EE 327 Signals and Systems 1 Name Solutions
Test 2 October 21, 2015

RULES

This is a closed book, closed notes test. You are, however, allowed one piece of paper (front side only) for notes and definitions, but no sample problems. The top half is the same as from the first test, and the bottom half contains the information added for the second test. You are also permitted to use a calculator. Additionally, a table of common Laplace Transforms has been provided at the end of this test.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	10	
2	20	
3	10	
4	10	
5	25	
6	30	
Total	100/105	

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PROBLEM 1 (10 Points)

Determine the Laplace transform for the following signals. Write the Laplace transforms as rational functions of "s" and simplify where possible.

A. $x(t) = 2e^{3t}\cos(t)u(t)$ (5 Points)

$$X(s) = 2\frac{s-3}{(s-3)^2+(1)^2} = \frac{2s-6}{(s-3)^2+1} = \frac{2s-6}{s^2-6s+10}$$

B. If $x(t) = 10\sin(3t)u(t)$

and $v(t) = 3x(2t)$,

then find the Laplace transform of $v(t)$. (5 Points)

$$X(s) = 10\frac{3}{s^2+3^2} = \frac{30}{s^2+9}$$

$$V(s) = 3\left(\frac{1}{2}\right)X\left(\frac{s}{2}\right) = \frac{3}{2}\frac{30}{\left(\frac{s}{2}\right)^2+9} = \frac{45}{\frac{s^2}{4}+9} = \frac{180}{s^2+36}$$

PROBLEM 2 (20 Points)

Determine the transfer function for the following systems. You must express the transfer functions as rational functions of "s" and simplify where appropriate.

A. $y + 5\dot{y} + 2\dddot{y} + 4x + 3\dot{x} = 0$ (10 Points)

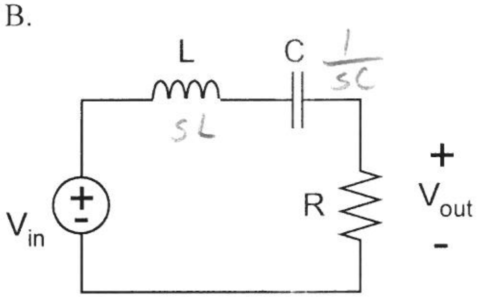
$$2\dddot{y} + 5\dot{y} + y = -3\dot{x} - 4x$$

$$2s^3Y(s) + 5sY(s) + Y(s) = -3sX(s) - 4X(s)$$

$$Y(s)\left[2s^3 + 5s + 1\right] = -X(s)\left[3s + 4\right]$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{-(3s+4)}{2s^3 + 5s + 1}$$

B. (10 Points)



Voltage Divider

$$V_{out} = V_{in}\frac{R}{sL + \frac{1}{sC} + R}$$

$$V_{out} = V_{in}\frac{sRC}{s^2LC + sRC + 1}$$

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{sRC}{s^2LC + sRC + 1}$$

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PROBLEM 3 (10 Points)

For each of the following systems, determine if the system is stable, marginally stable, or unstable. For all systems, determine how many poles are unstable. You must write your answers on the lines provided, and you must justify your answer to receive full credit. Hint – what constitutes a stable pole? a marginally stable pole? an unstable pole? For the purposes of this problem, a pole should be considered only one of those three options.

A. $H(s) = \dfrac{(s-1)}{(s+2)}$ (5 Points)

Stable, Marginally Stable, or Unstable? *Stable*

Number of Unstable Poles? *0*

pole $= -2 \rightarrow$ in the Left-Half Plane

B. $H(s) = \dfrac{s}{s^5 + 2s^4 + 2s^3 + 3s + 1}$ (5 Points)

Stable, Marginally Stable, or Unstable? *Unstable*

Number of Unstable Poles? *2*

$$
\begin{array}{cc|ccc}
+ & s^5 & 1 & 2 & 3 \\
+ & s^4 & 2 & 0 & 1 \\
+ & s^3 & 2 & \frac{5}{2} & \\
- & s^2 & -\frac{5}{2} & 1 & \\
+ & s^1 & \frac{33}{10} & & \\
+ & s^0 & 1 & &
\end{array}
$$

$$\frac{(-\frac{5}{2})(\frac{5}{2}) - (2)(1)}{-\frac{5}{2}} = \frac{-\frac{25}{4} - \frac{8}{4}}{-\frac{5}{2}} = \frac{-\frac{33}{4}}{-\frac{5}{2}} = \frac{33}{10}$$

Two sign changes

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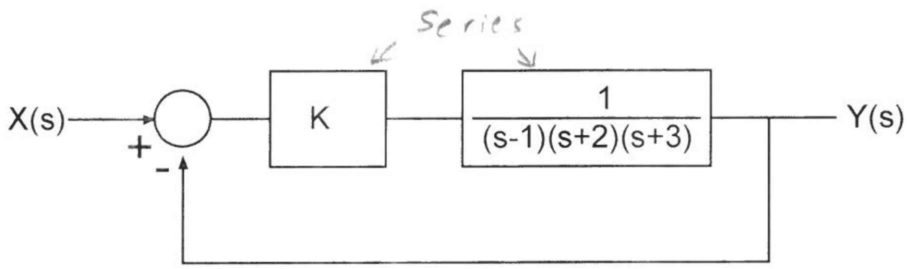
PROBLEM 4 (10 Points)

You are given a system defined by the following transfer function.

$$H(s) = \frac{1}{(s-1)(s+2)(s+3)}$$

Your task is to determine if you can design a controller, K, within the feedback system shown below to keep the entire new system stable. Clearly state whether or not it is possible to keep the new feedback system stable. Justify your answer. If it is possible to keep the system stable, determine what range of values of K will keep this system stable.



Feedback connection

Overall Transfer Function

$$H(s) = \frac{\frac{K}{(s-1)(s+2)(s+3)}}{1 + \frac{K}{(s-1)(s+2)(s+3)}} = \frac{K}{(s-1)(s+2)(s+3) + K}$$

$$H(s) = \frac{K}{(s^2+s-2)(s+3) + K} = \frac{K}{s^3 + 3s^2 + s^2 + 3s - 2s - 6 + K}$$

$$H(s) = \frac{K}{s^3 + 4s^2 + s + (K-6)}$$

s^3	1	1
s^2	4	$(K-6)$
s^1	$\frac{10-K}{4}$	
s^0	$K-6$	

$\rightarrow \frac{10-K}{4} > 0 \Rightarrow K < 10$

$\rightarrow K - 6 > 0 \Rightarrow K > 6$

$$\boxed{\text{Stable if } 6 < K < 10}$$

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PROBLEM 5 (25 Points)

A system is defined by the following transfer function.

$$H(s) = \frac{1}{s+1}$$

This system receives the following input signal.

$$X(s) = \frac{7s+2}{s^2+4}$$

A. Determine the Laplace-domain output, *Y(s)*, of the system. Write this as a rational function of "s." (5 Points)

$$Y(s) = \frac{1}{s+1} \cdot \frac{7s+2}{s^2+4} = \frac{7s+2}{(s+1)(s^2+4)} = \frac{7s+2}{s^3+s^2+4s+4}$$

C. Determine the *general form* of the time-domain solution, *y(t)*. (5 Points)

$$Y(s) = \frac{7s+2}{(s+1)(s^2+4)} = \frac{k_1}{s+1} + \frac{k_2 s + k_3}{s^2+4}$$

complex poles

$$y(t) = k_1 e^{-t} u(t) + k_4 \cos(2t + \theta)\, u(t)$$

D. Determine the exact form of the time-domain solution, *y(t)*. (More room on the next page) (15 Points)

$$Y(s) = \frac{7s+2}{(s+1)(s^2+4)} = \frac{k_1}{s+1} + \frac{k_2 s + k_3}{s^2+4}$$

$$k_1 = Y(s)(s+1)\Big|_{s=-1} = \frac{7s+2}{s^2+4}\Big|_{s=-1} = \frac{-5}{5} = -1$$

Problem 5 Work Page

Brute Force Method

$$\frac{k_1(s^2+4) + (k_2 s + k_3)(s+1)}{(s+1)(s^2+4)}$$

Equate Numerators

$$7s + 2 = k_1 s^2 + 4k_1 + k_2 s^2 + (k_2 + k_3)s + k_3$$

Equate like powers of "s"

$$s^2 \rightarrow 0 = k_1 + k_2 \Rightarrow k_2 = -k_1 = 1$$

$$s^0 \rightarrow 2 = 4k_1 + k_3 \Rightarrow k_3 = 2 - 4k_1 = 2 - (4)(-1) = 6$$

$$Y(s) = \frac{-1}{s+1} + \frac{s+6}{s^2+4} = \frac{-1}{s+1} + \frac{s}{s^2+4} + \frac{3(2)}{s^2+4}$$

$$y(t) = -e^{-t}u(t) + \cos(2t)u(t) + 3\sin(2t)u(t)$$

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PROBLEM 6 (30 Points)

A second-order, continuous-time system is defined by the following transfer function.

$$H(s) = \frac{50}{s^2 + (2.5)s + 25}$$

This system receives a step input.

A. What is the steady-state output, $y_{ss}(t)$, resulting from a step input? (5 Points)

$$y_{ss}(t) = \lim_{t \to \infty} y(t) = \lim_{s \to 0} s\,Y(s) = \lim_{s \to 0} s\,H(s)\frac{1}{s} = \lim_{s \to 0} H(s)$$

$$\lim_{s \to 0} \frac{50}{s^2 + 2.5s + 25} = \frac{50}{25} = \boxed{2}$$

B. What is the natural frequency, ω_n, for this system? (3 Points)

$$\omega_n = \sqrt{25} = 5 \text{ rad/sec}$$

C. What is the damping ratio, ζ, for this system? (3 Points)

$$2\zeta\omega_n = 2.5$$

$$\zeta = \frac{2.5}{(2)(5)} = \frac{2.5}{10} = 0.25$$

D. Is this system under damped, critically damped, or over damped? Why? (2 Points)

Underdamped because $\zeta < 1$

(complex Poles)

E. What is the (dominant) time constant for this system? (3 Points)

$$\tau = \frac{1}{\zeta \omega_n} = \frac{1}{(0.25)(5)} = \frac{4}{5} \text{ sec} = 0.8 \text{ sec}$$

F. Does this system overshoot the steady-state value if there is a step input? Explain why or why not. If so, what is the percent overshoot? (3 Points)

Yes because it is underdamped

$$P.O. \approx (100\%)\, e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 44.4\%$$

G. With a step input, does this system oscillate? Explain why or why not. If so, at what frequency? (3 Points)

Yes because it is underdamped

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = (5)\sqrt{1-(\tfrac{1}{4})^2} = 4.84 \text{ rad/sec}$$

H. What is the 5% settling time (if applicable, otherwise state how you would find this value)? (3 Points)

$$t_s \approx 3\tau = (3)(0.8 \text{ sec}) = 2.4 \text{ sec}$$

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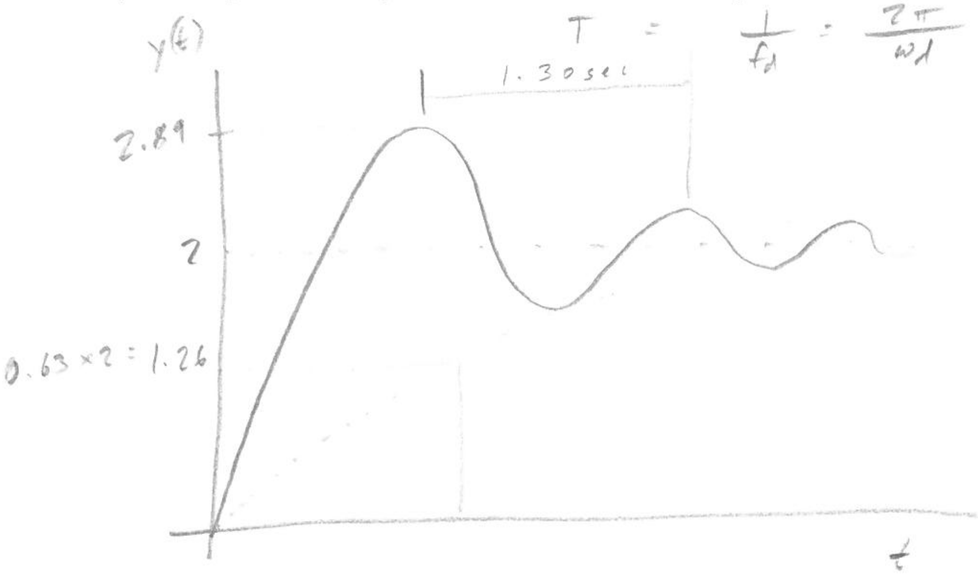
I. Sketch the step response of this system using the previous parts as a guide. **Label all important points and provide as much detail as possible**. (5 Points)

$$T = \frac{1}{f_d} = \frac{2\pi}{\omega_d} = \frac{2\pi}{4.84} = 1.30 \text{ sec}$$



Max Value →

$$P.O. = 100\% \frac{y_{max} - y_{ss}}{y_{ss}}$$

$$y_{max} = \frac{P.O.}{100\%} y_{ss} + y_{ss} = 2.89$$