EE 327 Signals and Systems 1 | Name Solutions
Test 3 | November 20, 2013

RULES

This is a closed book, closed notes test. You are, however, allowed one piece of paper (front side and half of the back side only) for notes and definitions, but no sample problems. You must staple your definitions sheet to the back of your test when you hand your test in. You are also permitted to use a calculator. Additionally, tables of common transforms have been provided at the end of this test.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

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PROBLEM 1 (25 Points)

A system is defined by the following transfer function.

$$H(s) = \frac{3{,}000{,}000s}{s^2 + 3{,}010s + 30{,}000}$$

Determine all zeros, poles, and corner frequencies, and write them on the lines that have been provided. Also, plot the frequency response (Bode plot – both magnitude and phase) of this system on the semilog paper on the following page. Only asymptotic responses are required. Clearly label all important points and all slopes. Clearly indicate which trace is the overall magnitude and phase (summation of the parts).

Zeros 0

Poles $-10,\ -3000$

Corner Frequencies $[0],\ 10,\ 3000$ rad/sec

$$H(s) = \frac{3{,}000{,}000(s)}{(s+10)(s+3{,}000)} = \frac{3{,}000{,}000}{(10)(3{,}000)} \frac{s}{\left(\frac{s}{10}+1\right)\left(\frac{s}{3{,}000}+1\right)}$$

$$H(j\omega) = \frac{100\,(j\omega)}{\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{3{,}000}+1\right)}$$

Zero = 0

poles = $-10,\ -3{,}000$

$20\log|100| = 40\text{dB}$

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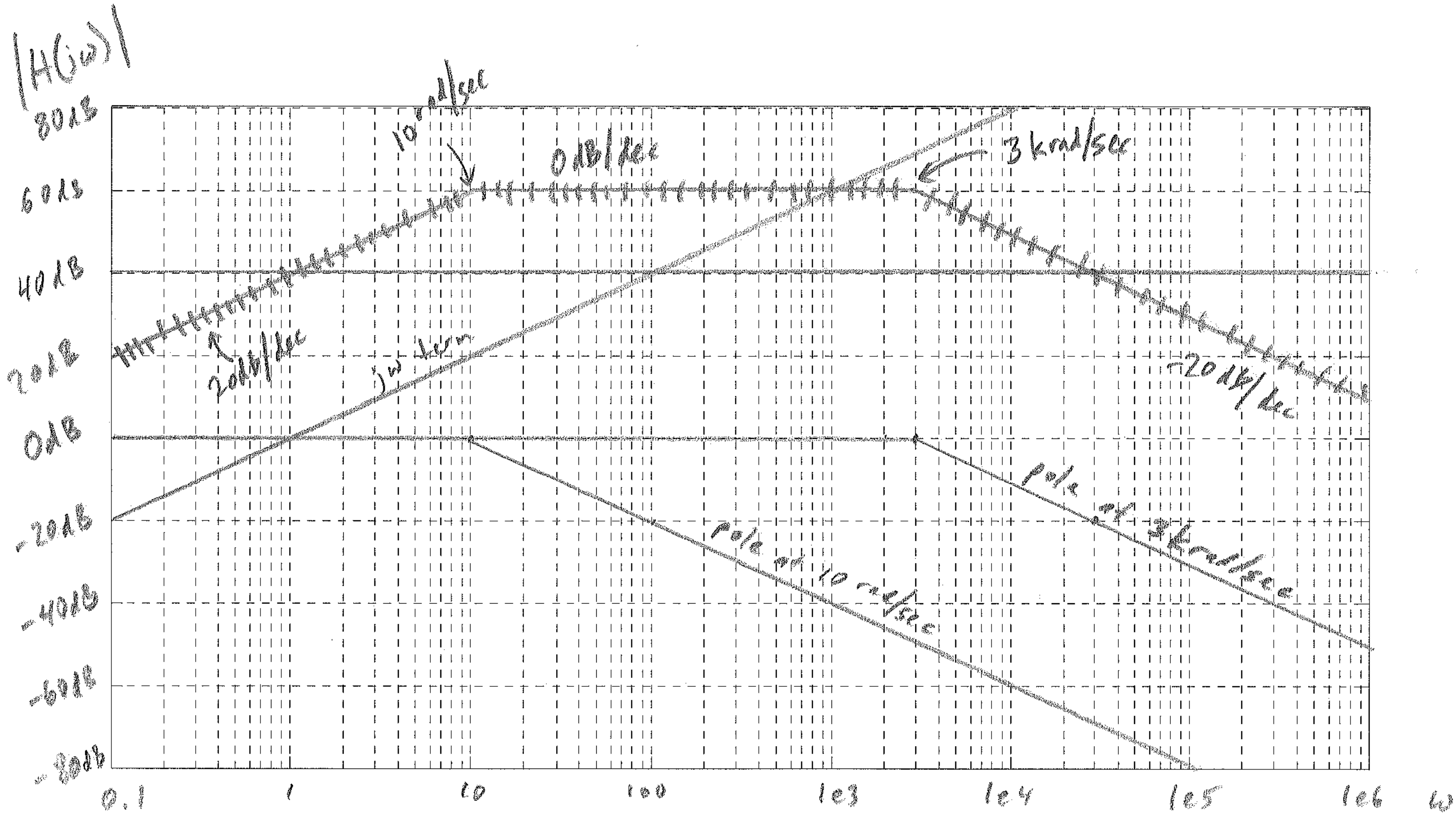

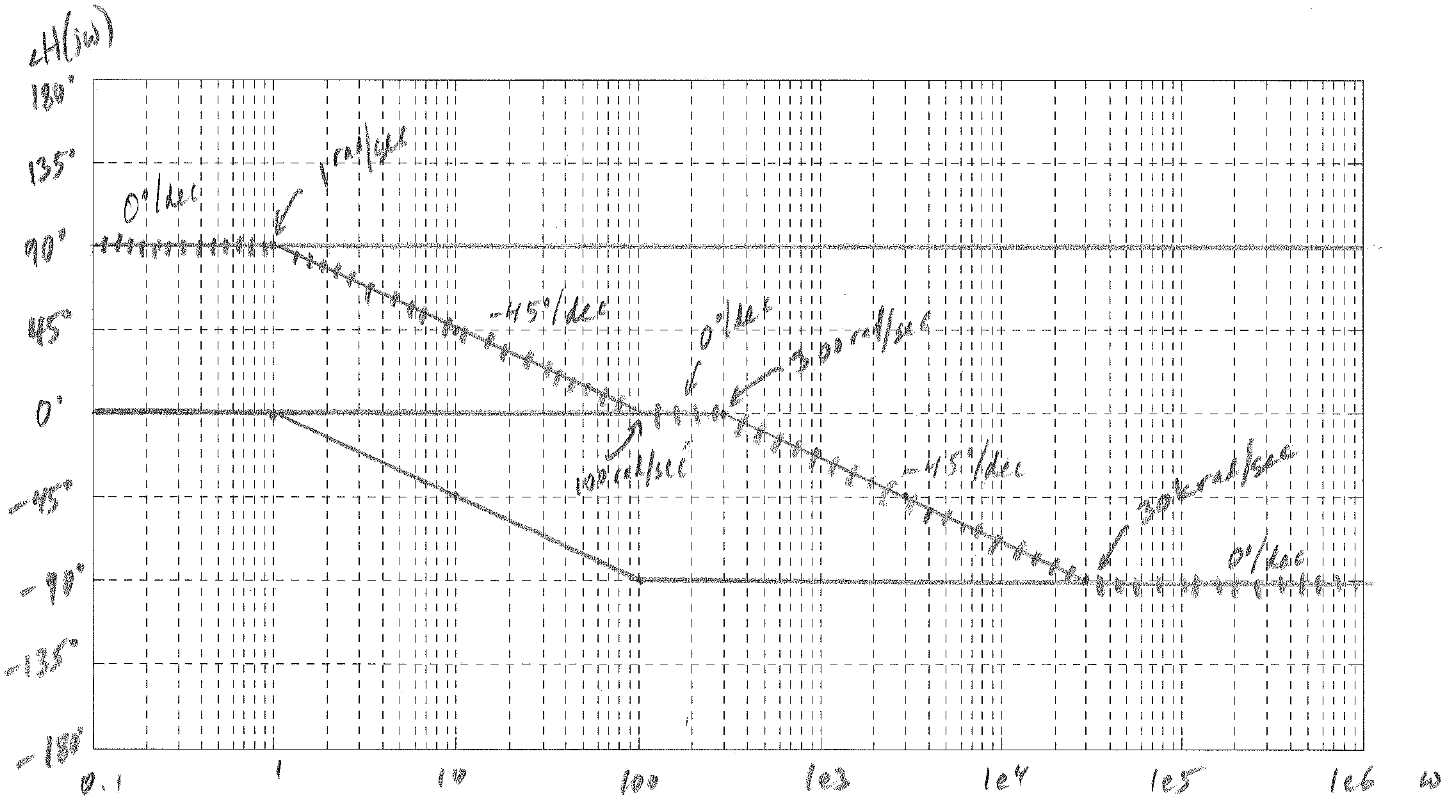

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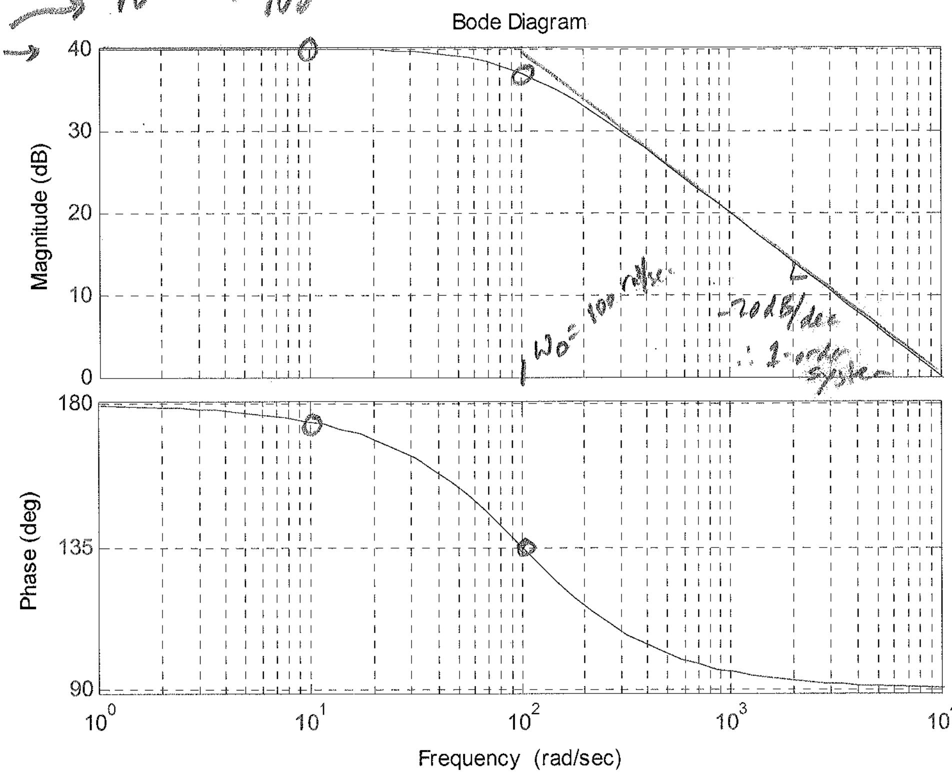
PROBLEM 2 (25 Points)

For the following continuous-time filter, whose frequency response is shown in the following figure, determine the following (see below). You *must* write your answer on the lines provided below. Write all values as scalar numbers (*not* dB values). Only determine the Q of the filter if that is applicable (i.e. there is a Q); otherwise, write N/A for not applicable. (Point values are indicated in parentheses below.) The following page is for extra work.

Passband gain = 40dB → $10^{40/20} = 100$



Total phase change of −90° ∴ one pole

Type of filter (i.e. filtering operation) (3) Lowpass

Order of the filter (2) First-order

Determine ω_0 (corner or center freq.) (2) 100 rad/sec

Determine Q (if applicable) (2) N/A (First-order systems do not have Q)

Passband gain (2) 100

Transfer function of the filter (4) $H(s) = \dfrac{-100}{\frac{s}{100}+1} = \dfrac{-10000}{s+100}$ ← Negative sign because a constant phase offset of 180°

Steady-state response to $x(t) = 10\cos(10t) + \cos(100t + 90^\circ)$ (10) (line below)

@ $\omega = 10$ rad/sec
$|H(j10)| = 40\text{dB} \rightarrow 10^{40/20} = 100$
$\angle H(j10) \approx 175^\circ$
@ $\omega = 100$ rad/sec
$|H(j100)| = 37\text{dB}$ (down from 40dB by 3dB → at corner freq) → ∴ $10^{37/20} \approx 70.7$
$\angle H(j100) = 135^\circ$

$y(t) = (10)(100)\cos(10t + 175^\circ) + (70.7)\cos(100t + 90^\circ + 135^\circ)$
$y(t) = 1000\cos(10t + 175^\circ) + (70.7)\cos(100t + 225^\circ)$ or -135°

(PROBLEM 2 Work Page)

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PROBLEM 3 (25 Points)

A system is defined by the following transfer function.

$$H(z) = \frac{z}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}$$

This system receives the following input.

$$X(z) = \frac{\frac{1}{4}z}{z+\frac{1}{4}}$$

A. Determine the z-domain output, *Y(z)*, of the system, *H(z)*, in response to the input, *X(z)*. (5 Points)

$$Y(z) = H(z)X(z) = \frac{z}{(z+\frac{1}{2})(z-\frac{1}{4})} \cdot \frac{\frac{1}{4}z}{(z+\frac{1}{4})} =$$

$$= \frac{1}{4}\frac{z^2}{(z+\frac{1}{2})(z-\frac{1}{4})(z+\frac{1}{4})}$$

← degree = 2
← degree = 3
∴ Can use Partial Fraction Expansion

B. Determine the general form of the time-domain solution, *y[n]*. (5 Points)

$$Y(z) = \frac{k_1}{z+\frac{1}{2}} + \frac{k_2}{z-\frac{1}{4}} + \frac{k_3}{z+\frac{1}{4}}$$

$$= k_1 z^{-1}\frac{z}{z+\frac{1}{2}} + k_2 z^{-1}\frac{z}{z-\frac{1}{4}} + k_3 z^{-1}\frac{z}{z+\frac{1}{4}}$$

$$\therefore y[n] = k_1\left(-\frac{1}{2}\right)^{n+1}u[n-1] + k_2\left(\frac{1}{4}\right)^{n-1}u[n-1] + k_3\left(-\frac{1}{4}\right)^{n-1}u[n-1]$$

C. Determine the complete solution for the time-domain response, *y[n]*. (10 Points)

$$Y(z) = \frac{\frac{1}{4} z^2}{(z+\frac{1}{2})(z-\frac{1}{4})(z+\frac{1}{4})} \quad \begin{matrix} \leftarrow \text{degree} = 2 \\ \leftarrow \text{degree} = 3 \end{matrix} \quad \therefore \text{ can use P.F.E.}$$

$$= \frac{k_1}{z+\frac{1}{2}} + \frac{k_2}{z-\frac{1}{4}} + \frac{k_3}{z+\frac{1}{4}}$$

Use Residue Method

$$k_1 = \frac{\frac{1}{4} z^2 \cancel{(z+\frac{1}{2})}}{\cancel{(z+\frac{1}{2})}(z-\frac{1}{4})(z+\frac{1}{4})}\Bigg|_{z=-\frac{1}{2}} = \frac{\frac{1}{4}(\frac{1}{4})}{(\frac{-3}{4})(\frac{-1}{4})} = \frac{1}{3}$$

$$k_2 = \frac{\frac{1}{4} z^2 \cancel{(z-\frac{1}{4})}}{(z+\frac{1}{2})\cancel{(z-\frac{1}{4})}(z+\frac{1}{4})}\Bigg|_{z=\frac{1}{4}} = \frac{(\frac{1}{4})(\frac{1}{16})}{(\frac{3}{4})(\frac{1}{2})} = \frac{1}{24}$$

$$k_3 = \frac{\frac{1}{4} z^2 \cancel{(z+\frac{1}{4})}}{(z+\frac{1}{2})(z-\frac{1}{4})\cancel{(z+\frac{1}{4})}}\Bigg|_{z=-\frac{1}{4}} = \frac{(\frac{1}{4})(\frac{1}{16})}{(\frac{1}{4})(-\frac{1}{2})} = -\frac{1}{8}$$

$$Y(z) = \frac{1}{3} z^{-1} \frac{z}{z+\frac{1}{2}} + \frac{1}{24} z^{-1} \frac{z}{z-\frac{1}{4}} - \frac{1}{8} z^{-1} \frac{z}{z+\frac{1}{4}} \Rightarrow$$

$$y[n] = \underbrace{\frac{1}{3}}_{0.33} \left(-\frac{1}{2}\right)^{n-1} u[n-1] + \underbrace{\frac{1}{24}}_{0.04167} \left(\frac{1}{4}\right)^{n-1} u[n-1] - \underbrace{\frac{1}{8}}_{0.125} \left(-\frac{1}{4}\right)^{n-1} u[n-1]$$

D. Determine the final value of *y[n]* as n→∞. (5 Points)

All poles are inside the unit circle, so stable

$$\therefore \; y[\infty] = \lim_{z \to 1} (z-1) Y(z) = 0$$

Also, as $n \to \infty$ for $y[n]$ from Part C, all terms decay to zero

PROBLEM 5 (25 Points)

A. Find the transfer function of the following discrete-time system. Write your answers as rational functions of "z," and simplify where applicable. (10 Points)

$$y[n]+\frac{1}{4}y[n-2]=x[n]-x[n-1]$$

- Take z transform
- Set all initial conditions to 0

$$\Rightarrow Y(z)+\frac{1}{4}z^{-2}Y(z)=X(z)-z^{-1}X(z)$$

$$Y(z)\left(1+\frac{1}{4}z^{-2}\right)=X(z)\left(1-z^{-1}\right)$$

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1-z^{-1}}{1+\frac{1}{4}z^{-2}}\cdot\frac{z^2}{z^2}=\frac{z(z-1)}{z^2+\frac{1}{4}}$$

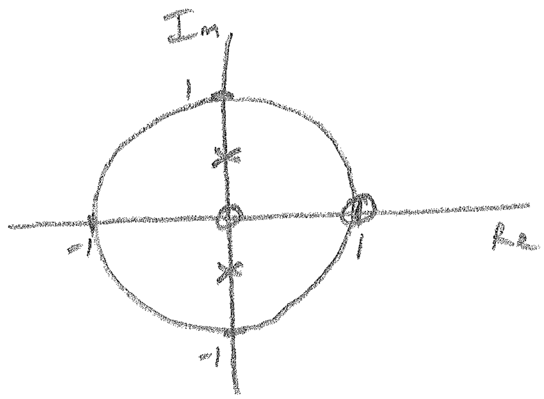
B. For discrete-time systems, describe the condition for stability in terms of poles and zeros. (5 Points)

All poles must lie within the unit circle on the Real-Imaginary plane in order to be stable

C. Plot the poles and zeros of the system defined in Part A on the Real-Imaginary plane. (5 Points)

$$H(z) = \frac{z(z-1)}{z^2 + \frac{1}{4}} \leftarrow \text{zeros} = 0, 1$$

$$\text{poles} = \frac{-0 \pm \sqrt{0-(4)(1)(\frac{1}{4})}}{2(1)} = \frac{\pm\sqrt{-1}}{2} = \pm\frac{j}{2}$$



D. Is the system of Part A stable, marginally stable, or unstable. You must justify your answer to receive full credit. (5 Points)

Stable → Both poles are inside the unit circle.

Problem 3.c. Alternate Method

$$Y(z) = \frac{\frac{1}{4}z^2}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)}$$

$$\frac{Y(z)}{z} = \frac{\frac{1}{4}z}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)} = \frac{k_1}{z+\frac{1}{2}} + \frac{k_2}{z-\frac{1}{4}} + \frac{k_3}{z+\frac{1}{4}}$$

$$k_1 = \left.\frac{\frac{1}{4}z}{\left(z-\frac{1}{4}\right)\left(z+\frac{1}{4}\right)}\right|_{z=-\frac{1}{2}} = -\frac{2}{3}$$

$$k_2 = \left.\frac{\frac{1}{4}z}{\left(z+\frac{1}{2}\right)\left(z+\frac{1}{4}\right)}\right|_{z=\frac{1}{4}} = \frac{1}{6}$$

$$k_3 = \left.\frac{\frac{1}{4}z}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{4}\right)}\right|_{z=-\frac{1}{4}} = \frac{1}{2}$$

$$\frac{Y(z)}{z} = \frac{-\frac{2}{3}}{z+\frac{1}{2}} + \frac{\frac{1}{6}}{z-\frac{1}{4}} + \frac{\frac{1}{2}}{z+\frac{1}{4}}$$

$$Y(z) = -\frac{2}{3}\,\frac{z}{z+\frac{1}{2}} + \frac{1}{6}\,\frac{z}{z-\frac{1}{4}} + \frac{1}{2}\,\frac{z}{z+\frac{1}{4}}$$

$$y[n] = \underbrace{\left(-\frac{2}{3}\right)}_{-0.667}\left(-\frac{1}{2}\right)^n u[n] + \underbrace{\left(\frac{1}{6}\right)}_{0.167}\left(\frac{1}{4}\right)^n u[n] + \left(\frac{1}{2}\right)\left(-\frac{1}{4}\right)^n u[n]$$

This expression is exactly the same as the other expression given by the other method (even though they may initially appear different).