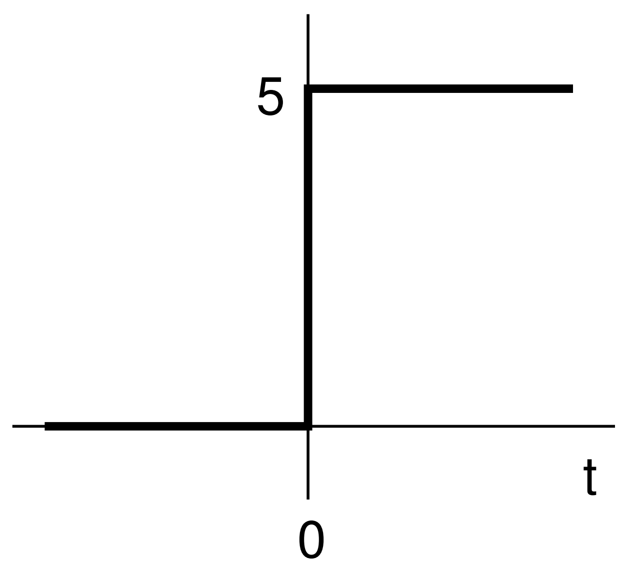
Method 2 (by inspection)

Simply shift the origin to t_d = 2

3. Amplitude Scaling

- Multiply the entire signal by a constant value
- $y(t) = Bx(t)$

ex. Sketch $y(t) = 5u(t)$



4. Addition of Signals

- Point-by-point addition of multiple signals
- Move from left to right (or vice versa), and add the value of each signal together to achieve the final signal
- $y(t) = x_1(t) + x_2(t)$

- Graphical solution
  – Plot each individual portion of the signal (break into parts)
  – Add the signals point by point

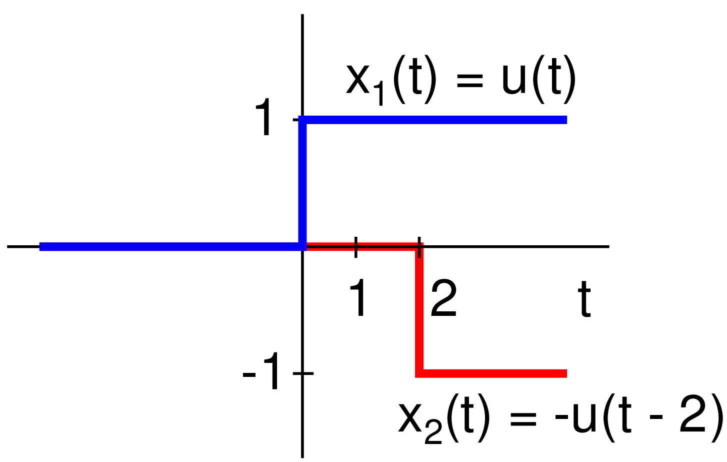
4. Addition of Signals

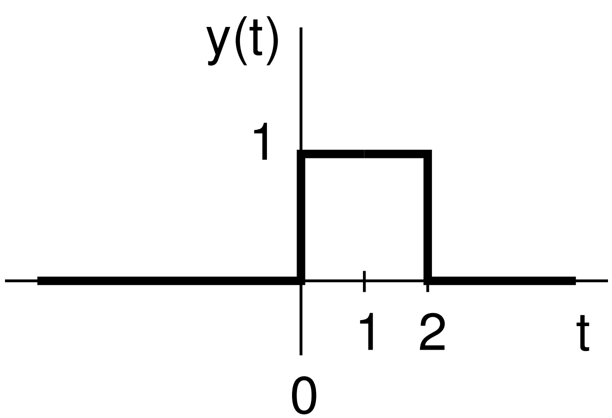
ex. Sketch $y(t) = u(t) – u(t – 2)$

First, plot each of the portions of this signal separately

- $x_1(t) = u(t)$ → Simply a step signal
- $x_2(t) = –u(t-2)$ → Delayed step signal, multiplied by -1

Then, move from one side to the other, and add their instantaneous values





5. Multiplication of Signals

- Point-by-point multiplication of the values of each signal
- $y(t) = x_1(t)x_2(t)$

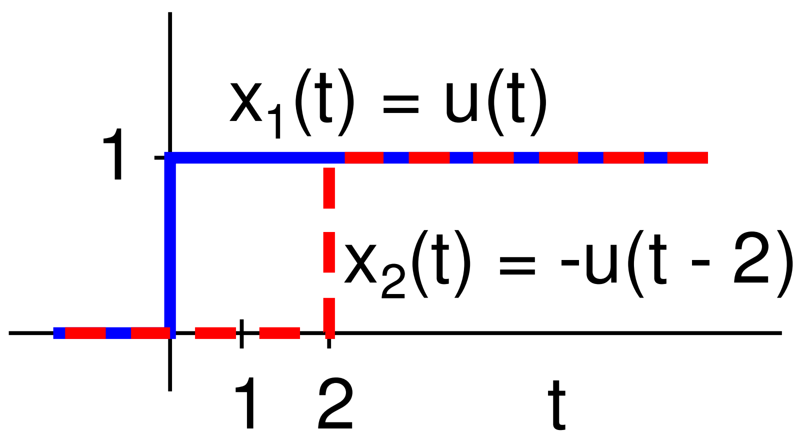
- Graphical solution
  - Plot each individual portion of the signal (break into parts)
  - Multiply the signals point by point

5. Multiplication of Signals

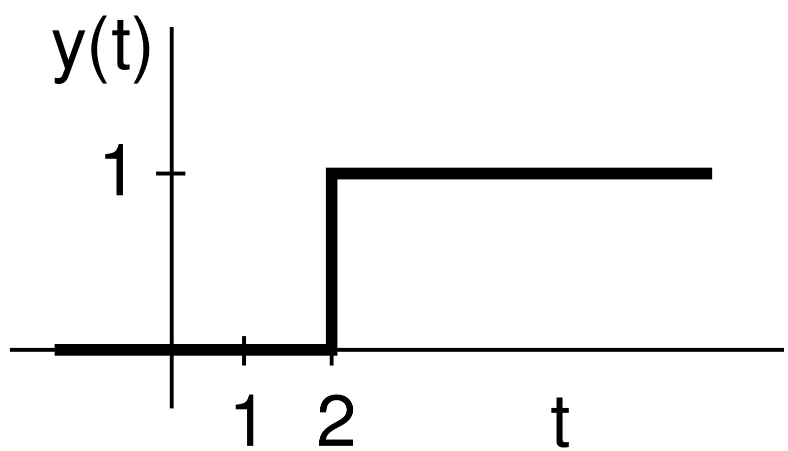
ex. Sketch $y(t) = u(t) \cdot u(t-2)$

First, plot each of the portions of this signal separately

- $x_1(t) = u(t)$ → Simply a step signal
- $x_2(t) = u(t\text{-}2)$ → Delayed step signal



Then, move from one side to the other, and multiply instantaneous values



6. Time Scaling

- Speed up or slow down a signal
- Multiply the time in the argument by a constant
- $y(t) = x(at)$

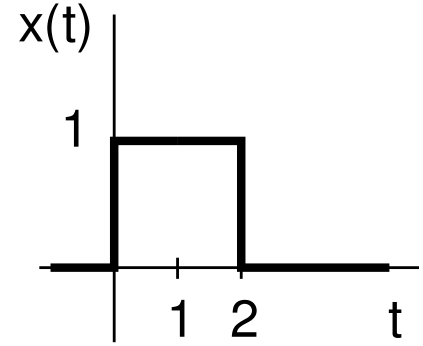
  $|a| > 1$ → Speed up x(t) by a factor of "a"

  $|a| < 1$ → Slow down x(t) by a factor of "a"
- Key → Replace all instances of "t" with "at"

6. Time Scaling

ex. Let $x(t) = u(t) - u(t-2)$
Sketch $y(t) = x(2t)$

First, plot x(t)

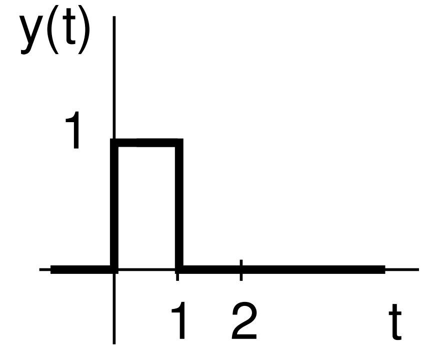


Replace all t's with 2t

$y(t) = x(2t) = u(2t) - u(2t-2)$

Turns on at
$2t \geq 0$
$t \geq 0$
No change

Turns on at
$2t - 2 \geq 0$
$t \geq 1$

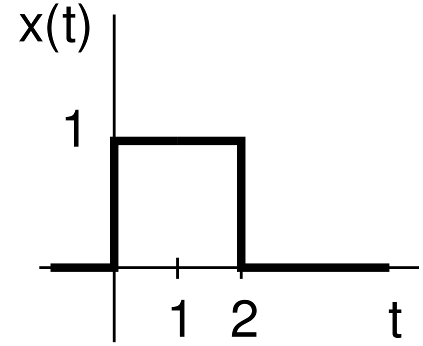


This has effectively "sped up" x(t) by a factor of 2
(What occurred at t=2 now occurs at t=2/2=1)

6. Time Scaling

ex. Let $x(t) = u(t) - u(t - 2)$
Sketch $y(t) = x(t/2)$

First, plot x(t)

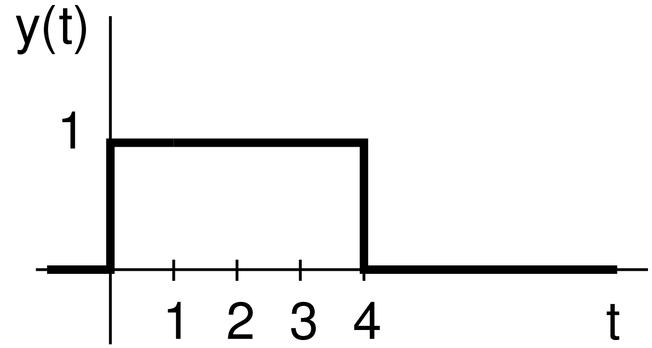


Replace all t's with t/2

$y(t) = x(t/2) = u(t/2) - u((t/2) - 2)$

Turns on at
$t/2 \geq 0$
$t \geq 0$
No change

Turns on at
$t/2 - 2 \geq 0$
$t \geq 4$



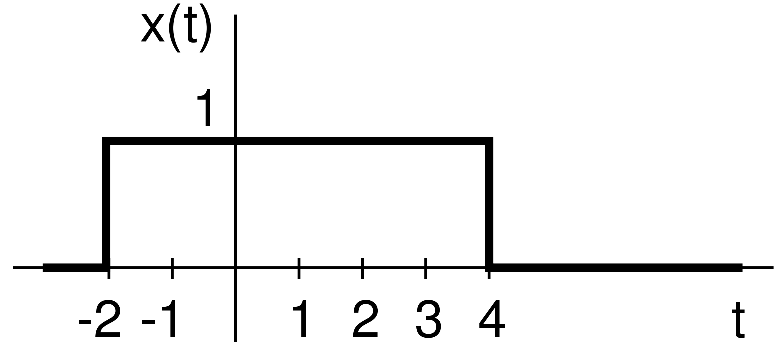
This has effectively "slowed down" x(t) by a factor of 2
(What occurred at t=1 now occurs at t=2)

Combinations of Operations

- Combinations of operations on signals
  - Easier to Determine the final signal in stages
  - Create intermediary signals in which one operation is performed

ex. Time Scale and Time Shift

Let $x(t) = u(t + 2) - u(t - 4)$
Sketch $y(t) = x(2t - 2)$



Can perform either operation first

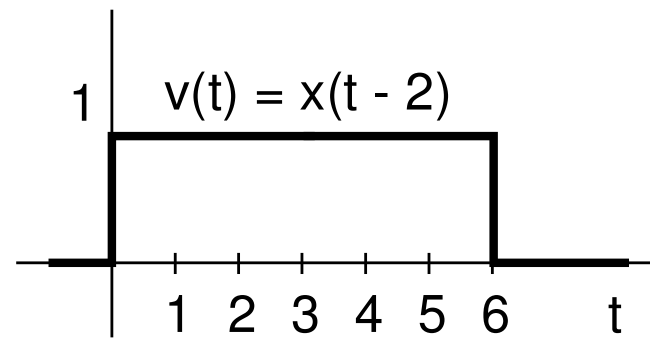
Method 1 → Shift then scale

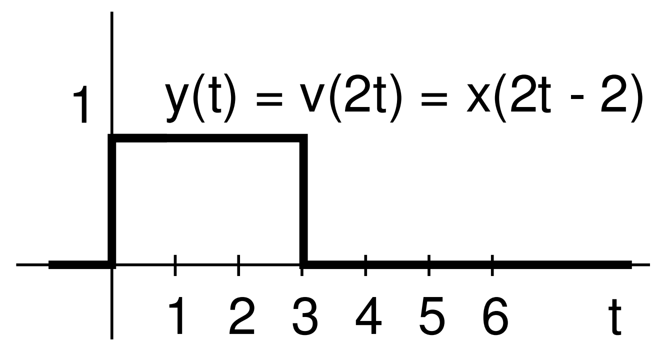
Let $v(t) = x(t - b)$ → Time shifted version of $x(t)$

Then $y(t) = v(at) = x(at - b)$

Replace "t" with the argument of "v"

Match up "a" and "b" to what is given in the problem statement



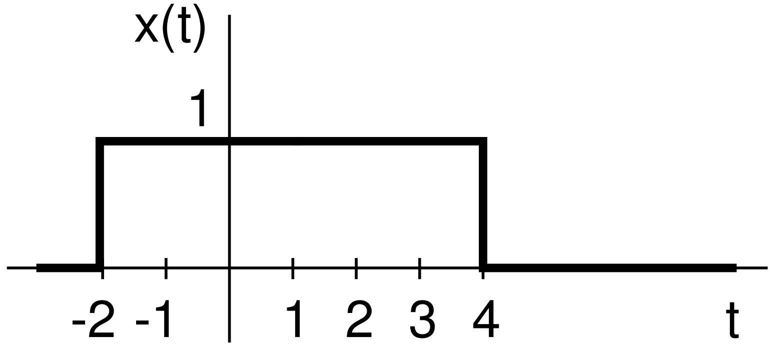


$at - b = 2t - 2$
(Match powers of t)
$a = 2$
$b = 2$

Therefore, shift by 2, then scale by 2

ex. Time Scale and Time Shift

Let $x(t) = u(t + 2) - u(t - 4)$
Sketch $y(t) = x(2t - 2)$



Can perform either operation first

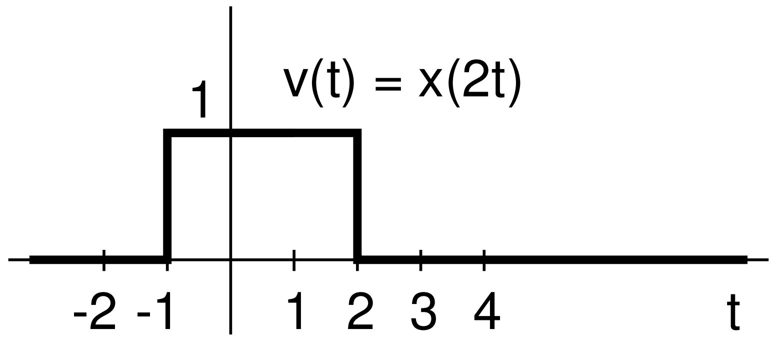
Method 2 → Scale then shift

Let $v(t) = x(at)$ → Time scaled version of $x(t)$

Then $y(t) = v(t - b) = x(a(t - b)) = x(at - ab)$

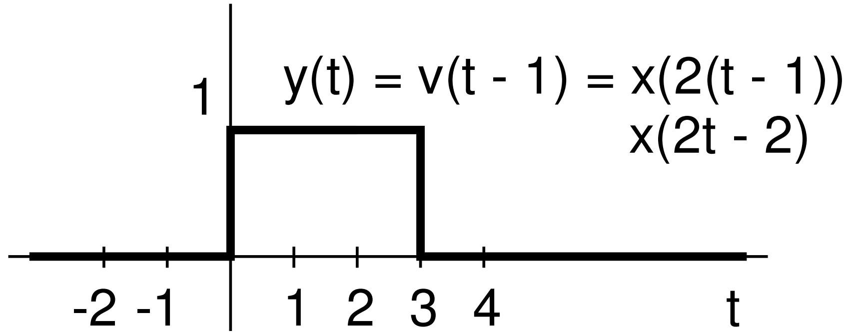
Replace "t" with the argument of "v"

Match up "a" and "b" to what is given in the problem statement



$at - ab = 2t - 2$
(Match powers of t)
$a = 2$
$ab = 2$, $b = 1$

Therefore, scale by 2, then shift by 1



ex. Time Scale and Time Shift

- Note – The results are the same
- Note – The value of b in Method 2 is a scaled version of the time delay
  - $t_d = 2$
  - Time scale factor = 2
  - New scale factor = 2/2 = 1