EE 327 Signals and Systems 1
Test 3

Name Solutions
November 19, 2014

RULES

This is a closed book, closed notes test. You are, however, allowed one piece of paper (front side and half of the back side only) for notes and definitions, but no sample problems. The front side should be the same as from the second test, and the top half of the reverse side contains the information added for the third test. You must staple your definitions sheet to the back of your test when you hand your test in. You are also permitted to use a calculator. Additionally, tables of common transforms have been provided at the end of this test.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	25	
2	30	
3	20	
4	25	
Total	100	

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PROBLEM 1 (25 Points)

A system is defined by the following transfer function.

$$H(s) = \frac{1{,}000s^2 + 10{,}000s}{s^2 + 100s + 1{,}000{,}000}$$

Determine all zeros, poles, and corner frequencies, and write them on the lines that have been provided. Also, plot the frequency response (Bode plot – both magnitude and phase) of this system on the semilog paper on the following page. Only asymptotic responses are required. Clearly label all important points and all slopes. Clearly indicate which trace is the overall magnitude and phase (summation of the parts).

Zeros $0, -10$

Poles $-50 + j998.7, -50 - j998.7$

Corner Frequencies $[0], 10, 1000$ rad/sec

$$H(j\omega) = \frac{1{,}000(j\omega)^2 + 10{,}000j\omega}{(j\omega)^2 + 100j\omega + 1{,}000{,}000}$$

Factor the denominator

$$\frac{-100 \pm \sqrt{(100)^2 - (4)(1)(1{,}000{,}000)}}{2} = -50 \pm \frac{1}{2}\sqrt{10{,}000 - 4{,}000{,}000}$$

$$= -50 \pm \frac{1}{2}\sqrt{-3{,}990{,}000}$$

$$= -50 \pm j998.7$$ ← complex poles

$$H(j\omega) = \frac{\frac{(10)(1{,}000)}{(1{,}000{,}000)}(j\omega)\left(\frac{j\omega}{10} + 1\right)}{\left(\frac{j\omega}{1000}\right)^2 + \frac{j\omega}{10{,}000} + 1}$$

↖ ω_n

$$H(j\omega) = \frac{(0.01)(j\omega)\left(\frac{j\omega}{10} + 1\right)}{\left(\frac{j\omega}{1000}\right)^2 + \left(\frac{j\omega}{10{,}000}\right) + 1}$$

Constant term $= 0.01$ → positive

$20\log|0.01| = -40\text{dB}$

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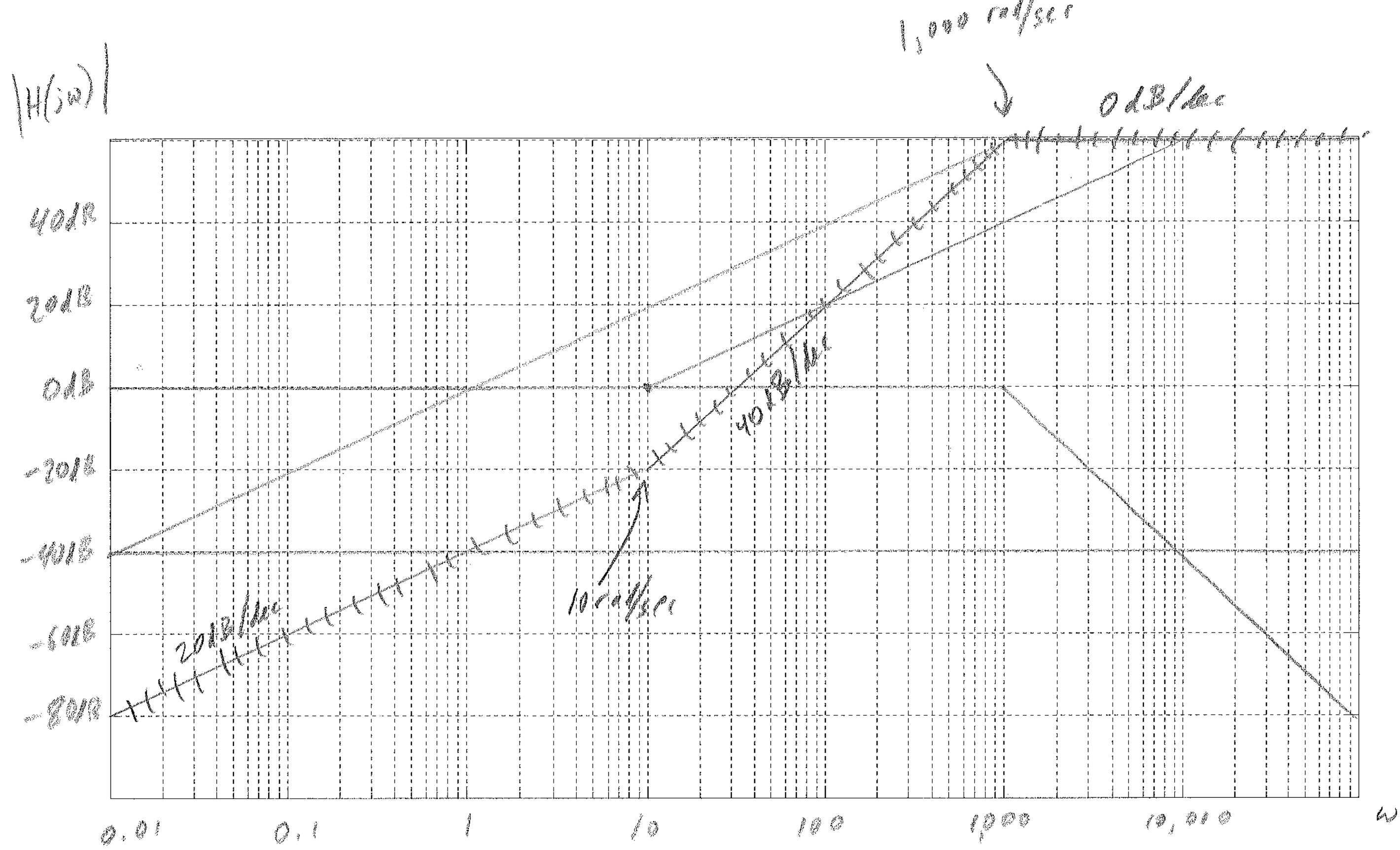

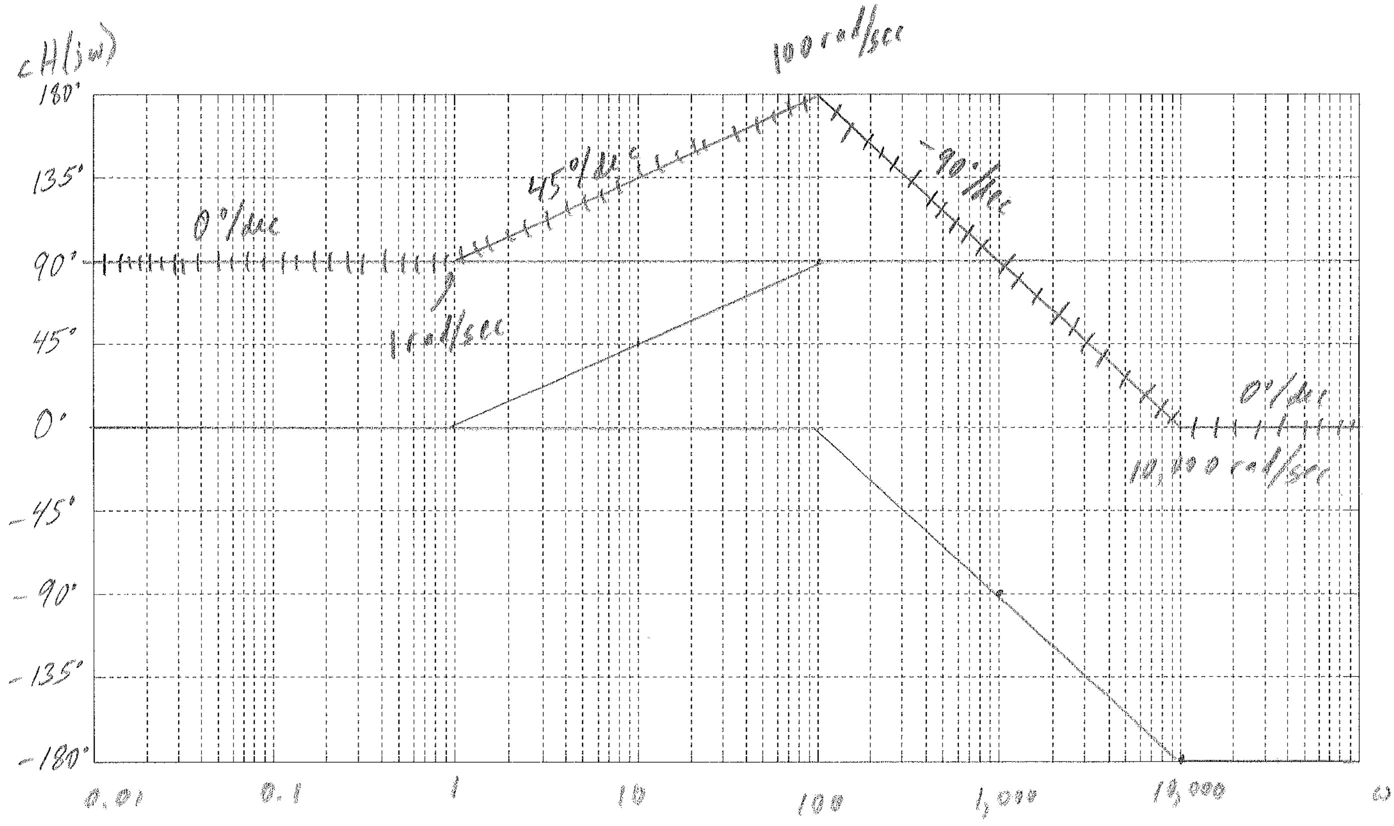

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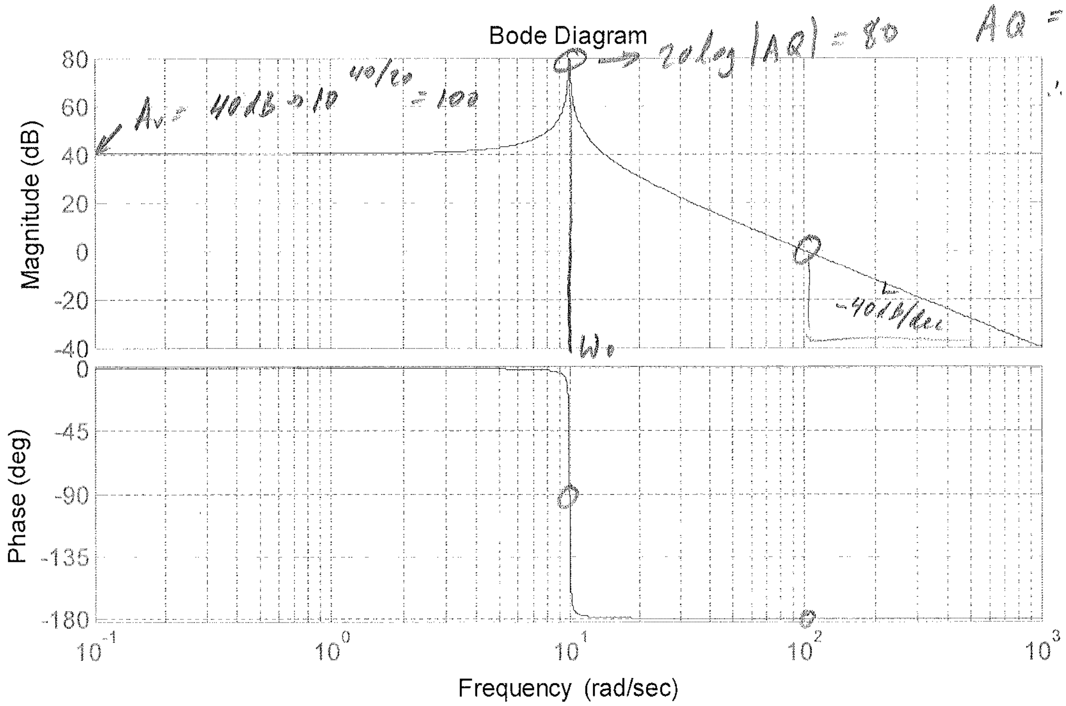
PROBLEM 2 (30 Points)

For the following continuous-time filter, whose frequency response is shown in the following figure, determine the following (see below). You *must* write your answer on the lines provided below. Write all values as scalar numbers (*not* dB values). Only determine the Q of the filter if it is applicable (for example, only if there is a Q); otherwise, write N/A for not applicable. (Point values are indicated in parentheses below.)



$AQ = 10^{80/20} = 10{,}000$

$\therefore Q = \frac{10{,}000}{A} = \frac{10{,}000}{100} = 100$

Filtering operation *Lowpass Filter* (3)

Order of the filter *2nd order* (3)

Determine ω_0 *10 rad/sec* (3)

Determine Q (if applicable) *100* (3)

Passband gain *100* (3)

$H(s) = A_v \dfrac{1}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q\omega_0} + 1}$

Transfer function of the filter (5) $H(s) = \dfrac{100}{\left(\frac{s}{10}\right)^2 + \frac{s}{1000} + 1} = \dfrac{10{,}000}{s^2 + 0.1s + 100}$

Steady-state response to $x(t) = 10\cos(10t) + \cos(100t + 90^\circ)$
(write on the line below)

$y(t) = 100{,}000\cos(10t - 90^\circ) + \cos(100t - 90^\circ)$ (10)

$\omega = 10 \to |H(j10)| = 80dB \to 10{,}000$, $\angle H(j10) = -90^\circ$

$\omega = 100 \to |H(j100)| = 0dB \to 1$, $\angle H(j100) = -180^\circ$

Problem 2 Work Page

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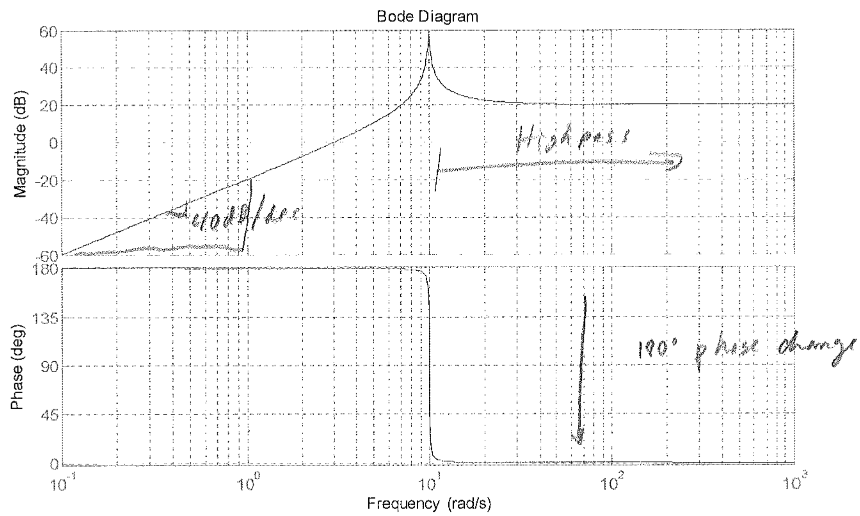
PROBLEM 3 (20 Points)

For each system below, determine the filtering operation (e.g. lowpass, etc.) and the order of the filter (e.g. first order, etc.). Write your answers on the lines provided. Point values are shown in parentheses.

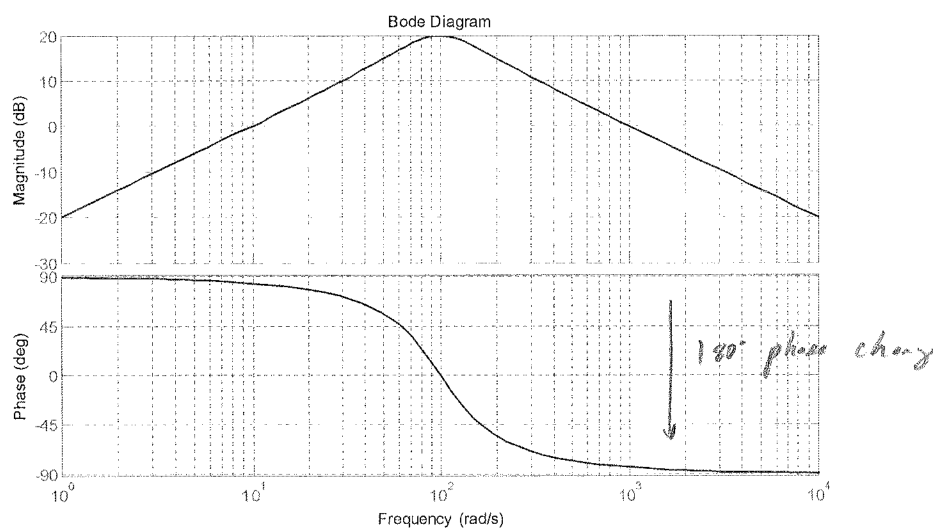
A.



Filtering Operation *Highpass* (2)

Filter Order *2nd order* (2)

B.



Filtering Operation *Bandpass* (2)

Filter Order *2nd order* (2)

C. $H(s) = \dfrac{1000s}{s^2 + 1000s + 100}$ ← Canonical form for Bandpass Filter

Filtering Operation Bandpass (2)

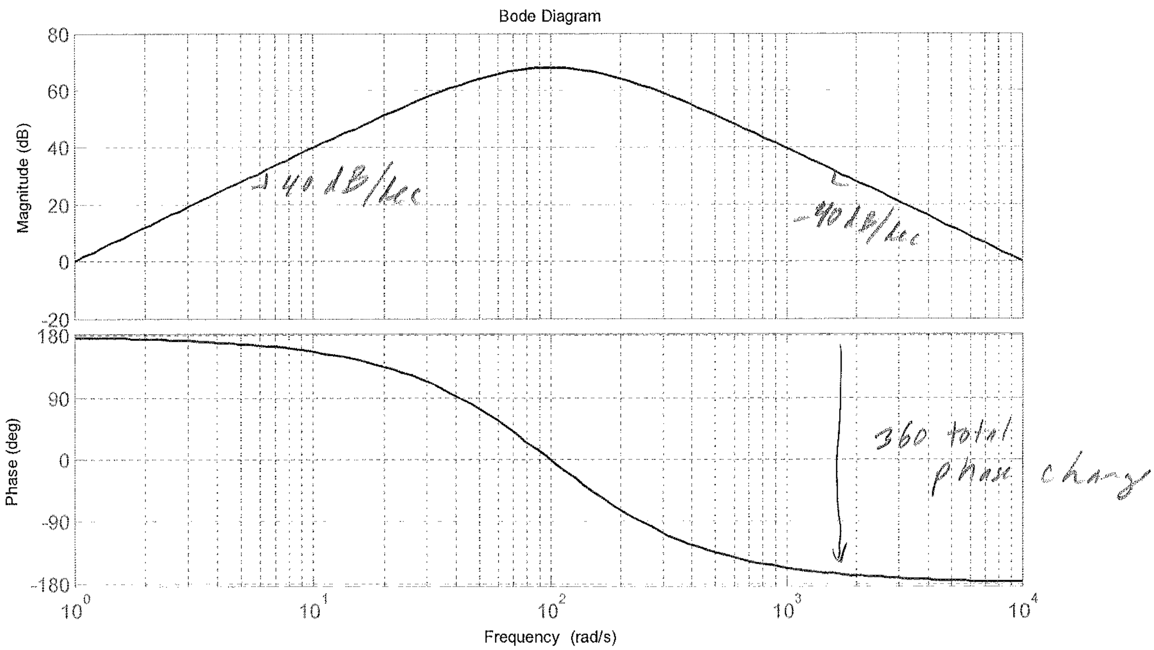
Filter Order 2nd order (2)

D. $H(s) = \dfrac{1000}{(s+10)^3} = \left(\dfrac{10}{s+10}\right)\left(\dfrac{10}{s+10}\right)\left(\dfrac{10}{s+10}\right)$ ← cascade of 3 first order filters each which are lowpass filters

Filtering Operation Lowpass (2)

Filter Order 3rd order (2)

E.



Filtering Operation Bandpass (2)

Filter Order 4th order (2)

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PROBLEM 4 (25 Points)

Find the following transforms.

A. Find the z transform of x[n]. Write as a rational function of 'z' and simplify where applicable. (3 Points)

$$x[n] = 2\delta[n] + 4\delta[n-1] + 6\delta[n-2]$$

$$X(z) = 2 + 4z^{-1} + 6z^{-2} = \frac{2z^2 + 4z + 6}{z^2}$$

B. Find the z transform of x[n]. Write as a rational function of 'z' and simplify where applicable. (3 Points)

$$x[n] = 5\left(\frac{1}{2}\right)^n u[n]$$

$$X(z) = \frac{5z}{z - \frac{1}{2}}$$

C. Let $v[n]$ have a z transform of $V(z) = \dfrac{z}{z - 0.5}$

Find the z transform of $x[n] = (2)^n v[n]$. Write as a rational function of 'z' and simplify where applicable. (4 Points)

$$X(z) = \frac{\frac{z}{2}}{\frac{z}{2} - \frac{1}{2}} = \frac{z}{z-1}$$

Alternate method

Recognize $V(z) = \dfrac{z}{z - \frac{1}{2}} \Leftrightarrow \left(\frac{1}{2}\right)^n u[n]$

Then $(2)^n \left(\frac{1}{2}\right)^n u[n] = \left[\frac{2}{2}\right]^n u[n] = (1)^n u[n] = u[n] \Leftrightarrow \dfrac{z}{z-1}$

D. Find the inverse z transform of X(z). (15 Points)

$$X(z)=\frac{4z}{\left(z+\frac{1}{2}\right)\left(z+\frac{1}{3}\right)} \quad \begin{matrix}\leftarrow \text{order} = 1 \\ \leftarrow \text{order} = 2\end{matrix}$$

$$X(z) = \frac{k_1}{z+\frac{1}{2}} + \frac{k_2}{z+\frac{1}{3}}$$

$$k_1 = X(z)\left(z+\tfrac{1}{2}\right)\Big|_{z=-\frac{1}{2}} = \frac{4z}{z+\frac{1}{3}}\Big|_{z=-\frac{1}{2}} = \frac{-2}{-\frac{1}{2}+\frac{1}{3}} = \frac{-2}{\frac{-3+2}{6}} = 12$$

$$k_2 = X(z)\left(z+\tfrac{1}{3}\right)\Big|_{z=-\frac{1}{3}} = \frac{4z}{z+\frac{1}{2}}\Big|_{z=-\frac{1}{3}} = \frac{-\frac{4}{3}}{-\frac{1}{3}+\frac{1}{2}} = \frac{-\frac{4}{3}}{\frac{-2+3}{6}} = -8$$

$$X(z) = \frac{12}{z+\frac{1}{2}} - \frac{8}{z+\frac{1}{3}}$$

check $\rightarrow$ $$\frac{12\left(z+\frac{1}{3}\right)-8\left(z+\frac{1}{2}\right)}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} = \frac{12z+4-8z-4}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} = \frac{4z}{\left(z+\frac{1}{2}\right)\left(z-\frac{1}{3}\right)} \checkmark$$

$$X(z) = \frac{12z}{z+\frac{1}{2}}z^{-1} - \frac{8z}{z+\frac{1}{3}}z^{-1}$$

$$x[n] = 12\left(-\tfrac{1}{2}\right)^{n-1}u[n-1] - 8\left(-\tfrac{1}{3}\right)^{n-1}u[n-1]$$