EE 327 Signals and Systems 1
Test 1

Name Solutions

September 23, 2016

**RULES**

This is a closed book, closed notes test. You are, however, allowed one half of one piece of paper (front side only) for notes and definitions, but no sample problems. You must staple your equations sheet to the back of your test when you hand your test in.

You are permitted to use a calculator.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

| Problem | Value | Score |
|---|---|---|
| 1 | 10 | |
| 2 | 10 | |
| 3 | 20 | |
| 4 | 30 | |
| 5 | 30 | |
| Total | 100 | |

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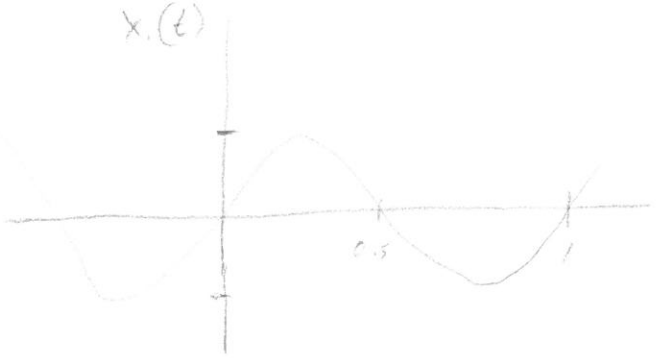
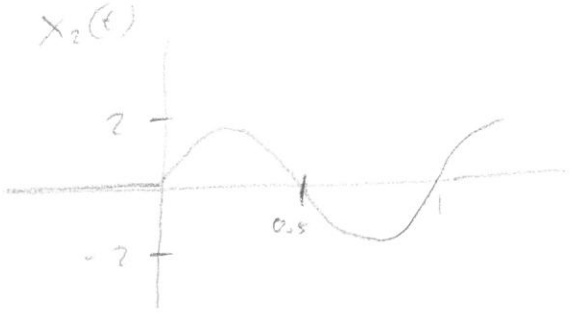
**PROBLEM 1** (10 Points)

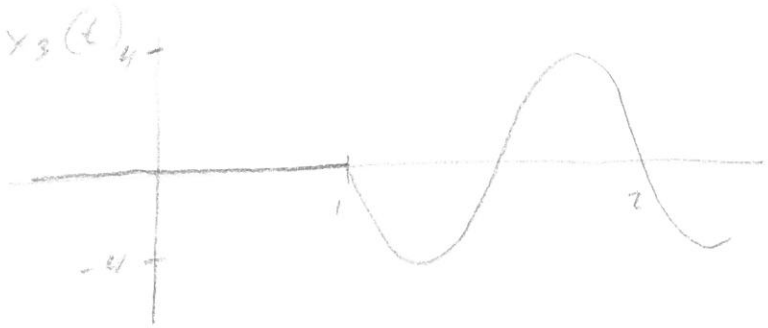
Sketch the following waveform.

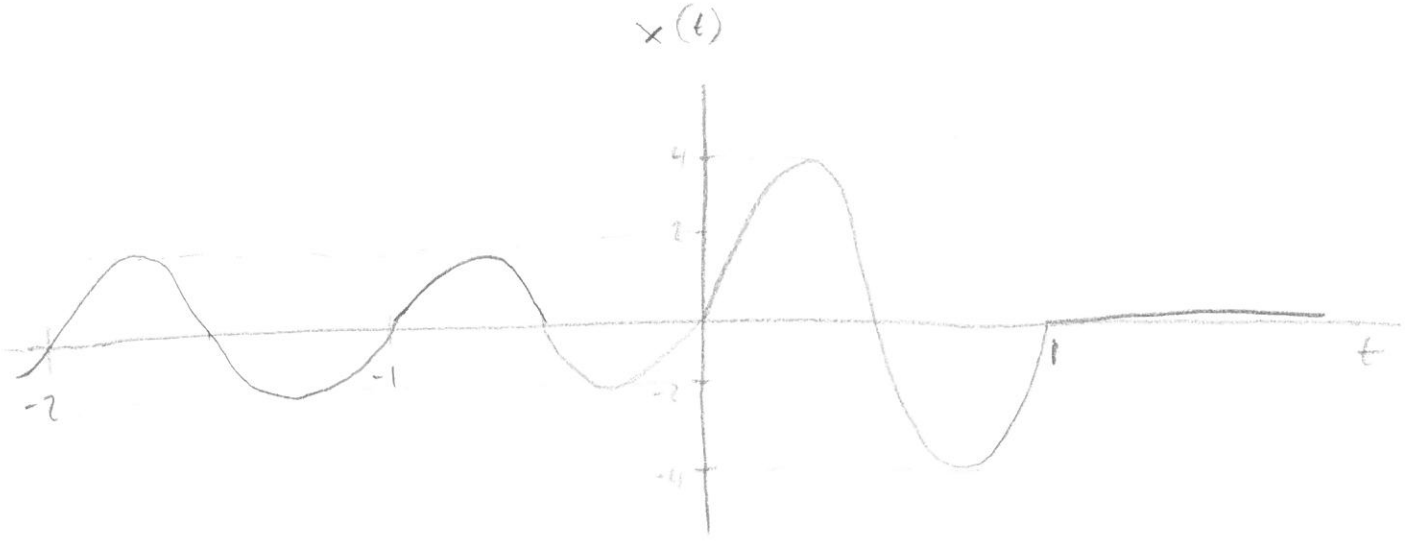
Amplitude $\omega = 2\pi \Rightarrow f = \frac{\omega}{2\pi} = 1\,\text{sec}$

$$x(t) = \underbrace{2\sin(2\pi t)}_{x_1(t)} + \underbrace{2\sin(2\pi t)u(t)}_{x_2(t)} - \underbrace{4\sin(2\pi t)u(t-1)}_{x_3(t)}$$



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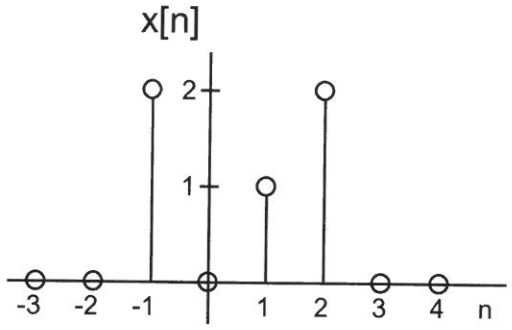
**PROBLEM 2** (10 Points)

You are given the following discrete-time signal, x[n].




Plot the following signal.

$$y[n] = x[-n+3]$$

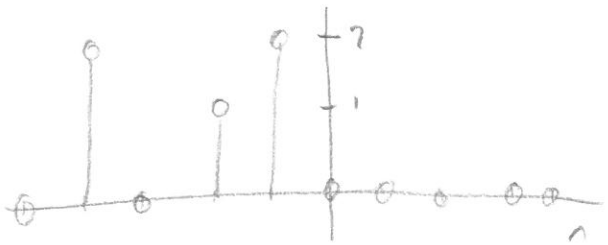
Let $v[n] = x[n+b]$

And $y[n] = v[an] = x[an+b]$

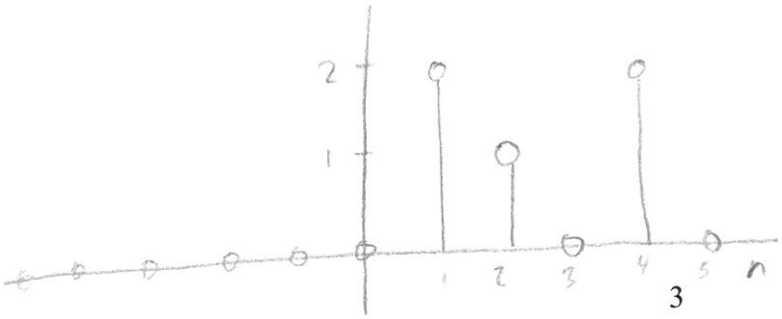
$\Rightarrow b = 3$

$a = -1$

$v[n] = x[n+3]$




Then $y[n] = v[-n] = x[-n+3]$




Double Check

$y[0] = x[-0+3] = x[3] = 0$

$y[1] = x[-1+3] = x[2] = 2$

$y[2] = x[-2+3] = x[1] = 1$

$y[3] = x[-3+3] = x[0] = 0$

$y[4] = x[-4+3] = x[-1] = 2$

Agrees

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**PROBLEM 3** (20 Points)

Determine the properties of the following system. Choose one property from each column, and circle the appropriate property. You must show your work to receive partial credit.

(4 Points Each)

$$y(t) = \sin(t)e^{x(t)}$$

| Column 1 | Column 2 | Column 3 | Column 4 | Column 5 |
|---|---|---|---|---|
| (Causal) | Has Memory | (Stable (BIBO)) | Linear | Time Invariant |
| OR | OR | OR | OR | OR |
| Non Causal | (Memoryless) | Unstable | (Nonlinear) | (Time Varying) |

Only looks at present value of input

For $|x(t)| \le A \mapsto \sin(t)\, e^{|x(t)|} \le e^{|x(t)|}$ Bounded

Homogeneity

$x(t) \mapsto \sin(t)\, e^{x(t)} = y(t)$

$ax(t) \mapsto \sin(t)\, e^{ax(t)} \neq a\,y(t)$

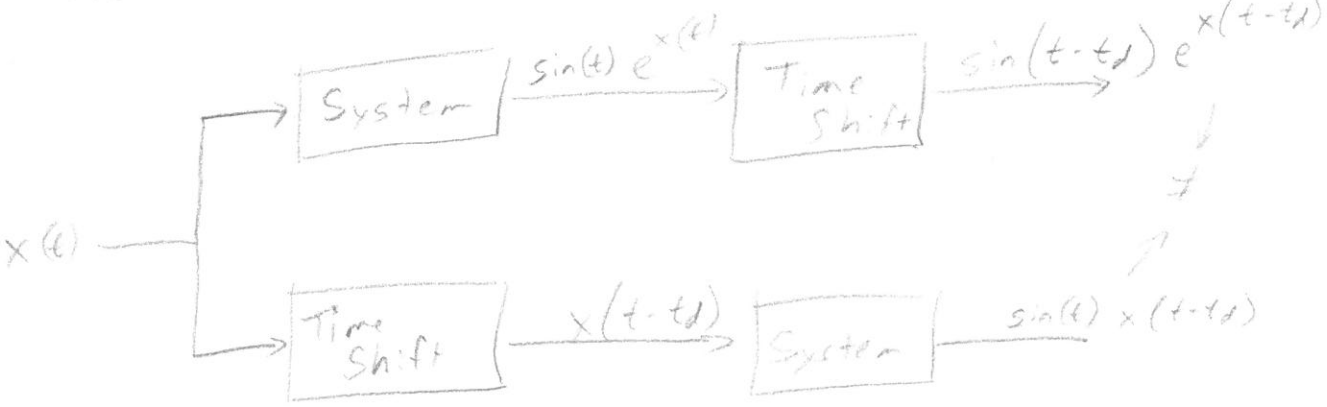
Nonlinear

Additivity

$x_1(t) \mapsto \sin(t)\, e^{x_1(t)} = y_1(t)$

$x_2(t) \mapsto \sin(t)\, e^{x_2(t)} = y_2(t)$

$x_1(t) + x_2(t) \mapsto \sin(t)\, e^{x_1(t)+x_2(t)} \neq y_1(t) + y_2(t)$



**PROBLEM 4** (30 Points)

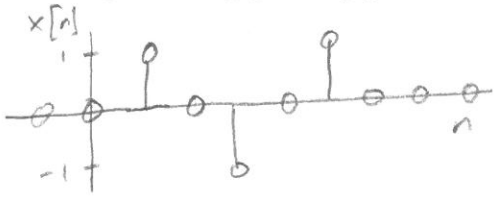
An input signal, x[n], is applied to a system with impulse response h[n] as given by

$$x[n] = \sin\left(\frac{n\pi}{2}\right)(u[n] - u[n-6])$$

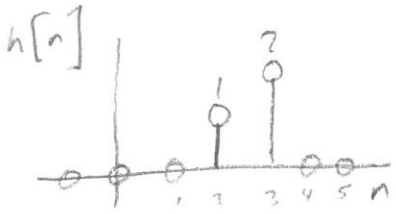
→ only nonzero from 0 to 5, so evaluate $\sin(\frac{n\pi}{2})$ over this interval

$$h[n] = (n-1)(u[n-2] - u[n-4])$$

A. Sketch plots of x[n] and h[n]. (5 Points)




N = 1




M = 2

N + M = 1 + 2 = 3

B. Determine the output, y[n], for the given input and impulse response. Write an expression for y[n] and also sketch y[n]. (25 Points)

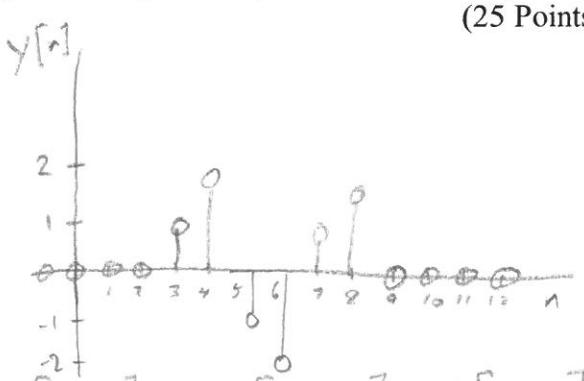
1 0 -1 0 1

1 2

1 0 -1 0 1

2 0 -2 0 2

1 2 -1 -2 1 2




$$y[n] = \delta[n-3] + 2\delta[n-4] - \delta[n-5] - 2\delta[n-6] + \delta[n-7] + 2\delta[n-8]$$

– Extra Room for Part B –

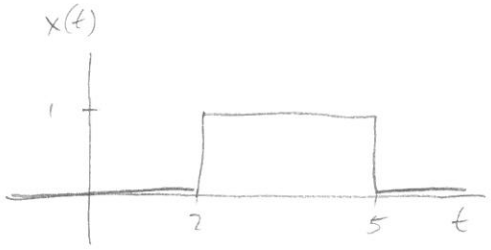
**PROBLEM 5** (30 Points)

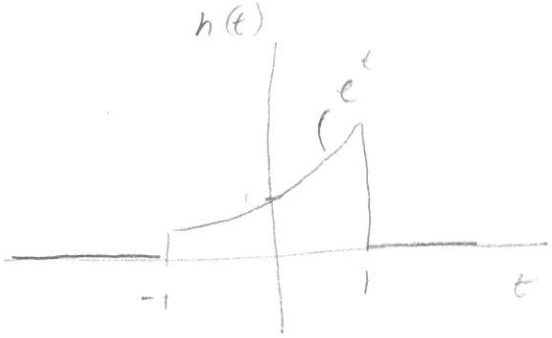
An input signal, x(t), is applied to a system with impulse response h(t) as given by

$$x(t) = u(t-2) - u(t-5)$$

$$h(t) = e^{t}\left(u(t+1) - u(t-1)\right)$$

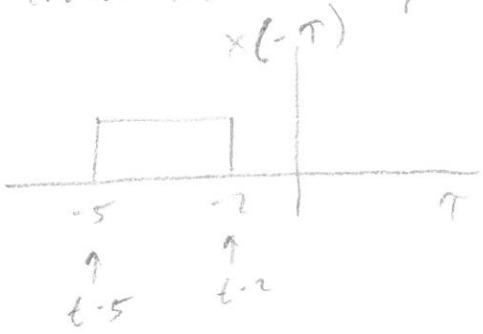
A. Sketch plots of x(t) and h(t). (5 Points)

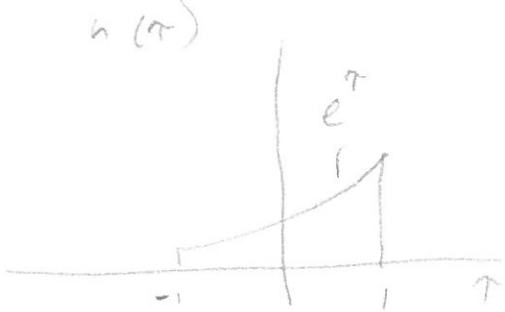



B. Determine the output, y(t), for the given input and impulse response. Write an <u>expression</u> for y(t). You do NOT need to include a sketch of the output signal. (25 Points)

Choose x(t) to flip and shift



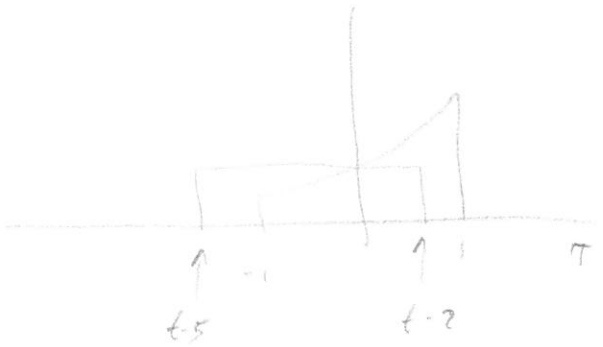

For $t < 1$

No overlap

$\therefore\ y(t) = 0$

– Problem 5 Work Page –

For $1 \le t < 3$




$$\int_{-1}^{t-2} e^{\tau}\, d\tau = e^{\tau}\Big|_{-1}^{t-2} = e^{t-2} - e^{-1}$$

For $3 \le t < 4$


$$\int_{-1}^{1} e^{\tau}\, d\tau = e^{\tau}\Big|_{-1}^{1} = e^{1} - e^{-1}$$

For $4 \le t < 6$


$$\int_{t-5}^{1} e^{\tau}\, d\tau = e^{\tau}\Big|_{t-5}^{1} = e^{1} - e^{t-5}$$

For $t \ge 6$ $y(t) = 0$

Summary

$$y(t) = \begin{cases} 0 & t < 1 \\ e^{t-2} - e^{-1} & 1 \le t < 3 \\ e^{1} - e^{-1} & 3 \le t < 4 \\ e^{1} - e^{t-5} & 4 \le t < 6 \\ 0 & t \ge 6 \end{cases}$$