EE 327 Signals and Systems 1 Name Solutions

Test 3 November 18, 2015

RULES

This is a closed book, closed notes test. You are, however, allowed one piece of paper (front side and half of the back side only) for notes and definitions, but no sample problems. The front side should be the same as from the second test, and the top half of the reverse side contains the information added for the third test. You must staple your definitions sheet to the back of your test when you hand your test in. You are also permitted to use a calculator. Additionally, tables of common transforms have been provided at the end of this test.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	25	
2	20	
3	30	
4	13	
5	12	
Total	100	

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PROBLEM 1 (25 Points)

A system is defined by the following transfer function.

$$H(s) = \frac{100s}{s^2 + 110s + 1000} = \frac{100s}{(s+10)(s+100)}$$

Determine all zeros, poles, and corner frequencies, and write them on the lines that have been provided. Also, plot the frequency response (Bode plot – both magnitude and phase) of this system on the semilog paper on the following page. Only asymptotic responses are required. Clearly label all important points and all slopes. Clearly indicate which trace is the overall magnitude and phase (summation of the parts).

Zeros: 0

Poles: -10, -100

Corner Frequencies: [0], 10, 100 rad/sec

$$H(j\omega) = \frac{100 j\omega}{(j\omega+10)(j\omega+100)} = \frac{1}{10}\frac{j\omega}{\left(\frac{j\omega}{10}+1\right)\left(\frac{j\omega}{100}+1\right)}$$

Zero = 0

Poles = -10, -100

Corner Freqs = [0], 10, 100 rad/sec

$20\log|0.1| = -20\text{dB}$

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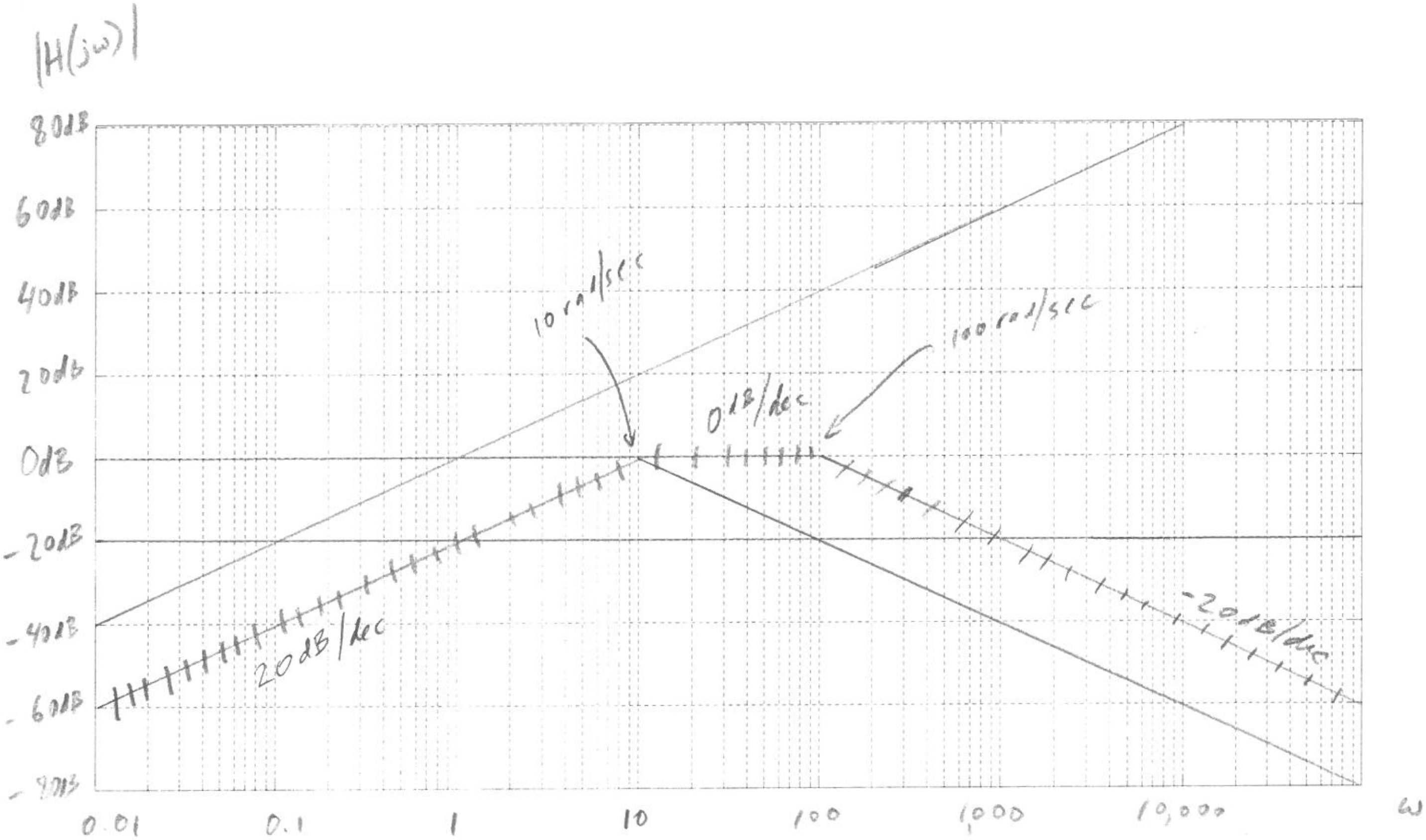

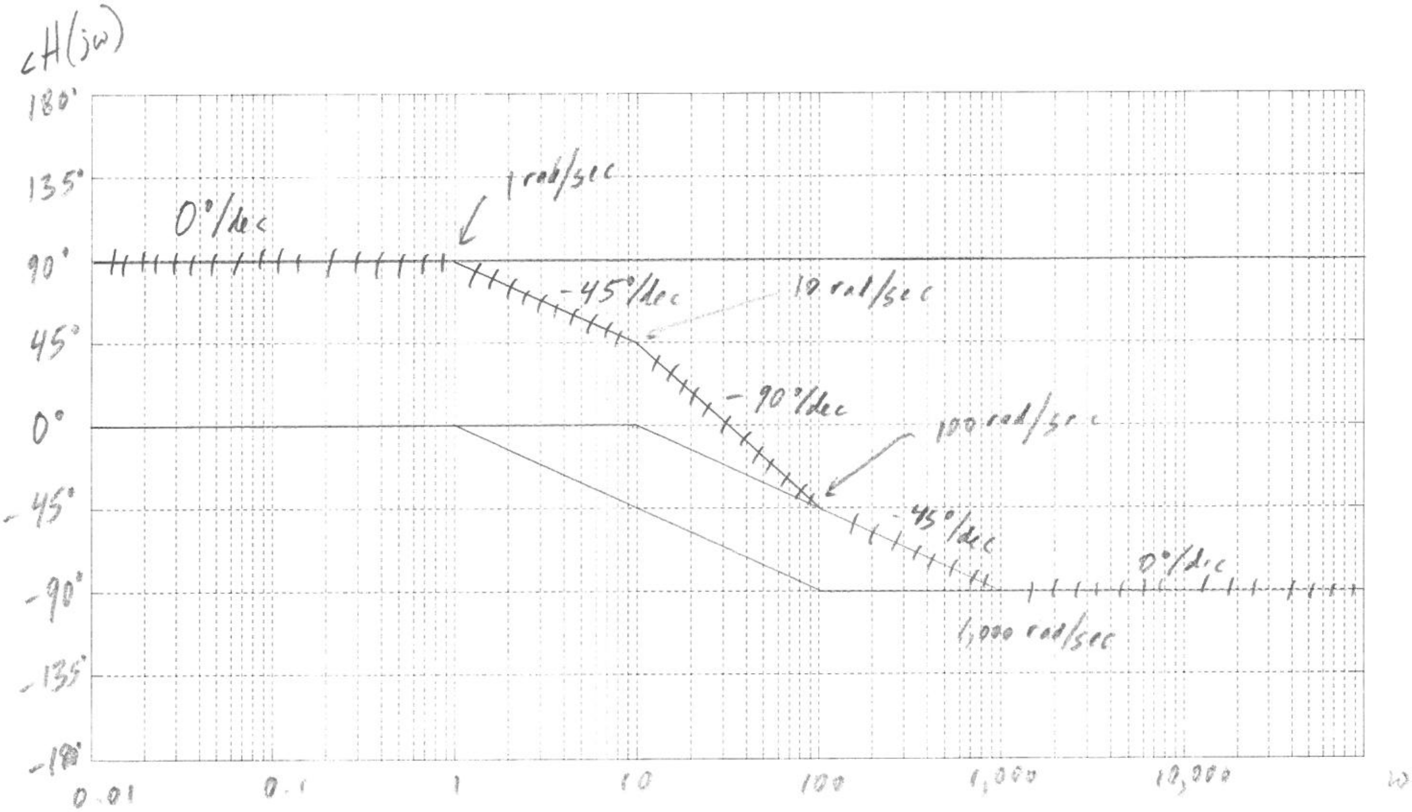

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PROBLEM 2 (20 Points)

A system is defined by the following transfer function.

$$H(z) = \frac{2z}{z - 0.4}$$

This system receives the following input.

$$X(z) = \frac{z}{z + 0.6}$$

A. Determine the z-domain output, *Y(z)*, of the system, *H(z)*, in response to the input, *X(z)*. (5 Points)

$$Y(z) = H(z)X(z) = \frac{2z^2}{(z-0.4)(z+0.6)}$$

$$= \frac{2z^2}{z^2 + 0.2z - 0.24}$$

B. Determine the complete solution for the time-domain response, *y[n]*. (More room on the next page.) (15 Points)

$$\frac{Y(z)}{z} = \frac{2z}{(z-0.4)(z+0.6)} = \frac{k_1}{z-0.4} + \frac{k_2}{z+0.6}$$

$$k_1 = \frac{Y(z)}{z}(z-0.4)\Big|_{z=0.4} = \frac{2z}{z+0.6}\Big|_{z=0.4} = 0.8$$

$$k_2 = \frac{Y(z)}{z}(z+0.6)\Big|_{z=-0.6} = \frac{2z}{z-0.4} = \frac{-1.2}{-1} = 1.2$$

$$\frac{Y(z)}{z} = \frac{0.8}{z-0.4} + \frac{1.2}{z+0.6}$$

$$Y(z) = \frac{0.8z}{z-0.4} + \frac{1.2z}{z+0.6}$$

$$y[n] = 0.8(0.4)^n u[n] + 1.2(-0.6)^n u[n]$$

Name *Solutions*

– Problem 2 Work Page –

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PROBLEM 3 (30 Points)
A discrete-time system is given by the following transfer function. (15 Points)

$$H(z) = \frac{5z}{z - 0.25}$$

Determine the frequency response of this system. Find analytic expressions for both the magnitude and phase and place these expressions on the lines provided. Both equations must be simplified as much as possible, and they may not include any terms with "j" in them. Point values are indicated in parentheses.

Magnitude Frequency Response (8) $H(e^{j\omega}) = \dfrac{5}{\sqrt{1.0625 - \frac{1}{2}\cos(\omega)}}$

Phase Frequency Response (7) $\angle H(e^{j\omega}) = \omega - \tan^{-1}\left(\dfrac{\sin(\omega)}{\cos(\omega) - 0.25}\right)$

$$H(e^{j\omega}) = \frac{5e^{j\omega}}{e^{j\omega} - 0.25}$$

$$|H(e^{j\omega})| = \sqrt{H(e^{j\omega})H^*(e^{j\omega})} = \left[\frac{5e^{j\omega}}{e^{j\omega} - \frac{1}{4}} \cdot \frac{5e^{-j\omega}}{e^{-j\omega} - \frac{1}{4}}\right]^{1/2} =$$

$$= \left[\frac{25}{1.0625 - \frac{1}{4}(e^{j\omega} + e^{-j\omega})}\right]^{1/2} = \frac{5}{\sqrt{1.0625 - \frac{1}{4}2\cos(\omega)}} =$$

$$= \frac{5}{\sqrt{1.0625 - \frac{1}{2}\cos(\omega)}}$$

$$\angle H(e^{j\omega}) = \tan^{-1}(5e^{j\omega}) - \tan^{-1}(e^{j\omega} - 0.25) =$$

$$= \tan^{-1}\left(\frac{\sin(\omega)}{\cos(\omega)}\right) - \tan^{-1}\left(\cos(\omega) + j\sin(\omega) - 0.25\right) =$$

$$= \omega - \tan^{-1}\left(\frac{\sin(\omega)}{\cos(\omega) - 0.25}\right)$$

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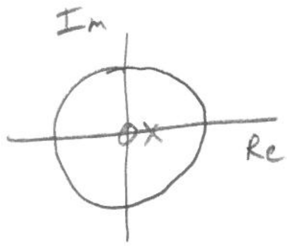
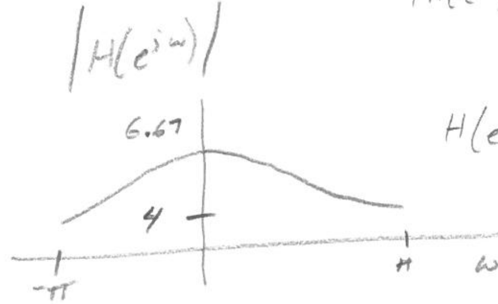
B. Sketch the magnitude frequency response from $-\pi$ to $+\pi$. Be sure to label all important points including all maximum and minimum values. (5 Points)

zero: 0

pole: $\frac{1}{4}$





$|H(e^{j0})| = \frac{5}{\sqrt{1.0625 - (\frac{1}{2})(1)}} = 6.67$

$H(e^{j\pi}) = \frac{5}{\sqrt{1.0625 + \frac{1}{2}}} = 4$

C. What type of filtering operation does this system perform? (2 Points)

Lowpass Filter (look from $\omega = 0$ to π)

D. Is this an FIR filter or an IIR filter. To receive full credit, you must explain why. (3 Points)

IIR → pole not at origin

E. Find the steady-state output, y[n], to the following input. (5 Points)

$$x[n] = 2\cos\left(\frac{\pi}{4}n\right)$$

$\omega = \frac{\pi}{4}$

$|H(e^{j\frac{\pi}{4}})| = \frac{5}{\sqrt{1.0625 - (0.5)(0.707)}} = 5.94$

$\angle H(e^{j\frac{\pi}{4}}) = \frac{\pi}{4} - \tan^{-1}\left(\frac{\sin(\frac{\pi}{4})}{\cos(\frac{\pi}{4}) - 0.25}\right) = -0.2115$ rads

$y[n] = (2)(5.94)\cos\left(\frac{\pi}{4}n - 0.2115\right)$

$y[n] = 11.88\cos\left(\frac{\pi}{4}n - 0.2115\right)$

PROBLEM 4 (13 Points)

A. With regard to poles and zeros, what is the condition for stability for discrete-time systems? (3 Points)

$|poles| < 1$

poles must be inside the unit circle

B. For the following discrete-time system, determine the transfer function, determine whether or not the system is stable, and determine if it is an FIR or IIR filter (if stable, otherwise write N/A for not applicable). You *must* write your answers on the lines provided. Your transfer function *must* be written as a rational function of z to receive full credit. The point values are indicated in parentheses. (10 Points)

$$y[n] = x[n] + 2x[n-2] - 3x[n-3]$$

Transfer Function (6) $H(z) = \dfrac{z^3 + 2z - 1}{z^3}$

Stable, Marginally Stable, or Unstable? (2) Stable

FIR or IIR Filter? (2) FIR

$$Y(z) = X(z) + 2z^{-2}X(z) - 3z^{-3}X(z)$$

$$Y(z) = X(z)\left(1 + 2z^{-2} - 3z^{-3}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = 1 + 2z^{-2} - 3z^{-3} = \frac{z^3 + 2z - 3}{z^3}$$

poles = 0, 0, 0

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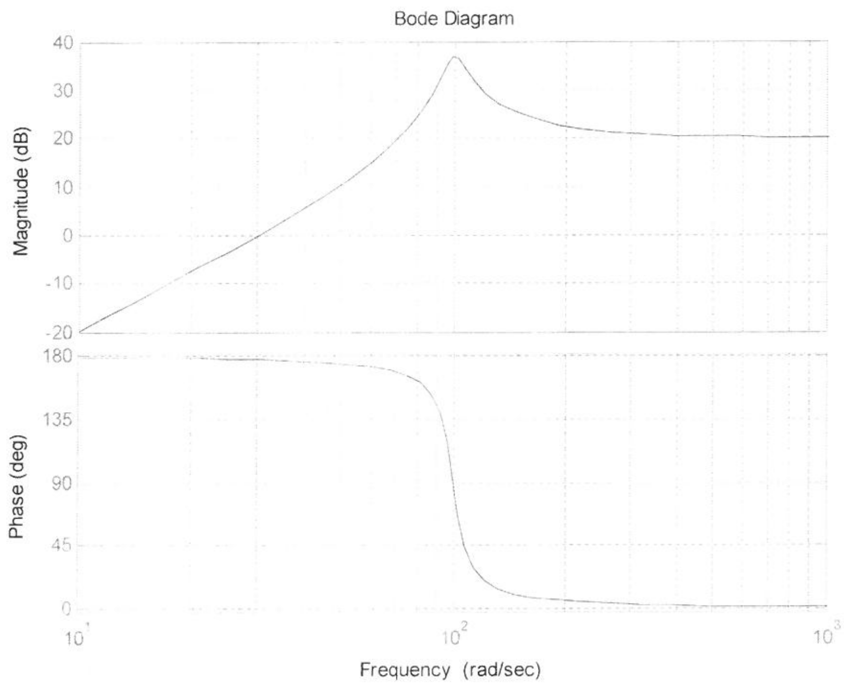
PROBLEM 5 (12 Points)

For each of the following systems, determine what type of filtering operation that system performs (e.g. lowpass, highpass, bandpass, bandstop, all-pass). Parts A and B are continuous-time systems, and Parts C and D are discrete-time systems.

A. (3 Points)

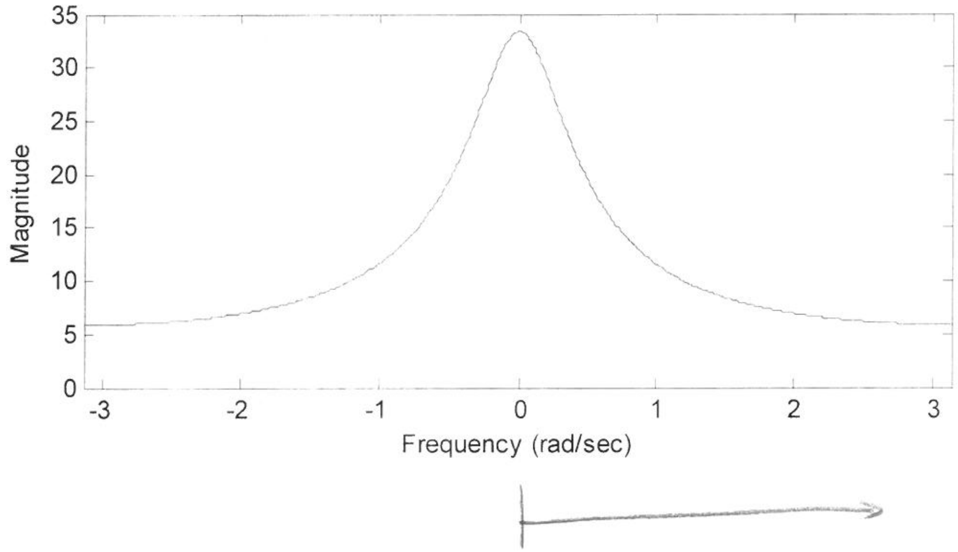


Highpass Filter

B. $H(s) = \frac{100s}{s^2 + 50s + 100}$ (3 Points)

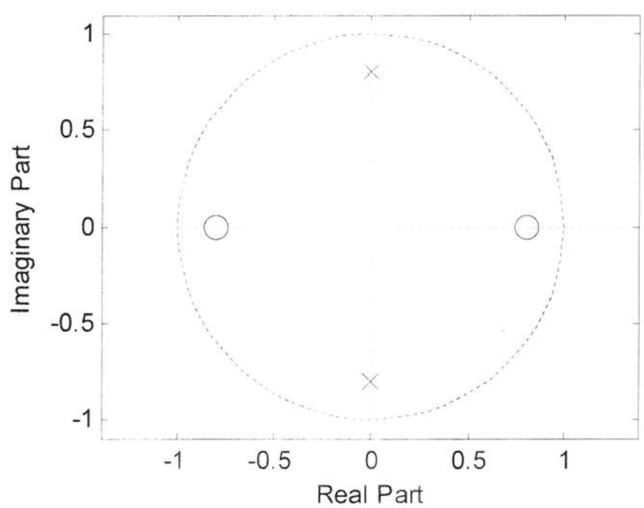
Bandpass Filter

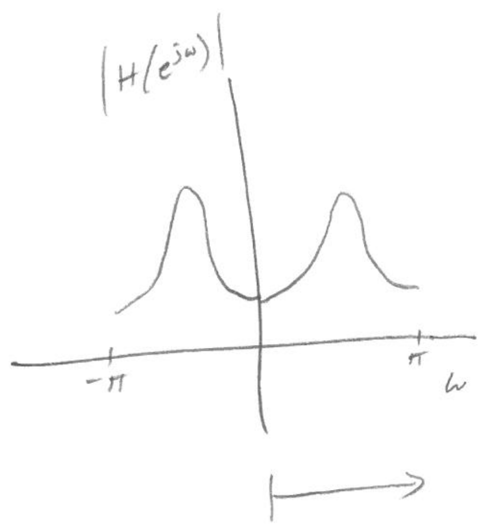
C. (3 Points)



Lowpass Filter

D. (3 Points)





Bandpass Filter