EE 327 Signals and Systems 1
Final Exam

Name Solutions

December 17, 2013

RULES

This is a closed book, closed notes test. You are, however, allowed one piece of paper (front and back) for notes and definitions, but no sample problems. You are also permitted to use a calculator. Additionally, tables of common transforms have been provided at the end of this test.

You have 2 hours to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	10	
2	10	
3	10	
4	30	
5	25	
6	10	
7	10	
Total	100/105	

PROBLEM 1 (10 Points)

Fill in the blank with the appropriate response. If not enough information is available to correctly fill in the blank, indicate that there is "not enough information." (1 Point Each)

A/an unit step function has a value of 0 until time t = 0 seconds, and then from time t = 0 to infinity, it has a value of 1.

A continuous-time system is unstable if all its poles are in the right-half plane.

If all the poles of a discrete-time system are located inside the unit circle, then the system is stable.

A/an allpass filter passes all frequencies.

You should use a/an bandstop / bandreject / notch filter to remove 60Hz noise, but keep everything else unattenuated.

Convolution in the time domain is equivalent to multiplication in the Laplace domain.

A/an transfer function is the Laplace transform of an impulse response.

The steady-state part of a system response is what is left over when the transient dies out.

bode() is the name of the function in Matlab to create a frequency response plot.

lsim() is the name of the function in Matlab to simulate state-space systems.

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PROBLEM 2 (10 Points)

A. Determine the properties of the following system. Choose one property from each column, and circle the appropriate property. (1 Point Each)

$$y[n] = x[-n]$$

Column 1	Column 2	Column 3	Column 4	Column 5
Causal	Has Memory	Stable (BIBO)	Linear	Time Invariant
OR	OR	OR	OR	OR
Non Causal	Memoryless	Unstable	Nonlinear	Time Varying

Linearity

Homogeneity

$x[n] \mapsto x[-n] = y[n]$

$ax[n] \mapsto ax[-n] = ay[n]$ ✓

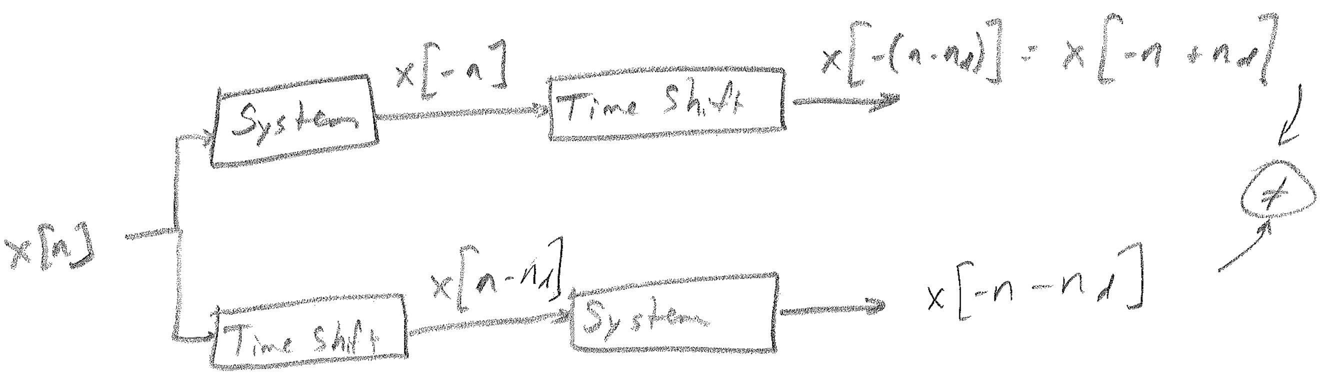
Additivity

$x_1[n] \mapsto x_1[-n] = y_1[n]$

$x_2[n] \mapsto x_2[-n] = y_2[n]$

$x_1[n] + x_2[n] \mapsto x_1[-n] + x_2[-n]$

$= y_1[n] + y_2[n]$ ✓



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B. Determine the properties of the following system. Choose one property from each column, and circle the appropriate property. (1 Point Each)

$$y(t) = x(t)x(t+1)$$

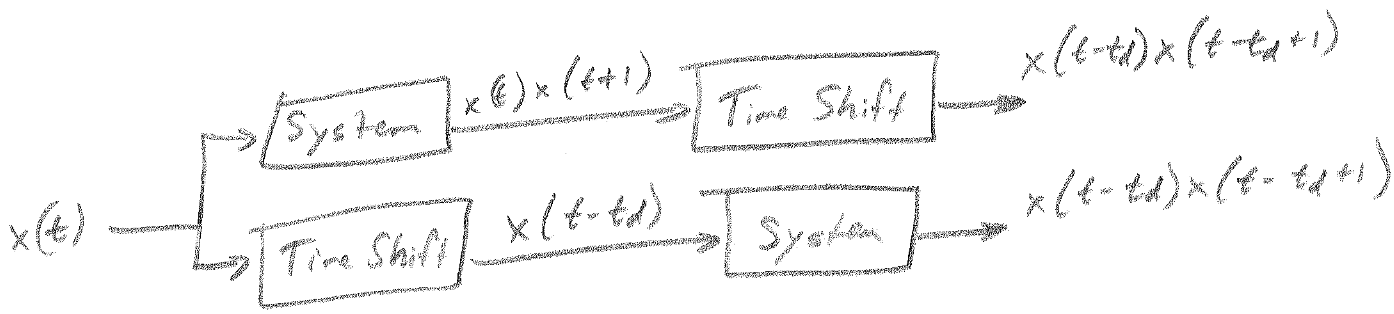
Column 1	Column 2	Column 3	Column 4	Column 5
Causal	Has Memory	Stable (BIBO)	Linear	Time Invariant
OR	OR	OR	OR	OR
Non Causal	Memoryless	Unstable	Nonlinear	Time Varying

Linearity

Homogeneity ×

$$x(t) \mapsto x(t)\,x(t+1) = y(t)$$

$$a\,x(t) \mapsto [a\,x(t)][a\,x(t+1)] = a^2 y(t)$$



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PROBLEM 3 (10 Points)

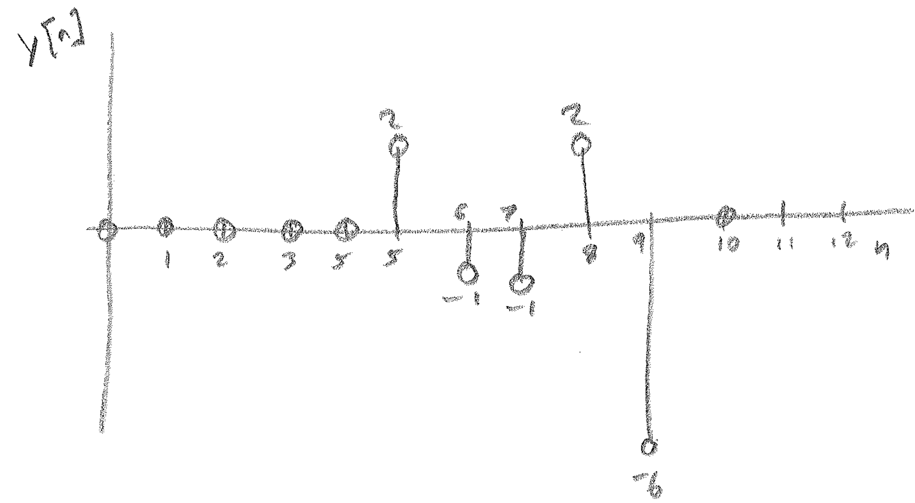
Determine the output of a system if the input and impulse response are given by the following. You must write an <u>expression</u> for the output and also provide a <u>sketch</u> of the output.

$x[n] = 2\delta[n-3] - \delta[n-4] + 3\delta[n-5]$ $N = 3$

$h[n] = \delta[n-2] - 2\delta[n-4]$ $M = 2$ $N + M = 5$

2	−1	3		
1	0	−2		
2	−1	3		
	0	0	0	
		−4	2	−6
2	−1	−1	2	−6

$y[n] = 2\delta[n-5] - \delta[n-6] - \delta[n-7] + 2\delta[n-8] - 6\delta[n-9]$



PROBLEM 4 (30 Points)

Determine a <u>transfer function</u> for each of the following systems. Write all transfer functions as rational functions of either "s" or "z" – whichever is appropriate. Simply where needed. (5 Points Each)

A. $h(t) = 3e^{-4t}\cos(100t)u(t)$ $\quad h(t) \Leftrightarrow H(s)$

$$H(s) = \frac{3(s+4)}{(s+4)^2 + (100)^2} = \frac{3s+12}{s^2+8s+10{,}016}$$

B. $h[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3]$

$$H(z) = 1 + 2z^{-1} + 2z^{-2} + z^{-3} = \frac{z^3 + 2z^2 + 2z + 1}{z^3}$$

C. $\dddot{y} - \ddot{y} + 4\dot{y} = 2x$

$$s^3 Y(s) - s^2 Y(s) + 4s Y(s) = 2 X(s)$$

$$Y(s)\left(s^3 - s^2 + 4s\right) = 2X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s^3 - s^2 + 4s} = \frac{2}{(s)\left(s^2 - s + 4\right)}$$

D. For this system, when the input is $X(s) = \dfrac{1}{s+10}$, the output is $Y(s) = \dfrac{2}{s^2 + 11s + 10}$.

$$Y(s) = H(s)X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{\dfrac{2}{s^2 + 11s + 10}}{\dfrac{1}{s+10}} = \frac{2(s+10)}{s^2 + 11s + 10} =$$

$$= \frac{2\cancel{(s+10)}}{(s+1)\cancel{(s+10)}} = \frac{2}{s+1}$$

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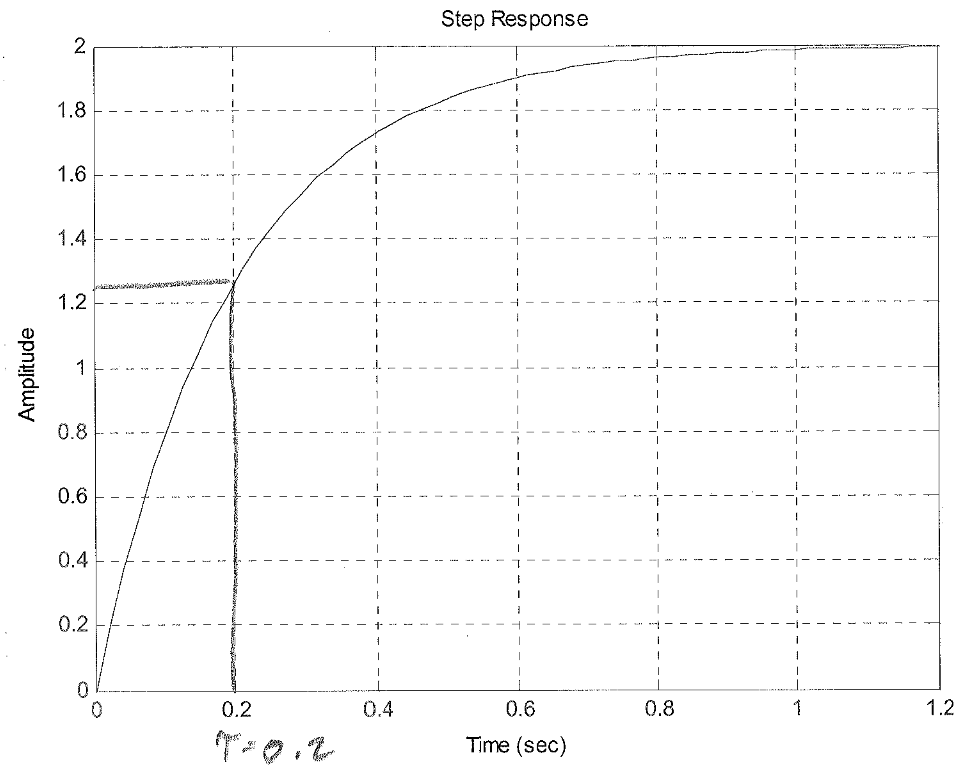
E. The following plot is the step response of a first-order system.



← Final Value = 2

$\tau \rightarrow$ time to get to 63% of Final value

$\therefore$ time to get to 1.26

$$H(s) = A \frac{1}{s + \frac{1}{\tau}} = A \frac{1}{\quad} = A \frac{1}{s + \frac{1}{\tau}}$$

$$\frac{A}{\frac{1}{\tau}} = \text{Final Value} = 2$$

$$A = \frac{2}{\tau} = \frac{2}{0.2} = 10$$

$$H(s) = \frac{10}{s + \frac{1}{0.2}} = \frac{10}{s + 5}$$

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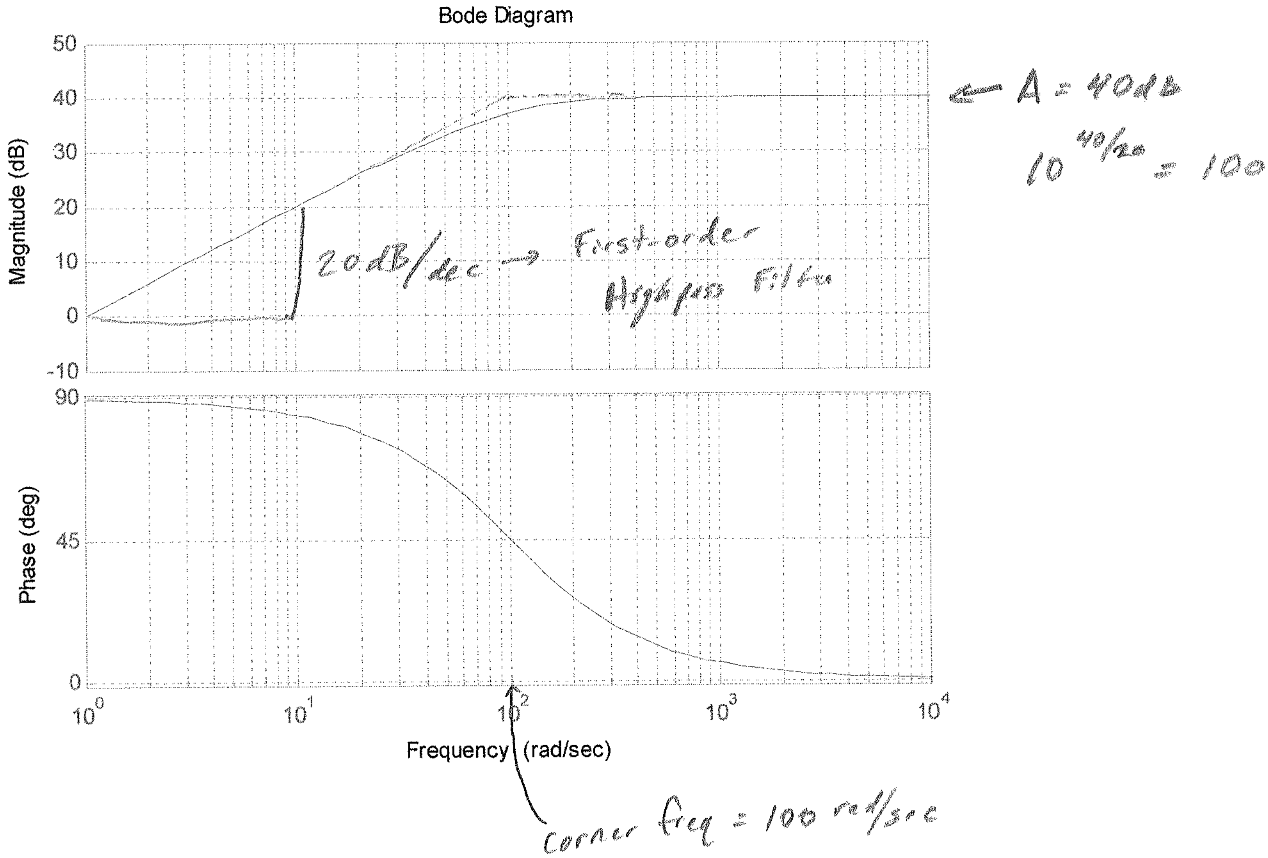
F. The following plot is the frequency response of a system.



First-order highpass filter

$$H(s) = A \frac{\frac{s}{\omega_c}}{\frac{s}{\omega_c}+1} = (100)\frac{\frac{s}{100}}{\frac{s}{100}+1} = \frac{s}{\frac{s}{100}+1}$$

$$= \frac{100\,s}{s+100}$$

Name *Solutions*

PROBLEM 5 (25 Points)

A discrete-time system is given by the following transfer function.

$$H(z) = \frac{z+0.5}{z-0.5}$$

Determine the frequency response of this system (find analytic expressions for both the magnitude and phase and place these expressions on the lines provided). Provide a rough sketch of the magnitude response of this system and be sure to include the values of all local maxima and minima. Additionally, find the steady-state response (y[n]) to the following sinusoidal input (and place your answer on the line that has been provided).

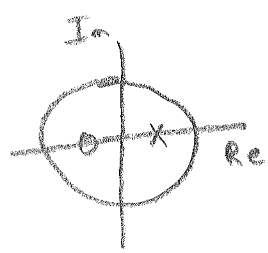
$$x[n] = 2\cos\left(\frac{\pi}{2}n\right)$$

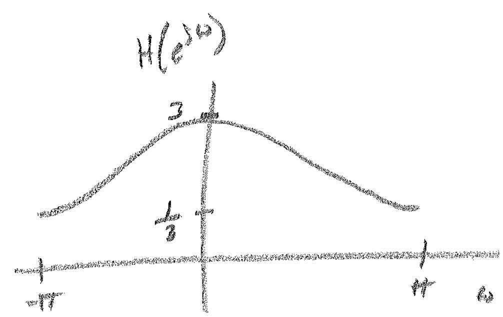
Finally, determine what type of filtering operation that the filtering operation performs, and determine if this is an FIR or IIR filter (with a brief explanation of how you know).

Again, be sure to place your answers in the designated areas (even though your work may be outside that area), or else you will not receive credit.

Magnitude Frequency Response $|H(e^{j\omega})| = \sqrt{\dfrac{1.25 + \cos(\omega)}{1.25 - \cos(\omega)}}$

Magnitude Frequency Response Sketch (Sketch in area below)



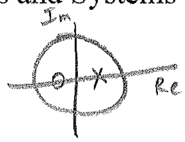


Phase Frequency Response $\angle H(e^{j\omega}) = \tan^{-1}\left(\dfrac{\sin(\omega)}{\cos(\omega)+\frac{1}{2}}\right) - \tan^{-1}\left(\dfrac{\sin(\omega)}{\cos(\omega)-\frac{1}{2}}\right)$

Steady-State Output $y(t) = 2\cos\left(\frac{\pi}{2}n - 0.9273\right)$

Type of Filtering Operation *Lowpass Filter*

FIR or IIR (and how you know) *IIR – The pole is not at the origin*



Work Page for Problem 5

$$H(z) = \frac{z+0.5}{z-0.5} \longrightarrow H(e^{j\omega}) = \frac{e^{j\omega}+0.5}{e^{j\omega}-0.5}$$

$$|H(e^{j\omega})|^2 = \sqrt{\frac{e^{j\omega}+\frac{1}{2}}{e^{j\omega}-\frac{1}{2}} \cdot \frac{e^{-j\omega}+\frac{1}{2}}{e^{-j\omega}-\frac{1}{2}}} = \sqrt{\frac{1+\frac{1}{2}e^{j\omega}+\frac{1}{2}e^{-j\omega}+\frac{1}{4}}{1-\frac{1}{2}e^{j\omega}-\frac{1}{2}e^{-j\omega}+\frac{1}{4}}} =$$

$$= \sqrt{\frac{1.25+\cos(\omega)}{1.25-\cos(\omega)}}$$

Mag at max $\rightarrow \omega = 0$ $\quad |H(e^{j\omega})| = \sqrt{\frac{1.25+\cos(0)}{1.25-\cos(0)}} = \sqrt{\frac{2.25}{0.25}} = 3$

Mag at min $\rightarrow \omega = \pi$ $\quad |H(e^{j\omega})| = \sqrt{\frac{1.25+\cos(\pi)}{1.25-\cos(\pi)}} = \sqrt{\frac{0.25}{2.25}} = \frac{1}{3}$

$$\angle H(e^{j\omega}) = \angle\left(\frac{\cos(\omega)+j\sin(\omega)+0.5}{\cos(\omega)+j\sin(\omega)-0.5}\right) =$$

$$= \tan^{-1}\left(\frac{\sin(\omega)}{\cos(\omega)+0.5}\right) - \tan^{-1}\left(\frac{\sin(\omega)}{\cos(\omega)-0.5}\right)$$

Magnitude and Phase at $\omega = \frac{\pi}{2}$

$$|H(e^{j\frac{\pi}{2}})| = \sqrt{\frac{1.25}{1.25}} = 1 \qquad \angle H(e^{j\frac{\pi}{2}}) = \tan^{-1}\left(\frac{1}{\frac{1}{2}}\right) - \tan^{-1}\left(\frac{1}{-\frac{1}{2}}\right) =$$

$$= \tan^{-1}(2) - \tan^{-1}(-2)$$

$$= 1.1071 - (\pi - \tan^{-1}(2)) =$$

$$= 1.1071 - 2.0344 = -0.9273$$

Steady-state Output

$$y(t) = 2|H(e^{j\frac{\pi}{2}})|\cos\left(\frac{\pi}{2}n + \angle H(e^{j\frac{\pi}{2}})\right) =$$

$$= 2\cos\left(\frac{\pi}{2}n - 0.9273\right)$$

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PROBLEM 6 (10 Points)

A continuous-time systems is defined by the following differential equation

$$\dddot{y} - \ddot{y} + 4\dot{y} = 2v$$

where y is the output, and v is the input. Determine a set of state-space equations that model this system.

Let $x_1 = y$ | $\dot{x}_1 = \dot{y} = x_2$

$x_2 = \dot{y}$ | $\dot{x}_2 = \ddot{y} = x_3$

$x_3 = \ddot{y}$ | $\dot{x}_3 = \dddot{y} = \ddot{y} - 4\dot{y} + 2v$

$= x_3 - 4x_2 + 2v$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} v$$

$$[y] = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]\, v$$

PROBLEM 7 (10 Points)

A system is defined by the following state-space equations.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} v$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1] v$$

Determine a transfer function of this system.

$$H(s) = C(sI - A)^{-1} B + D$$

$$(sI - A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} s-2 & -1 \\ 1 & s-2 \end{bmatrix}$$

$$(sI - A)^{-1} = \begin{bmatrix} s-2 & 1 \\ -1 & s-2 \end{bmatrix} \frac{1}{(s-2)^2 + 1} \qquad \begin{bmatrix} s-2 & 1 \\ -1 & s-2 \end{bmatrix} \frac{1}{s^2 - 4s - 3}$$

$$H(s) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s-2 & 1 \\ -1 & s-2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{s^2 - 4s - 3} + [1] =$$

$$= \begin{bmatrix} s-2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{s^2 - 4s - 3} + [1] =$$

$$= \frac{s-1}{s^2 - 4s - 3} + 1 = \frac{s-1}{s^2 - 4s - 3} + \frac{s^2 - 4s - 3}{s^2 - 4s - 3} =$$

$$= \frac{s^2 - 3s - 4}{s^2 - 4s - 3}$$