EE 327 Signals and Systems 1 Name Solutions
Test 1 September 26, 2016

RULES

This is a closed book, closed notes test. You are, however, allowed one half of one piece of paper (front side only) for notes and definitions, but no sample problems. You must staple your equations sheet to the back of your test when you hand your test in.

You are permitted to use a calculator.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	15	
2	10	
3	20	
4	10	
5	20	
6	25	
Total	100	

EE 327 Signals and Systems 1
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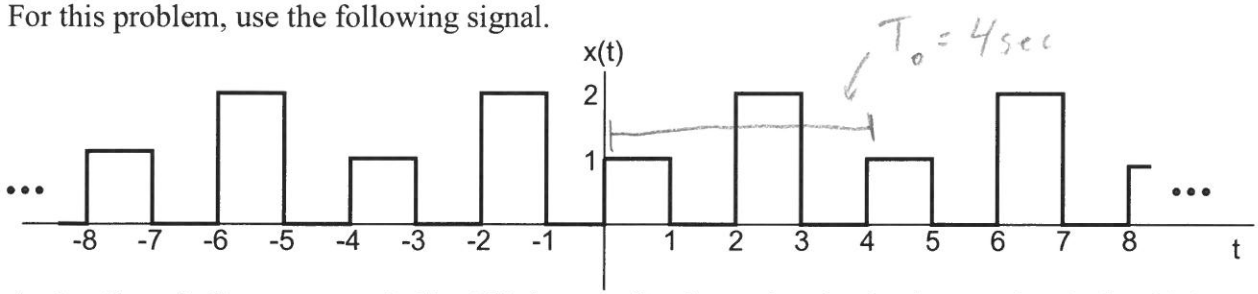
PROBLEM 1 (15 Points)

For this problem, use the following signal.



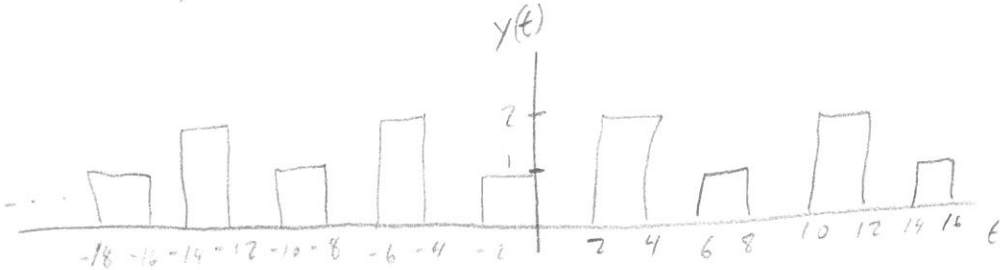
A. Is x(t) periodic or non-periodic. If it is periodic, determine the fundamental period. If it is non-periodic, explain how you know it is non-periodic. (5 Points)

Periodic

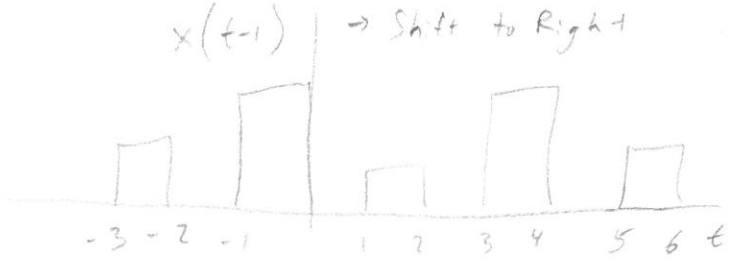
$T_0 = 4$ sec

B. Sketch the following signal. $y(t) = x\left(-\frac{t}{2}\right)$ (5 Points)

Time Scaling + Time Reversal

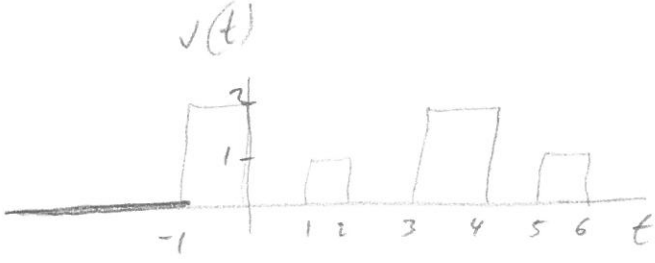


C. Sketch the following signal. $v(t) = x(t-1)u(t+1)$ (5 Points)



multiply by u(t+1)

→



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PROBLEM 2 (10 Points)

Sketch the following waveform.

$$y[n] = 2\left(\frac{1}{2}\right)^n \underbrace{(u[n+2] - u[n-2])}$$

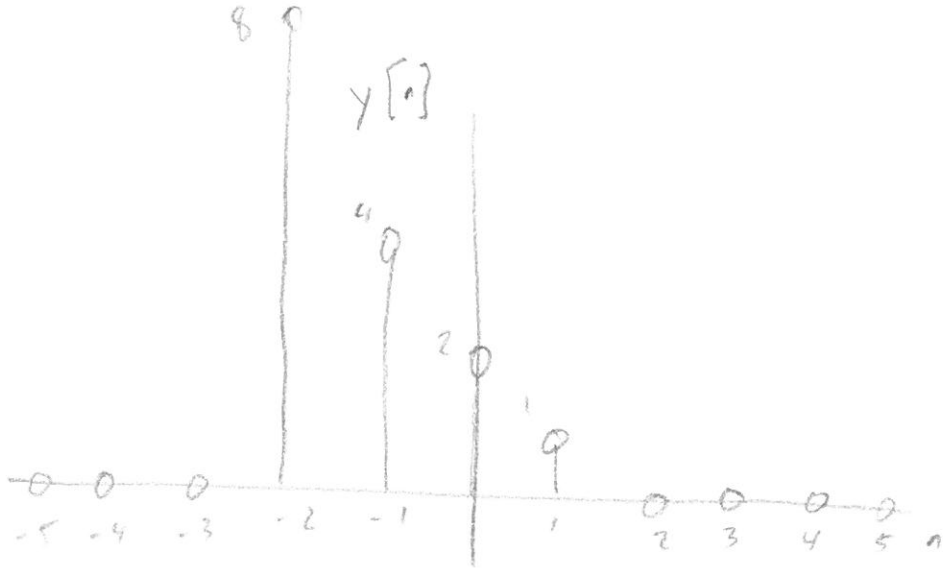
only nonzero for sample values $n = -2, -1, 0, 1$

$$y[-2] = 2\left(\frac{1}{2}\right)^{-2} = 8$$

$$y[-1] = 2\left(\frac{1}{2}\right)^{-1} = 4$$

$$y[0] = 2\left(\frac{1}{2}\right)^{0} = 2$$

$$y[1] = 2\left(\frac{1}{2}\right)^{1} = 1$$



PROBLEM 3 (20 Points)

Determine the properties of the following system. Choose one property from each column, and circle the appropriate property. You must show your work to receive partial credit.

(4 Points Each)

$$y[n] = x[n]u[n]$$

Column 1	Column 2	Column 3	Column 4	Column 5
(Causal)	Has Memory	(Stable (BIBO))	(Linear)	Time Invariant
OR Non Causal	~~OR~~ (Memoryless)	OR Unstable	OR Nonlinear	~~OR~~ (Time Varying)

Only looks to the present value of the input

If $|x[n]| \le A \mapsto |x[n]|\,u[n] \le A$

Linearity

Homogeneity

$x[n] \mapsto x[n]\,u[n]$

$a\,x[n] \mapsto a\,x[n]\,u[n]$ ✓

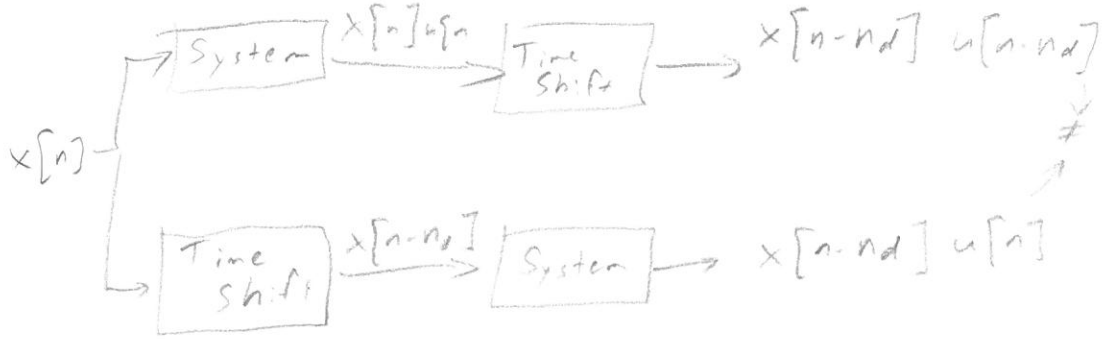
Additivity

$x_1[n] \mapsto x_1[n]\,u[n] = y_1[n]$

$x_2[n] \mapsto x_2[n]\,u[n] = y_2[n]$

$x_1[n] + x_2[n] \mapsto (x_1[n] + x_2[n])\,u[n]$ ✓

Time Invariance



PROBLEM 4 (10 Points)
Answer the following questions. (2 Points Each)

A. A typical human ear can hear frequencies between 20Hz to 20kHz. If you need to sample a continuous-time audio sound to the discrete-time domain, how fast must you sample the sound? Provide your answer in Hz.

$f_s > 2 f_{max}$

$\therefore f_s > (2)(20\,kHz)$

$f_s > 40\,kHz$

B. What Matlab function should you use to plot a discrete-time waveform?

stem

C. Write the mathematical definition of the convolution sum (i.e. for discrete-time, and your answer should be an equation).

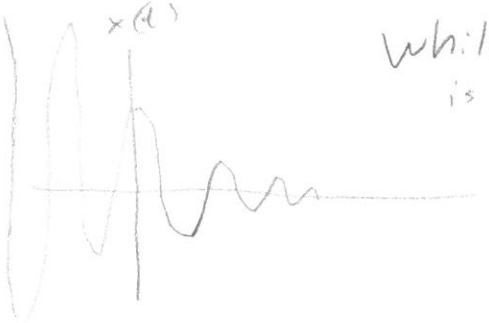
$$y[n] = \sum_{i=-\infty}^{\infty} x[i]\, h[n-i]$$

D. True or False about convolution.
$x(t) * h(t) = h(t) * x(t)$

True

E. Is the following waveform periodic or non-periodic? Why or why not?
$x(t) = e^{-t}\cos(2\pi t)$

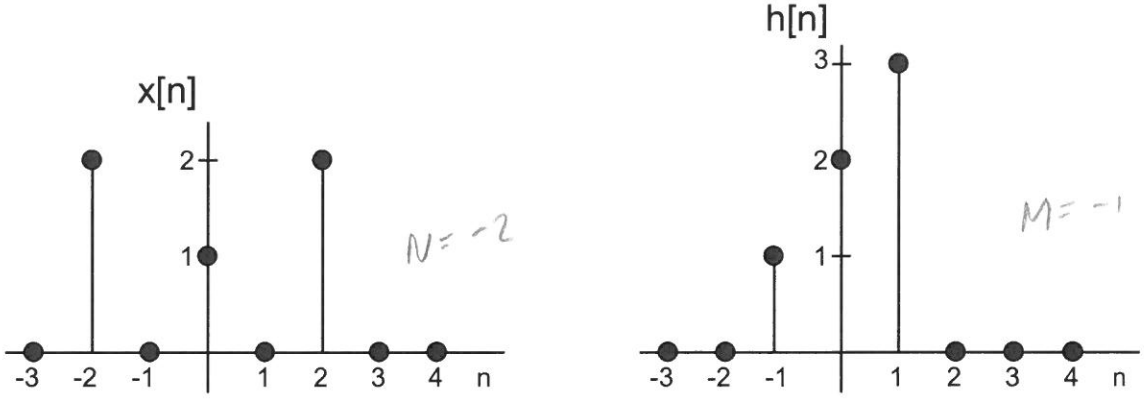
Non-Periodic



While it oscillates, there is no T for which $x(t) = x(t+T)$

PROBLEM 5 (20 Points)

An input signal, x[n], is applied to a system with impulse response h[n] as given by



Determine the output, y[n], for the given input and impulse response. Write an expression for y[n] and also sketch y[n].

$N + M = -2 - 1 = -3$

2 0 1 0 2

1 2 3

---

2 0 1 0 2

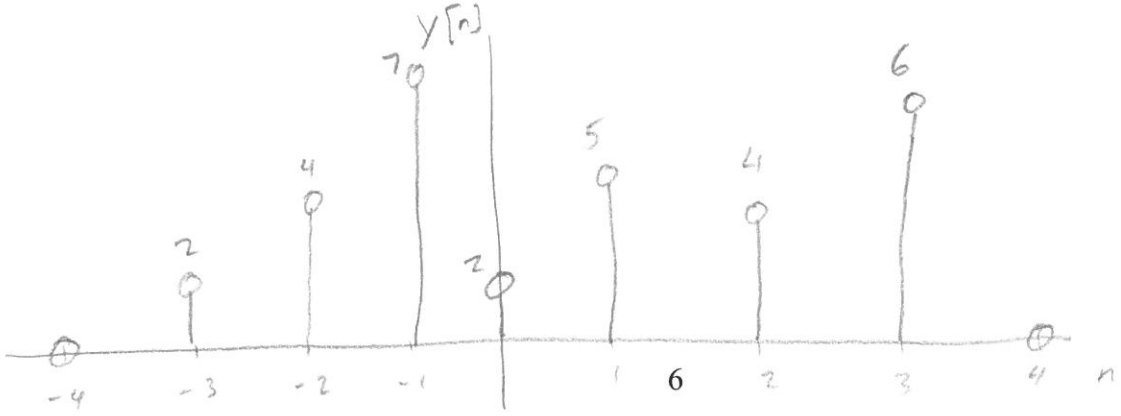
4 0 2 0 4

6 0 3 0 6

---

2 4 7 2 5 4 6

$y[n] = 2\delta[n+3] + 4\delta[n+2] + 7\delta[n+1] + 2\delta[n] + 5\delta[n-1] + 4\delta[n-2] + 6\delta[n-3]$



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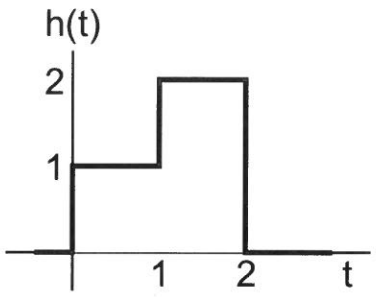
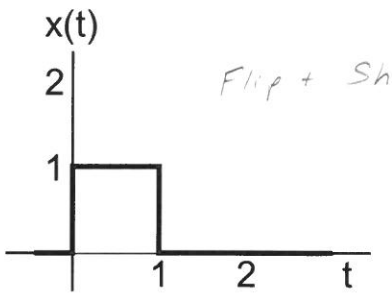
– Extra Room for Problem 5 –

PROBLEM 6 (25 Points)

An input signal, x(t), is applied to a system with impulse response h(t) as given by

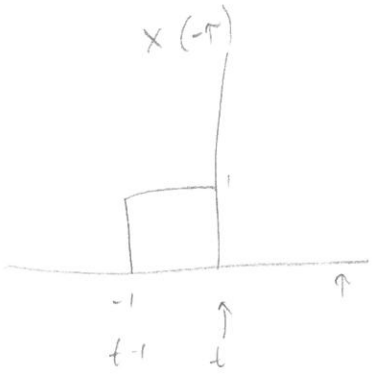


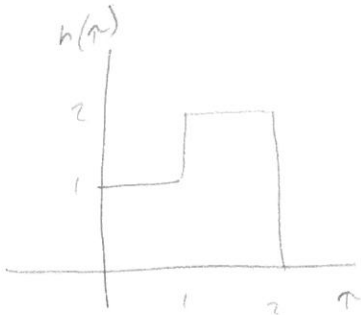


Flip + Shift

Determine the output signal. Write an expression for y(t) and also sketch y(t).

Hint. $\int_a^c \beta dx = \int_a^b \beta dx + \int_b^c \beta dx$



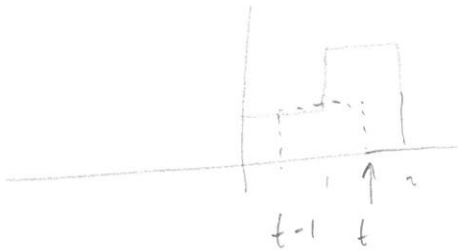


For $t < 0$ $\Rightarrow$ No overlap $\Rightarrow$ $y(t) = 0$

For $0 \le t < 1$

$$\int_0^t (1)(1)\, d\tau = \tau \Big|_0^t = t$$

For $1 \le t < 2$



$\int_{t-1}^{t} x(t-\tau)\, h(\tau)\, d\tau$ $\rightarrow$ Since $h(\tau)$ has different expressions over this range of τ, split up the integral

– Problem 6 Work Page –

$$\int_{t-1}^{1} x(t-\tau)\,h(\tau)\,d\tau = \int_{t-1}^{1} x(t-\tau)\,h(\tau)\,d\tau + \int_{1}^{t} x(t-\tau)\,h(\tau)\,d\tau =$$

$$= \int_{t-1}^{1} d\tau + \int_{1}^{t} 2\,d\tau = \tau\Big|_{t-1}^{1} + 2\tau\Big|_{1}^{t} =$$

$$= (1 - t + 1) + 2(t-1) = 2 - t + 2t - 2 =$$

$$= t$$

For $2 \le t < 3$

$$\int_{t-1}^{2} 2\,d\tau = 2\tau\Big|_{t-1}^{2} =$$

$$= 4 - 2(t-1) = -2t + 6$$

For $t \ge 3 \Rightarrow$ No overlap $\Rightarrow y(t) = 0$

$$y(t) = \begin{cases} 0 & \text{for } t < 0 \\ t & \text{for } 0 \le t < 2 \\ -2t + 6 & \text{for } 2 \le t < 3 \\ 0 & \text{for } t \ge 3 \end{cases}$$

Note, combined periods for $0 \le t < 1$ and $1 \le t < 2$ since they both evaluated to the same expression