EE 327 Signals and Systems 1
Test 1

Name Solutions

September 18, 2015

RULES

This is a closed book, closed notes test. You are, however, allowed one half of one piece of paper (front side only) for notes and definitions, but no sample problems. You must staple your equations sheet to the back of your test when you hand your test in.

You are permitted to use a calculator.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	10	
2	20	
3	20	
4	25	
5	25	
Total	100	

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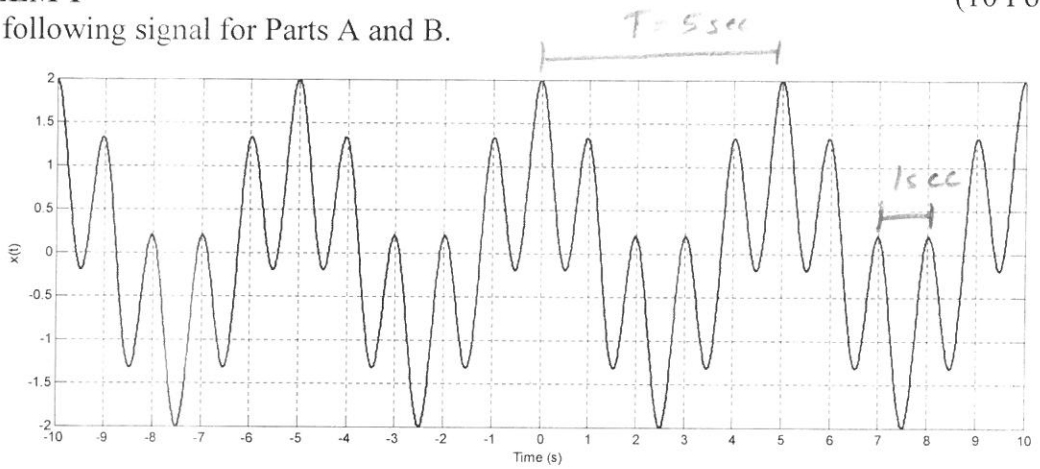
PROBLEM 1 (10 Points)

Use the following signal for Parts A and B.



A. Determine whether or not this signal is periodic. You must clearly justify your answer to receive full credit. If the signal is period, what is the fundamental frequency?

Yes. Periodic.

For $T = 5$ sec., $x(t) = x(t+T) \Rightarrow$ It repeats itself every 5 sec.

$T_0 = 5$ sec is the fundamental period

$f_0 = \frac{1}{T_0} = 0.2$ Hz

B. If you need to sample this signal, determine the condition on the sampling rate that guarantees that there will be no aliasing when sampling this particular signal.

The maximum frequency present in this signal is $T = 1$ sec $\Rightarrow f = 1$ Hz

Therefore, must sample at $f_s > 2 f_{max}$

$f_s > 2$ Hz

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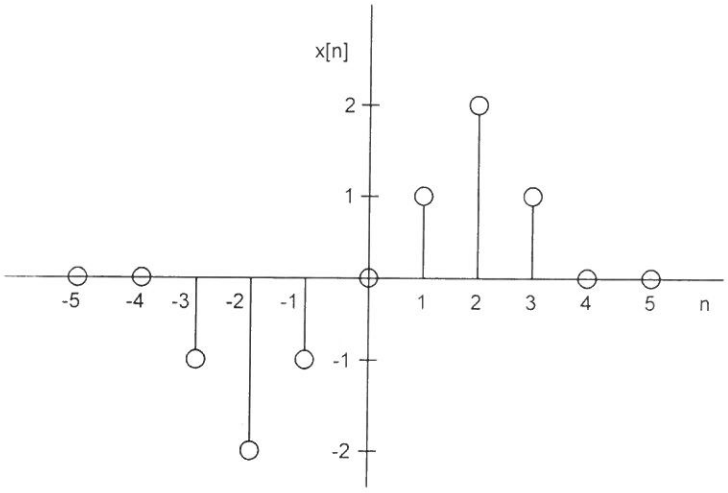
PROBLEM 2 (20 Points)

You are given the following discrete-time signal, x[n].



A. Plot the following signal. (10 Points)

$$y[n] = 3x[-n-2]$$

Let $v[n] = x[n+b]$

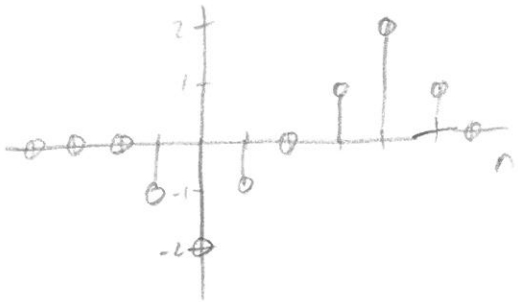
Let $y[n] = 3v[an] = 3x[an+b]$

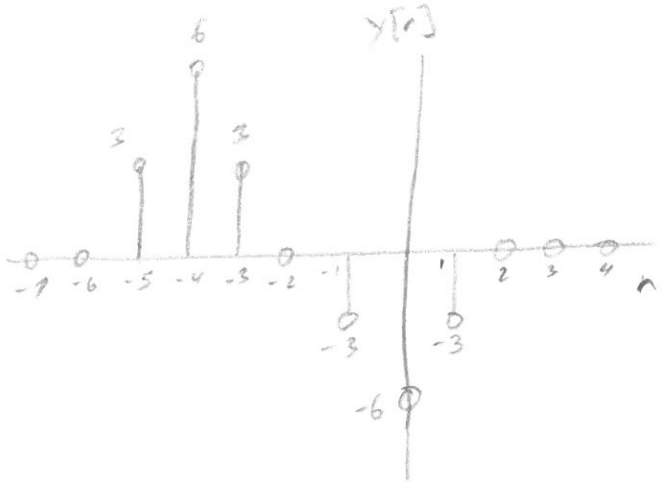
$\therefore \; an + b = -n - 2$

$a = -1, \; b = -2$

$v[n] = x[n-2]$ ← Time Shift

$y[n] = 3v[-n]$ ← Time Reversal, ↙ scale factor





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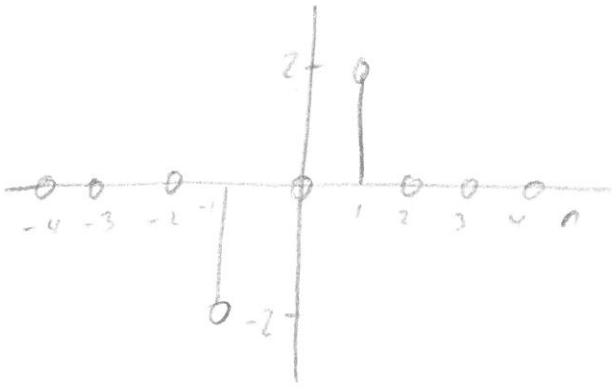
B. Plot the following signal. (10 Points)

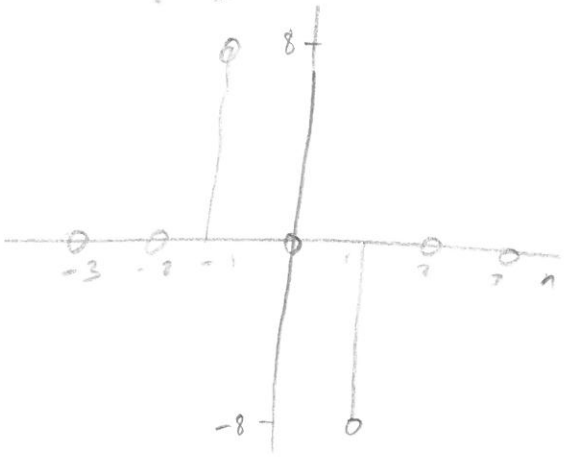
$$y[n] = -4x[2n]$$

$x[2n]$ → subsample → effectively remove some sample values (speed up)

$x[2n]$



$-4x[2n]$



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PROBLEM 3 (20 Points)

Determine the properties of the following system. Choose one property from each column, and circle the appropriate property. You must show your work to receive partial credit.

(4 Points Each)

$$y(t) = 10\cos\left(tx^2(t)\right)$$

Column 1	Column 2	Column 3	Column 4	Column 5
(Causal)	Has Memory	(Stable (BIBO))	Linear	Time Invariant
OR	OR	OR	OR	OR
Non Causal	(Memoryless)	Unstable	(Nonlinear)	(Time Varying)

$y(t)$ only depends on the present value of $x(t)$

$y(t) = 10\cos\left(tx^2(t)\right) \leq 10$ regardless of t or $x(t)$

$\Rightarrow$ The output is always bounded

Additivity Test

$x_1(t) \mapsto 10\cos\left(tx_1^2(t)\right) = y_1(t)$

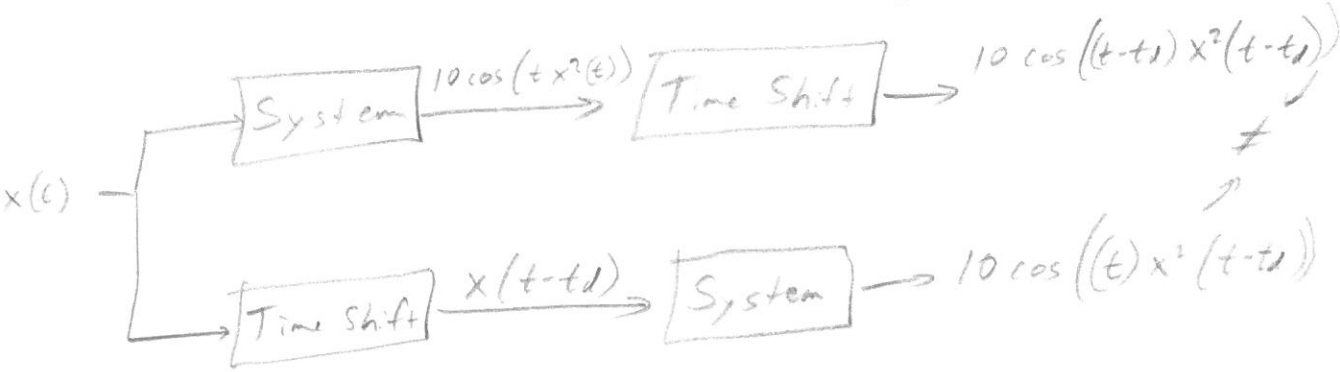
$x_2(t) \mapsto 10\cos\left(tx_2^2(t)\right) = y_2(t)$

$x_1(t) + x_2(t) \mapsto 10\cos\left(t\left(x_1(t) + x_2(t)\right)^2\right) =$

$= 10\cos\left(t\left(x_1^2(t) + 2x_1(t)x_2(t) + x_2^2(t)\right)\right)$

$\neq y_1(t) + y_2(t)$

$\therefore$ Nonlinear



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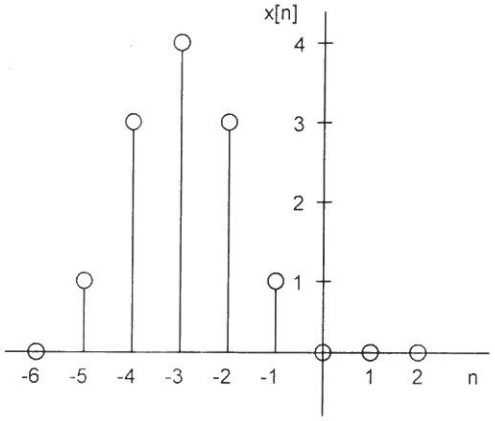
PROBLEM 4 (25 Points)

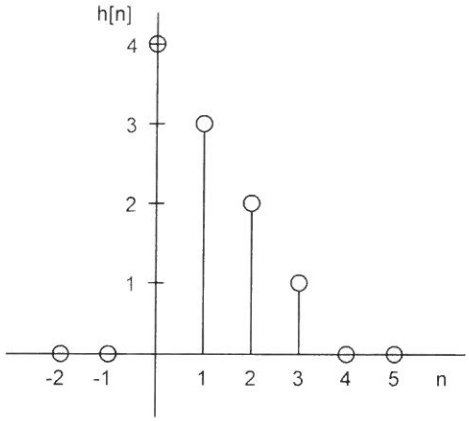
A. What is the mathematical definition of the convolution sum (i.e. an equation)? (3 Points)

$$y[n] = \sum_{i=-\infty}^{\infty} x[i]\, h[n-i]$$

B. Determine the output, y[n], of the system to the input signal, x[n], if the system is defined by the impulse response, h[n]. Write an expression for y[n] and also sketch y[n]. (20 Points)



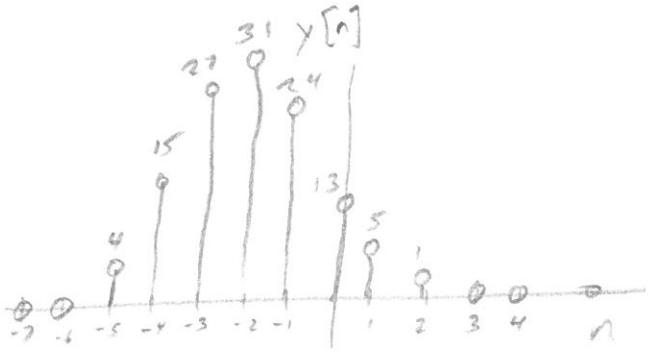
$N = -5$



$M = 0$

$N + M = -5 + 0 = -5$

	1	3	4	3	1		
	4	3	2	1			
4	12	16	12	4			
	3	9	12	9	3		
		2	6	8	6	2	
			1	3	4	3	1
4	15	27	31	24	13	5	1



$$y[n] = 4\delta[n+5] + 15\delta[n+4] + 27\delta[n+3] + 31\delta[n+2] + 24\delta[n+1] + 13\delta[n] + 5\delta[n-1] + \delta[n-2]$$

– Extra Room for Part B –

C. If you were to find y[n] using Matlab, provide the line(s) of code that would be used to write the result to variable `yy`. Assume that the coefficients of x[n] and h[n] have already been entered into the Matlab workspace and are represented by the variables `xx` and `hh`. If any necessary information is missing that is needed to find this output, describe what that information is and how it should be incorporated into your Matlab code. (2 Points)

```
yy= conv(xx, hh);
```

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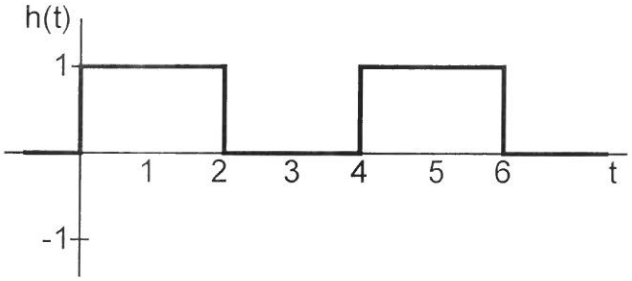
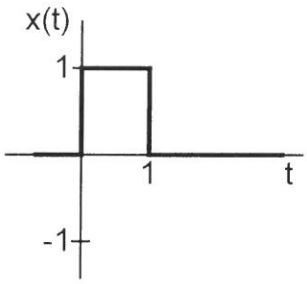
PROBLEM 5 (25 Points)

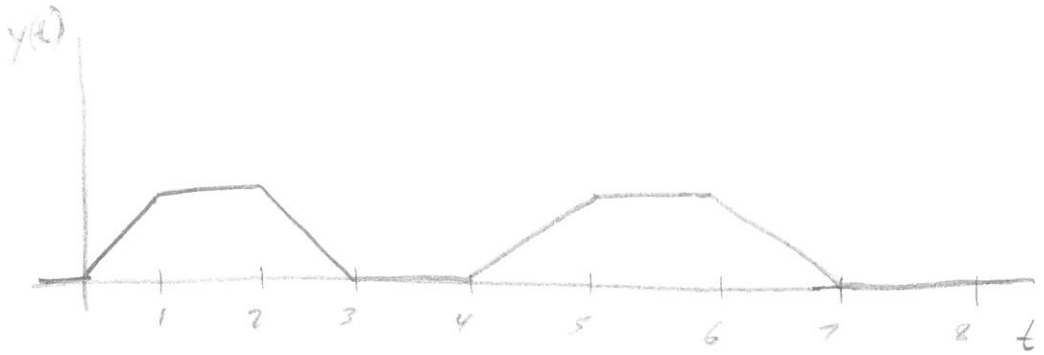
Use the impulse response, h(t), and input, x(t), shown below to find the output signal, y(t). Write your summary solution in the box provided below (you must write your solution here to receive full credit). You must also provide a sketch of y(t); be as detailed as possible. The sketch does not need to be in the answer box, but make sure that you clearly label your final sketch of y(t).





Answer

$$y(t) = \begin{cases} 0 & \text{for } t<0 \\ t & \text{for } 0 \le t < 1 \\ 1 & \text{for } 1 \le t < 2 \\ -t+3 & \text{for } 2 \le t < 3 \\ 0 & \text{for } 3 \le t < 4 \\ t-4 & \text{for } 4 \le t < 5 \\ 1 & \text{for } 5 \le t < 6 \\ -t+7 & \text{for } 6 \le t < 7 \\ 0 & \text{for } t \ge 7 \end{cases}$$



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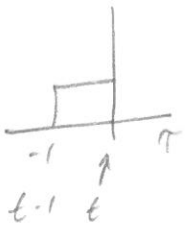
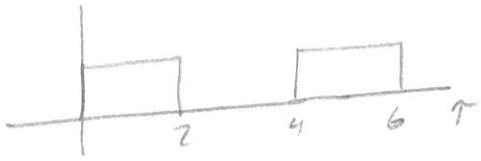
– Problem 5 Work Page –

Flip and Shift $x(t)$

$x(-\tau)$

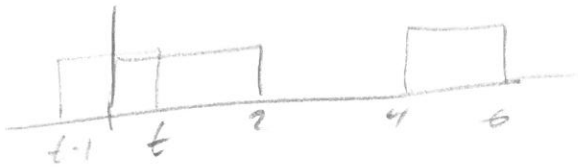


$h(\tau)$



For $t < 0$, No overlap $\Rightarrow y(t)$

For $0 \le t < 1$

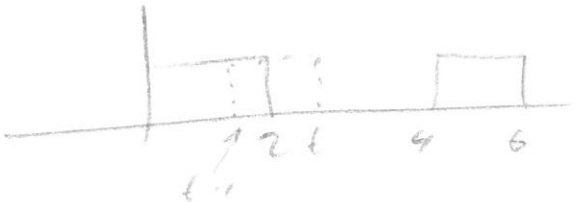


$$\int_0^t (1)(1)\,d\tau = \tau \Big|_0^t = t$$

For $1 \le t < 2$

$$\int_{t-1}^{t} d\tau = \tau \Big|_{t-1}^{t} = t - t + 1 = 1$$

For $2 \le t < 3$



$$\int_{t-1}^{2} d\tau = \tau \Big|_{t-1}^{2} = 2 - t + 1 = -t + 3$$

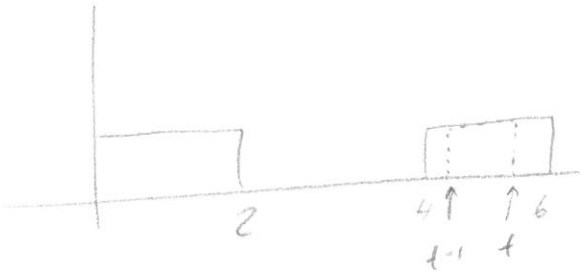
For $3 \le t < 4$, No overlap $\Rightarrow y(t)$

For $4 \le t < 5$

$$\int_4^t d\tau = \tau \Big|_4^t = t - 4$$

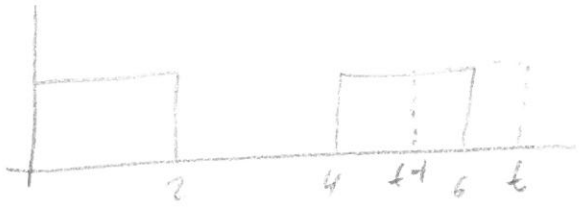
– Problem 5 Work Page –

For $5 \le t < 6$



$$\int_{t-1}^{t} d\tau = \tau \Big|_{t-1}^{t} = 1$$

For $6 \le t < 7$



$$\int_{t-1}^{6} d\tau = \tau \Big|_{t-1}^{6} = 6 - t + 1 = -t + 7$$

For $t > 7$, No overlap $\Rightarrow y(t)$