EE 327 Signals and Systems 1
Test 2

Name Solutions
October 19, 2016

RULES

This is a closed book, closed notes test. You are, however, allowed one piece of paper (front side only) for notes and definitions, but no sample problems. The top half is the same as from the first test, and the bottom half contains the information added for the second test. You are also permitted to use a calculator. Additionally, a table of common Laplace Transforms has been provided at the end of this test.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	10	
2	10	
3	30	
4	15	
5	25	
6	10	
Total	100	

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PROBLEM 1 (10 Points)

Determine the Laplace transform for the following signals. Write the Laplace transforms as rational functions of "s" and simplify where possible.

A. $x(t) = 2t\cos(3t)u(t)$ (2 Points)

Directly from the transform table

$$X(s) = 2\,\frac{s^2 - 9}{(s^2+9)^2} = \frac{2s^2 - 18}{s^4 + 18s^2 + 81}$$

B. $x(t) = 3 + 4e^{-2t} + t$ ← u(t) is implied because of the definition of the Laplace Transform (unilateral) (3 Points)

$$x(t) = 3u(t) + 4e^{-2t}u(t) + t\,u(t)$$

$$X(s) = \frac{3}{s} + \frac{4}{s+2} + \frac{1}{s^2} = \frac{3(s)(s+2) + 4s^2 + (s+2)}{(s^2)(s+2)} =$$

$$= \frac{7s^2 + 7s + 2}{s^3 + 2s^2}$$

C. Let $V(s) = \dfrac{1}{s+2}$ be the Laplace Transform of *v(t)*.

Find *W(s)* for $w(t) = tv(t)$ (5 Points)

$$v(t) = e^{-2t}u(t)$$

$$w(t) = te^{-2t}u(t)$$

$$\therefore W(s) = \frac{1}{(s+2)^2} = \frac{1}{s^2+4s+4}$$

from the L.T. Table

Alternate Method

$$V(s) = \frac{1}{s+2}$$

$$w(t) = tv(t) \Longleftrightarrow (-1)^1 \frac{d}{ds}\frac{1}{s+2}$$

$$\Rightarrow W(s) = (-1)(-1)(s+2)^{-2}$$

$$= \frac{1}{(s+2)^2} = \frac{1}{s^2+4s+4}$$

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PROBLEM 2 (10 Points)

A signal is given by the following.

$$X(s) = \frac{-2}{s^2 - 1}$$

A. Determine the initial value of this signal (the value at time t=0). (5 Points)

$$x(0) = \lim_{s \to \infty} sX(s) = \lim_{s \to \infty} \frac{-2s}{s^2 - 1} = \frac{\infty}{\infty}$$

Use L'Hopital's Rule

$$= \lim_{s \to \infty} \frac{-2}{2s} = \frac{-2}{\infty} = 0$$

B. If possible, determine the final value of this signal (as time approaches infinity). If it is not possible to determine the final value of the signal, state why this is the case, and also state how you would go about finding the final value if it were possible to do so. Justify your answer.
(5 Points)

$$X(s) = \frac{-2}{s^2 - 1} = \frac{-2}{(s+1)(s-1)} \qquad \text{pole} = \pm 1$$

There is a pole in the right-half plane, so no final value exists.

$\therefore$ We cannot apply the F.V.T.

How the F.V.T. would be used (if possible)

$$x(\infty) = \lim_{s \to 0} sX(s)$$

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PROBLEM 3 (30 Points)

Determine the transfer function for the following systems. You must express the transfer functions as rational functions of "s" and simplify where appropriate.

A. $2\dddot{y} + \ddot{y} + 4y = \dot{x} + 3x$ (10 Points)

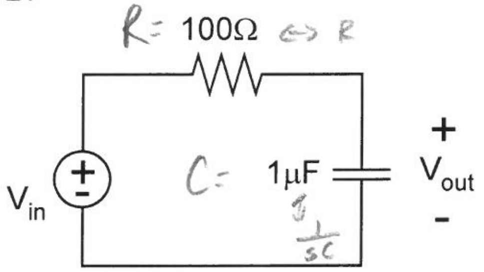
$$2s^3 Y(s) + s^2 Y(s) + 4Y(s) = sX(s) + 3X(s)$$

Note, all initial conditions have been set to 0

$$(2s^3 + s^2 + 4)Y(s) = (s+3)X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+3}{2s^3 + s^2 + 4}$$

B. (10 Points)



Voltage Divider

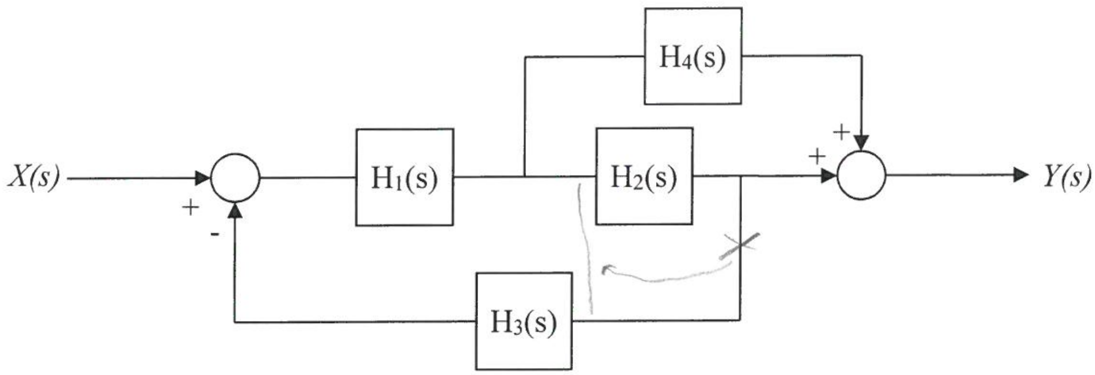
$$V_{out}(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} V_{in}(s)$$

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} = \frac{1}{sRC + 1}$$

$RC = 0.0001$ sec

$$\therefore H(s) = \frac{1}{0.0001s + 1}$$

C. Reduce the following block diagram to a single block. Write the transfer function of the resulting block. (10 Points)



where the transfer functions of the individual blocks are given by

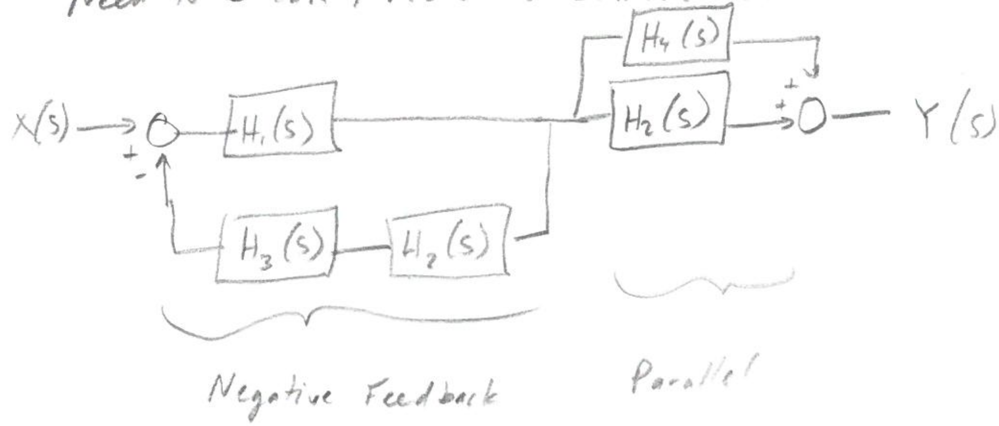
$H_1(s) = \frac{10}{s-1}$

$H_2(s) = s+1$

$H_3(s) = \frac{1}{s+1}$

$H_4(s) = 10$

Need to break / move a connection



$$X(s) - \boxed{\frac{H_1(s)}{1 + H_1(s)H_2(s)H_3(s)}} - \boxed{H_2(s) + H_4(s)} - Y(s)$$

$\hookrightarrow (s+1) + 10 = s+11$

$$\hookrightarrow \frac{\frac{10}{s-1}}{1 + \left(\frac{10}{s-1}\right)(s+1)\left(\frac{1}{s+1}\right)} = \frac{10}{s+9}$$

In series

$$\therefore H(s) = \frac{10}{s+9} \cdot (s+11) = \frac{10s+110}{s+9}$$

PROBLEM 4 (15 Points)

For each of the following systems, determine if the system is stable, marginally stable, or unstable. You must write your answers on the lines provided, and you must justify your answer to receive ANY credit.

A. $H(s) = \frac{(s+1)}{s^3}$ (5 Points)

poles = 0, 0, 0

Stable, Marginally Stable, or Unstable? Marginally Stable

All poles on the imaginary axis

B. $H(s) = \frac{1}{(s-1)^2}$ (5 Points)

poles = 1, 1

Stable, Marginally Stable, or Unstable? Unstable

Two poles are in the Right-Half Plane

C. $H(s) = \dfrac{(s-100)^2}{s^4 + 6s^3 + 13s^2 + 12s + 4}$ (5 Points)

Stable, Marginally Stable, or Unstable? Stable

s^4	1	13	4
s^3	6	12	
s^2	11	4	
s^1	$\frac{108}{11}$		
s^0	4		

$$\frac{(6)(13) - (1)(12)}{6} = 11$$

$$\frac{(11)(12) - (6)(4)}{11} = \frac{108}{11} = 9.82$$

No sign changes

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PROBLEM 5 (25 Points)

A system is defined by the following transfer function.

$$H(s) = \frac{80}{s^s + 12s + 20}$$

This system receives the following input signal.

$$X(s) = \frac{1}{s+10}$$

Determine the exact form of the time-domain output signal, *y(t)*. (More room on the next page)

$$Y(s) = H(s)\,X(s) = \frac{80}{s^2+12s+20} \cdot \frac{1}{s+10}$$

$$Y(s) = \frac{80}{\underbrace{(s^2+12s+20)}_{(s+2)(s+10)}(s+10)} = \frac{80}{(s+2)(s+10)^2}$$

$$Y(s) = \frac{80}{(s+2)(s+10)^2} = \frac{k_1}{s+2} + \frac{k_2}{s+10} + \frac{k_3}{(s+10)^2}$$

Use Residue Method for k_1 and k_3

$$k_1 = (s+2)\,Y(s)\Big|_{s=-2} = \frac{80}{(s+10)^2} = \frac{80}{64} = 1.25$$

$$k_3 = (s+10)^2\,Y(s)\Big|_{s=-10} = \frac{80}{s+2}\Big|_{s=-10} = \frac{80}{-8} = -10$$

Use Brute Force for k_2

$$\frac{k_1(s+10)^2 + k_2(s+2)(s+10) + k_3(s+2)}{(s+2)(s+10)^2}$$

Equate Numerators

$$80 = k_1(s^2+20s+100) + k_2(s^2+12s+20) + k_3(s+2)$$

Problem 5 Work Page

Equate Like powers of s

$s^0 \Rightarrow 80 = k_1(100) + k_2(20) + k_3(2)$

$80 = 125 + k_2(20) - 20$

$k_2 = \frac{-25}{20} = -1.25$

Check

$$\frac{(1.25)(s^2+20s+100) - (1.25)(s^2+12s+20) - (10)(s+2)}{(s+2)(s+10)^2} =$$

$$= \frac{80}{(s+2)(s+10)^2} \quad \checkmark$$

$$Y(s) = \frac{1.25}{s+2} - \frac{1.25}{s+10} - \frac{10}{(s+10)^2}$$

$$y(t) = 1.25e^{-2t}u(t) - 1.25e^{-10t}u(t) - 10te^{-10t}u(t)$$

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PROBLEM 6 (10 Points)

You are given a system defined by the following transfer function.

$$H(s)=\frac{20}{2s+10}$$

If this system has a step input, provide the following information on the lines that have been provided and sketch the time-domain output signal. Label as much as possible on your sketch. (Extra Credit on the following page)

Time Constant, τ 0.2 sec

Steady-State Output, y(∞) 2

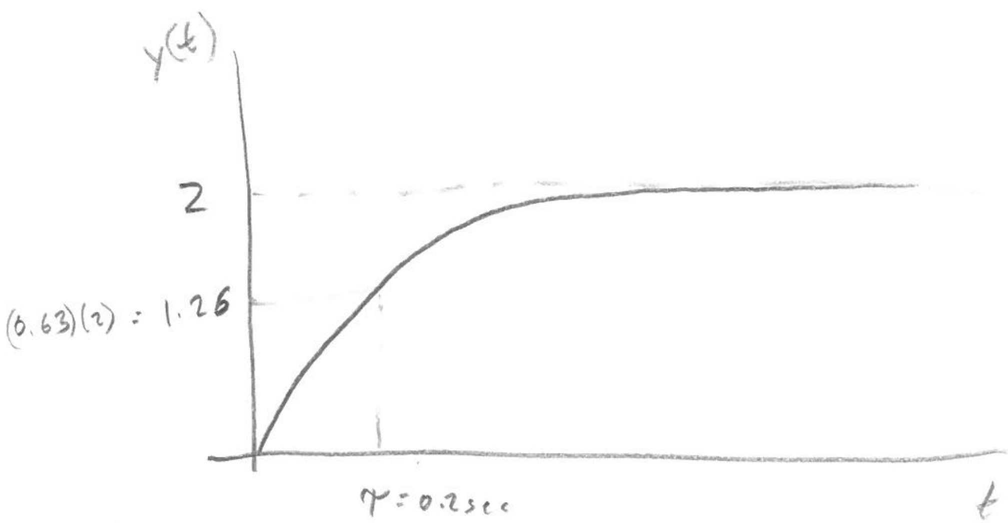
$$H(s)=\frac{20}{2s+10} \rightarrow \text{simplify} \rightarrow H(s)=\frac{10}{s+5}$$

pole $= -5$

$$\tau = \frac{-1}{\text{pole}} = 0.2 \text{ sec}$$

$$y(\infty)=\lim_{s\to 0} sY(s) = \lim_{s\to 0} sH(s)X(s) = \lim_{s\to 0} sH(s)\frac{1}{s} =$$

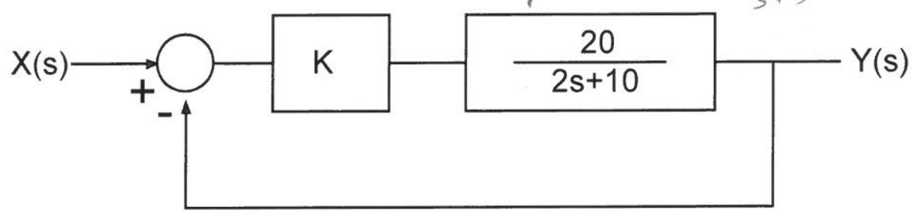
$$= \lim_{s\to 0} H(s) = \lim_{s\to 0}\frac{10}{s+5} = 2$$



EXTRA CREDIT. Your task is to determine if you can design a controller, K, within the feedback system shown below that causes the overall system to have a time constant of $\tau = 0.01\,\text{sec}$. Clearly state whether or not if this is possible. Justify your answer. If it is possible, determine the appropriate value(s) of K.

(5 Points)

simplifies to $\frac{10}{s+5}$



Simplify the block diagram (Negative Feedback Structure)

$$H(s) = \frac{\frac{10K}{s+5}}{1 + \frac{10K}{s+5}} = \frac{10K}{s + 5 + 10K}$$

pole $= -(5 + 10K)$

$$\tau = -\frac{1}{\text{pole}} = \frac{1}{5 + 10K} = 0.01\,\text{sec}$$

$$(\tau)(5 + 10K) = (\tau)(5)(1+2K) = 1$$

$$1+2K = \frac{1}{5\tau}$$

$$K = \frac{\frac{1}{5\tau} - 1}{2} = \frac{\frac{1}{(5)(0.01)} - 1}{2} = 9.5$$

Yes, if $K = 9.5$, then the overall system can achieve $\tau = 0.01$ sec