EE 327 Signals and Systems 1 Name Solutions

Test 1 September 19, 2014

RULES

This is a closed book, closed notes test. You are, however, allowed one half of one piece of paper (front side only) for notes and definitions, but no sample problems. You must staple your equations sheet to the back of your test when you hand your test in.

You are permitted to use a calculator.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	20	
2	5	
3	24	
4	20	
5	31	
Total	100	

EE 327 Signals and Systems 1 Name Solutions
Test 1 September 19, 2014

PROBLEM 1 (20 Points)

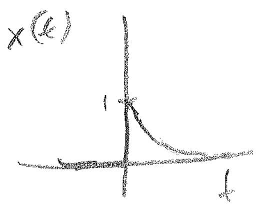
A. Given that

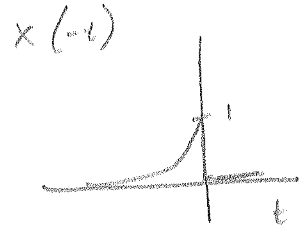
$$x(t) = e^{-t}u(t)$$

plot the following signal. Clearly label all the important points. Plot from t = –10 to t = 10.

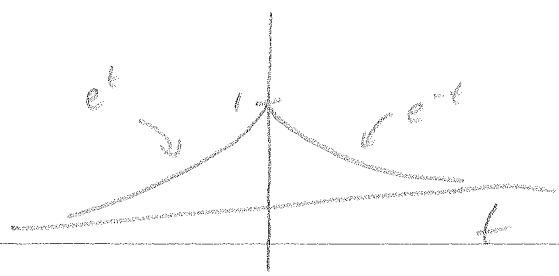
$$y(t) = x(t) + x(-t)$$

(10 Points)





$\therefore y(t) = x(t) + x(-t)$



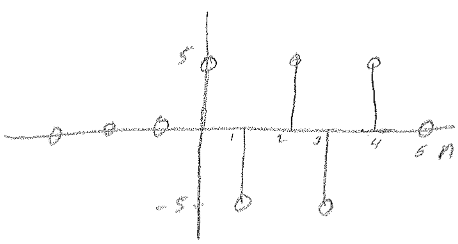
EE 327 Signals and Systems 1 Name Solutions
Test 1 September 19, 2014

B. Plot the following signal. Clearly label all the important points. Plot from n = –10 to n = 10.

$$x[n] = 5\cos(\pi\ n)(u[n] - u[n-5]) + n\delta[n+6]$$

(10 Points)

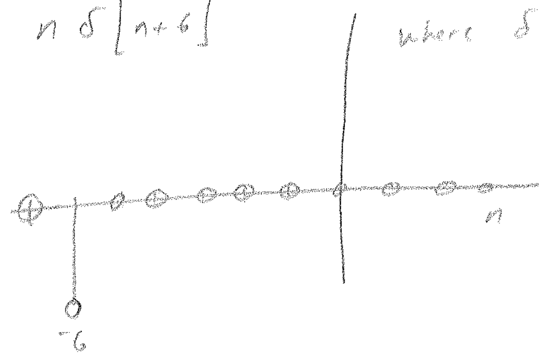
$5\cos(\pi n)\left(u[n] - u[n-5]\right)$



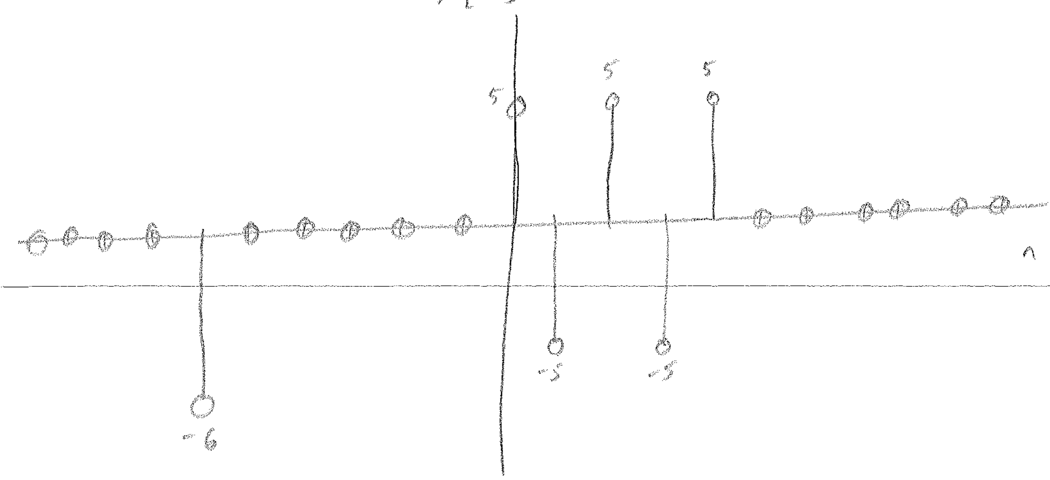
+

$n\,\delta[n+6]$

at $n = -6$
$n = -6$
where $\delta[n+6] = 1$



$x[n]$



EE 327 Signals and Systems 1 Name *Solutions*
Test 1 September 19, 2014

PROBLEM 2 (5 Points)

The following continuous-time signal is to be sampled.

$$x(t) = 10\cos(20t) + 11\sin\left(100t + \frac{\pi}{2}\right)$$

Determine the Nyquist rate, f_{NS}, for sampling this signal.

$\omega = 20$ rad/sec $\omega = 100$ rad/sec

For the Nyquist rate, you need to look to the highest frequency component, which is $\omega = 100$ rad/sec

$\therefore$ The highest frequency in Hz is

$$f_{highest} = \frac{\omega_{highest}}{2\pi} = \frac{100 \text{ rad/sec}}{2\pi} = 15.92 \text{ Hz}$$

$$f_{NS} = 2 f_{highest} = \boxed{31.82 \text{ Hz}}$$

EE 327 Signals and Systems 1 Name Solutions

Test 1 September 19, 2014

PROBLEM 3 (24 Points)

Determine the properties of the following system. Choose one property from each column, and circle the appropriate property.

present value past value

$$y(t) = x(t)x(t-1)$$

Column 1	Column 2	Column 3	Column 4	Column 5	Column 6
Causal	Has Memory	Stable (BIBO)	Linear	Time Invariant	LTI
OR	OR	OR	OR	OR	OR
Non Causal	Memoryless	Unstable	Nonlinear	Time Varying	Not LTI

Only looks to present and past values of input

if $|x(t)| \le M$

then $|y(t)| \le (M)(M) = M^2$ which is a finite bound

Linearity Tests

Homogeneity Test

$x(t) \mapsto x(t)x(t-1) = y(t)$

$a\,x(t) \mapsto a\,x(t)\,a\,x(t-1) = a^2 x(t)x(t-1) \neq a\,y(t)$

$\therefore$ Nonlinear

Additivity Test

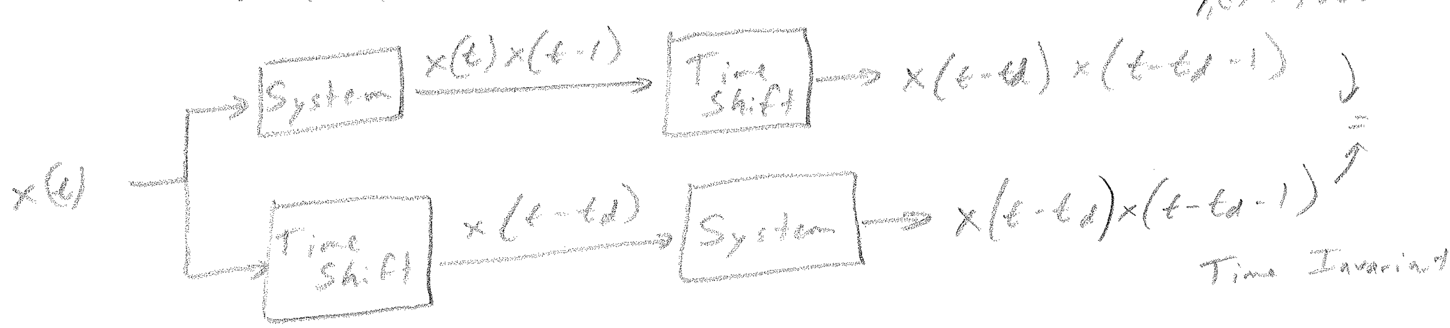
$x_1(t) \mapsto x_1(t)\,x_1(t-1) = y_1(t)$

$x_2(t) \mapsto x_2(t)\,x_2(t-1) = y_2(t)$

$x_1(t) + x_2(t) \mapsto \big(x_1(t) + x_2(t)\big)\big(x_1(t-1) + x_2(t-1)\big) =$

$= \underbrace{x_1(t)\,x_1(t-1)}_{y_1(t)} + \underbrace{x_2(t)\,x_2(t-1)}_{y_2(t)} + \underbrace{x_1(t)x_2(t-1) + x_2(t)x_1(t-1)}_{\text{not part of } y_1(t)+y_2(t)}$

$\therefore$ Nonlinear



Time Invariant

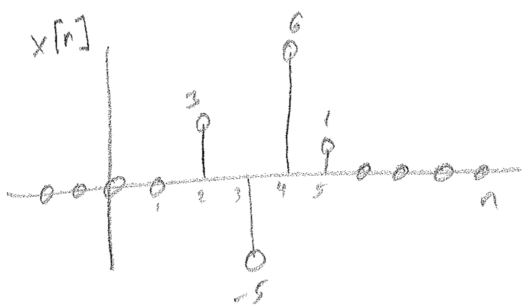
Name *Solutions*

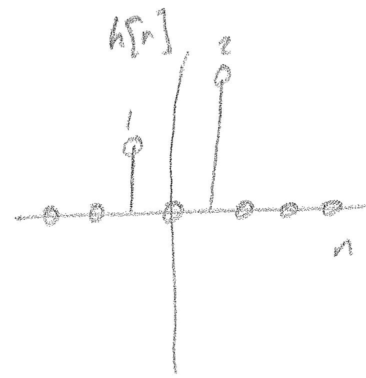
PROBLEM 4 (20 Points)

For the following problem, let

$$x[n] = 3\delta[n-2] - 5\delta[n-3] + 6\delta[n-4] + \delta[n-5]$$
$$h[n] = \delta[n+1] + 2\delta[n-1]$$

A. Sketch plots of both waveforms. (5 Points)



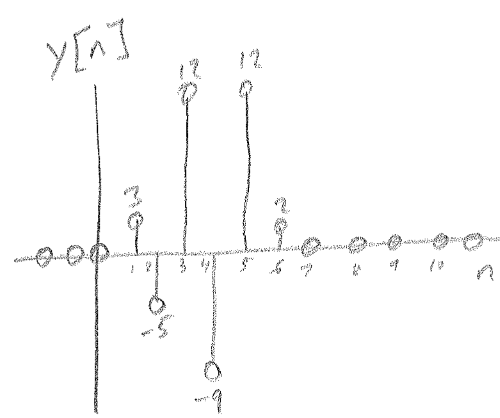


$N = 2$ ← First nonzero sample values → $M = -1$

B. Determine what the output, y[n], will be. Write an <u>expression</u> for y[n] and also <u>sketch</u> y[n]. (15 Points)

$N + M = 2 - 1 = 1$ → First nonzero sample value of $y[n]$ will be at $y[1]$

3	-5	6	1		
1	0	2			
3	-5	6	1		
	0	0	0	0	
		6	-10	12	2
3	-5	12	-9	12	2

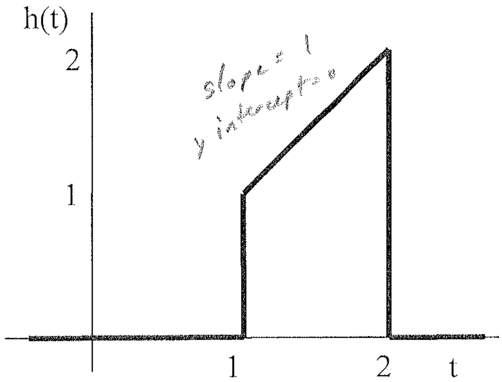


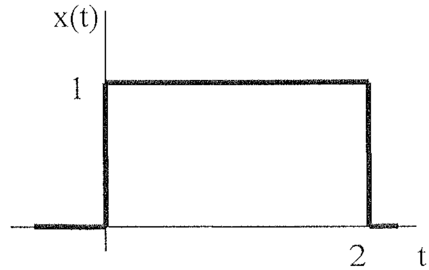
$y[n] = 3\delta[n-1] - 5\delta[n-2] + 12\delta[n-3] - 9\delta[n-4] + 12\delta[n-5] + 2\delta[n-6$

(Problem 4 Work Page)

PROBLEM 5 (30 Points)

Use the system definition, h(t), and input, x(t), shown below for the following problem.





A. Write an expression for h(t). (3 Points)

$h(t) = t\left(u(t-1) - u(t-2)\right)$

B. Write an expression for x(t). (3 Points)

$x(t) = u(t) - u(t-2)$

C. What is the *mathematical definition* of the convolution integral for continuous-time convolution (i.e., provide a mathematical equation)? (2 Points)

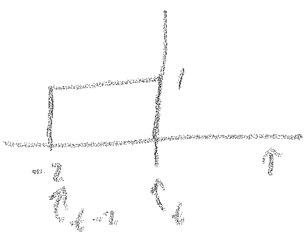
$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

EE 327 Signals and Systems 1 Name Solutions
Test 1 September 19, 2014

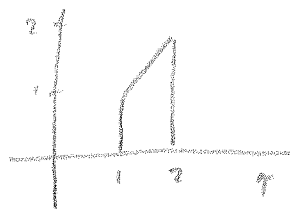
D. Determine the output, y(t), for the given input and impulse response. You do *not* need to sketch the output. (20 Points)

Choose $x(t)$ to flip and shift

$x(\tau)$



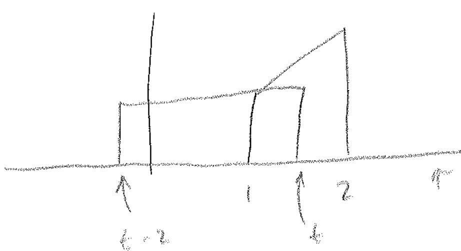
$h(\tau)$



For $t < 1$
No overlap of the nonzero portions of $x(t-\tau)$ and $h(\tau)$
$\therefore\ y(t) = 0$ over this time interval

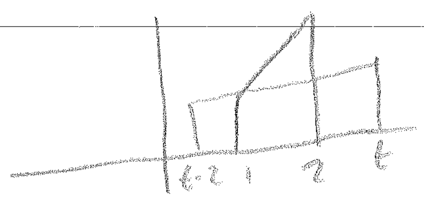
For $1 \le t < 2$

$$\int_1^t \tau \, d\tau = \frac{1}{2}\tau^2 \Big|_1^t = \frac{1}{2}t^2 - \frac{1}{2}$$



For $2 \le t < 3$

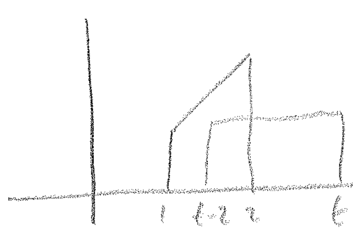
$$\int_1^2 \tau \, d\tau = \frac{1}{2}\tau^2 \Big|_1^2 = \frac{4}{2} - \frac{1}{2} = \frac{3}{2} = 1.5$$



For $3 \le t < 4$

$$\int_{t-2}^2 \tau \, d\tau = \frac{1}{2}\tau^2 \Big|_{t-2}^2 = \frac{4}{2} - \frac{(t-2)^2}{2} =$$

$$= \frac{4}{2} - \frac{1}{2}(t^2 - 4t + 4) = -\frac{1}{2}t^2 + 2t$$



For $t \ge 4$ No overlap
$y(t) = 0$

(Problem 5 Work Page)

Summary

$$y(t) = x(t) * h(t) = \begin{cases} 0 & t < 1 \\ \frac{1}{2}t^2 - \frac{1}{2} & 1 \le t < 2 \\ \frac{3}{2} & 2 \le t < 3 \\ -\frac{1}{2}t^2 + 2t & 3 \le t < 4 \\ 0 & t \ge 4 \end{cases}$$

E. If you were to find y(t) using Matlab, provide the line(s) of code that would be used to write the result to variable `yy`. Assume that the coefficients of the input and the impulse response have already been entered into the Matlab workspace and are represented by the variables `xx` and `hh`. If any necessary information is missing that is needed to find this output, describe what that information is and how it should be incorporated into your Matlab code. (3 Points)

T: sampling period

```
yy = conv(xx, T.hh);
```

or

```
yy = conv(T.xx, hh);
```