EE 327 Signals and Systems 1 | Name Solutions
Test 3 | November 16, 2016

RULES

This is a closed book, closed notes test. You are, however, allowed one piece of paper (front side and half of the back side only) for notes and definitions, but no sample problems. The front side should be the same as from the second test, and the top half of the reverse side contains the information added for the third test. You must staple your definitions sheet to the back of your test when you hand your test in. You are also permitted to use a calculator. Additionally, tables of common transforms have been provided at the end of this test.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

There is to be absolutely no cheating. Cheating will not be tolerated.

I you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Problem	Value	Score
1	25	
2	25	
3	10	
4	10	
5	30	
Total	100	

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PROBLEM 1 (25 Points)

A second-order, continuous-time system is defined by the following transfer function.

$$H(s) = \frac{2}{s^2 + 10s + 100}$$

This system receives a step input.

A. What is the steady-state output, $y_{ss}(t)$, resulting from a step input? (5 Points)

$$y(\infty) = \lim_{s \to 0} H(s) = \frac{2}{100} = 0.02$$

B. What is the natural frequency, ω_n, for this system? (3 Points)

$$H(s) = \frac{2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\therefore \omega_n = \sqrt{100} = 10 \text{ rad/sec}$$

C. What is the damping ratio, ζ, for this system? (3 Points)

$$2\zeta\omega_n = 10$$

$$\zeta = \frac{10}{2\omega_n} = \frac{1}{2}$$

D. Is this system under damped, critically damped, or over damped? Why? (2 Points)

Under damped

because $\zeta < 1$ (complex poles)

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E. What is the (dominant) time constant for this system? (3 Points)

$$\tau = \frac{1}{\zeta \omega_n} = \frac{1}{(0.5)(10)} = \frac{1}{5} = 0.2 \text{ sec}$$

F. Does this system overshoot the steady-state value if there is a step input? Explain why or why not. If so, what is the percent overshoot? (3 Points)

Yes → Under-damped

$$P.O. \approx (100\%)\, e^{-\zeta\pi/\sqrt{1-\zeta^2}} = 16.3\%$$

G. With a step input, does this system oscillate? Explain why or why not. If so, at what frequency? (3 Points)

Yes → Under-damped

$$\omega_d = \omega_n \sqrt{1-\zeta^2} = 8.66 \text{ rad/sec}$$

H. When plotted from 0 to 100 seconds, what is the maximum value that the output signal achieves over this time period? If you do not have enough information to provide an exact answer, state that you do not have enough information and provide an approximate value for the maximum value. (3 Points)

Maximum Value at the overshoot

$$P.O. = (100\%)\frac{Y_{max} - Y_{ss}}{Y_{ss}}$$

$$(Y_{ss})\left(\frac{P.O.}{100\%}\right) + Y_{ss} = Y_{max}$$

$$\therefore\ Y_{max} = (0.02)(0.163) + 0.02 = 0.02326$$

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PROBLEM 2 (25 Points)

A system is defined by the following transfer function.

$$H(s) = \frac{1{,}000s}{s^2 + 10s + 100}$$

$\rightarrow$ zero = 0

$\rightarrow \omega_n^2 = 100 \Rightarrow \omega_n = 10$ rad/sec

Determine all zeros, poles, and corner frequencies, and write them on the lines that have been provided. Also, plot the frequency response (Bode plot – both magnitude and phase) of this system on the semilog paper on the following page. Only asymptotic responses are required. Clearly label all important points and all slopes. Clearly indicate which trace is the overall magnitude and phase (summation of the parts).

$$\text{poles} = \frac{-10 \pm \sqrt{100-400}}{2} = -5 \pm \frac{j}{2}\sqrt{300}$$

$$= -5 \pm 5j\sqrt{3}$$

complex poles

Zeros 0

Poles $-5 \pm j5\sqrt{3}$

Corner Frequencies $\cancel{0}$, 10 rad/sec

$$H(j\omega) = \frac{1000\, j\omega}{(j\omega)^2 + 10 j\omega + 100} = 10 \frac{j\omega}{\left(\frac{j\omega}{10}\right)^2 + \frac{1}{10} j\omega + 1}$$

$$20\log_{10}|10| = 20\,\text{dB}$$

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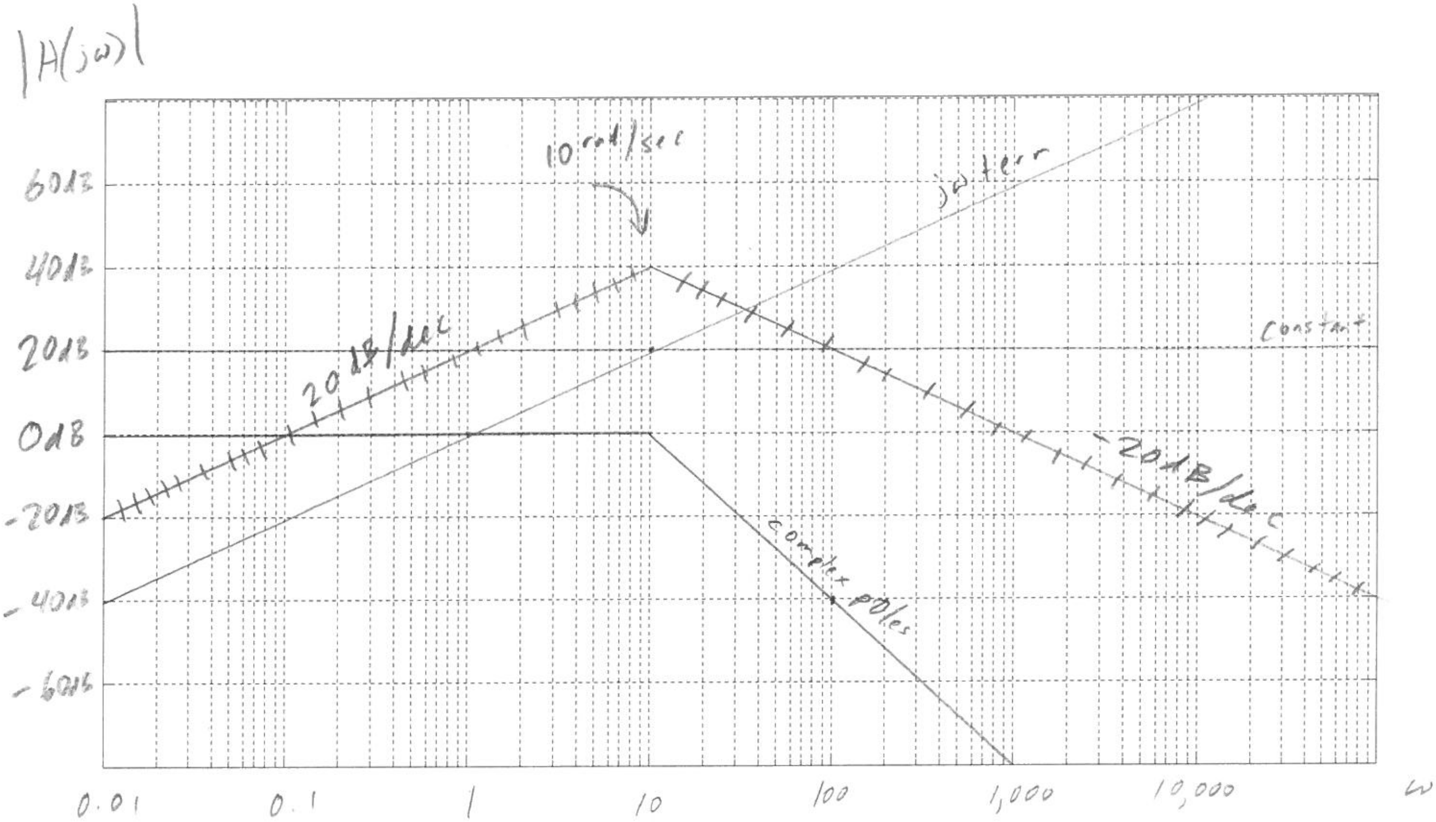
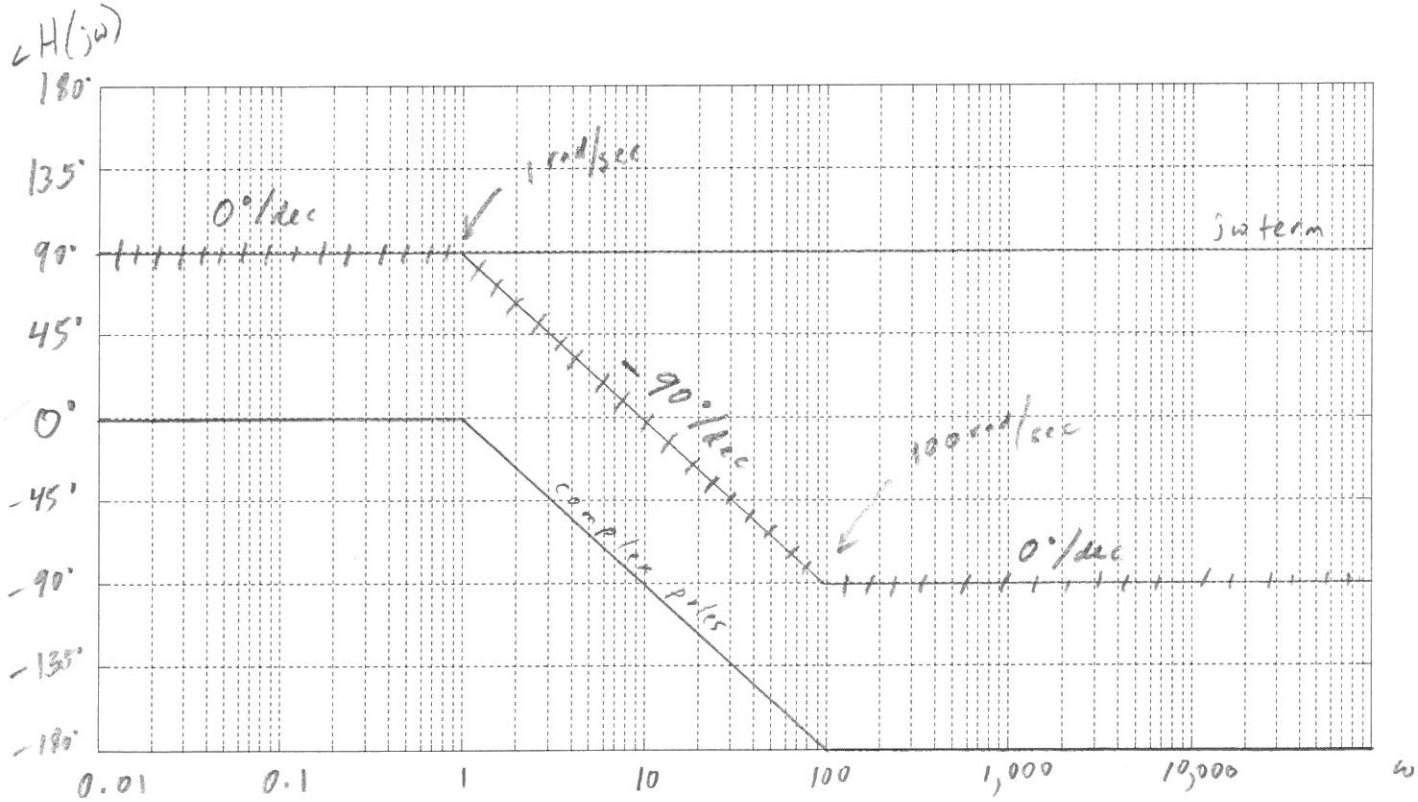
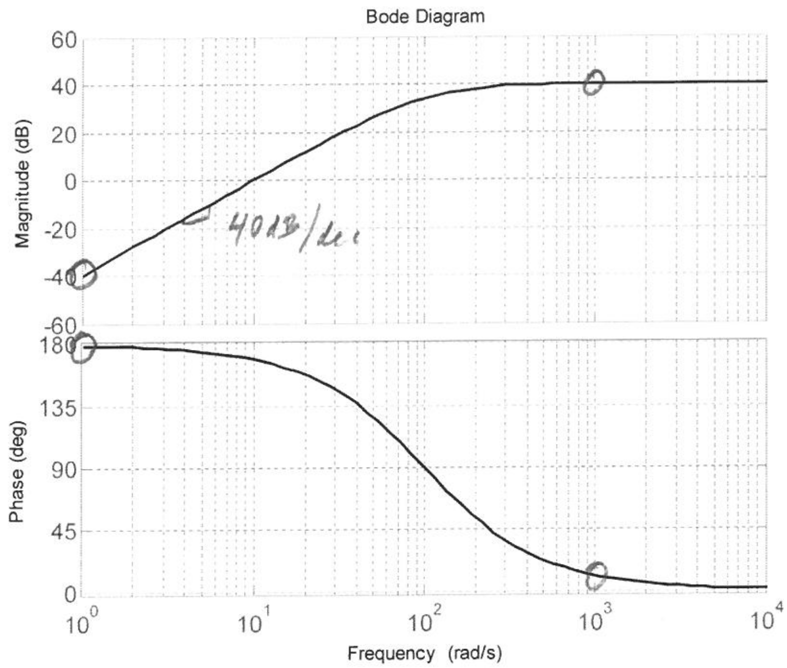




PROBLEM 3 (10 Points)

Use the following plot for each of the questions below.



Total phase change of −180°

A. What type of filtering operation does this system perform? (2 Points)

High pass

B. What is the order of this filter? (3 Points)

2nd-Order

C. What is the steady-state output of this system if the input is (5 Points)

$$x(t) = 20\cos(t) + 5\cos(1000t)$$

$\omega = 1,\ 1000$ $\quad |H(j)| = -40\text{dB} \Rightarrow 10^{-40/20} = 0.01$

$\angle H(j) \approx 180°$

$|H(j1000)| = 40\text{ dB} \Rightarrow 10^{40/20} = 100$

$\angle H(j1000) \approx 10°$

$y(t) = (0.01)(20)\cos(t + 180°) + (100)(5)\cos(1000t + 10°)$

$y(t) = 0.2\cos(t + 180°) + 500\cos(1000t + 10°)$

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– Problem 3 Work Page –

PROBLEM 4 (10 Points)

Determine the z transform for the following signals. Write the z transforms as rational functions of "z" and simplify where possible.

A. $x[n]=(10)\left(\frac{1}{2}\right)^n \cos(\pi n)u[n]$ (5 Points)

From the transform table

$$X(z) = (10)\frac{z^2 - \left(\frac{1}{2}\right)\cos(\pi)z}{z^2 - (2)\left(\frac{1}{2}\right)\cos(\pi)z + \left(\frac{1}{2}\right)^2} = 10\,\frac{z^2 + \frac{1}{2}z}{z^2 + z + \frac{1}{4}} =$$

$$= \frac{10z^2 + 5z}{z^2 + z + \frac{1}{4}}$$

B. If $W(z) = \frac{4z}{z-2}$ $\longleftrightarrow$ $w[n] = 4(2)^n u[n]$

and $v[n] = (n)w[n]$, $\rightarrow$ $v[n] = 4(n)(2)^n u[n]$

then find the z transform of $v[n]$. (5 Points)

$$V(z) = 4\,\frac{2z}{(z-2)^2} = \frac{8z}{(z-2)^2} = \frac{8z}{z^2 - 4z + 4}$$

Alternate Method

$\rightarrow$ Use z Transform Properties

$$V(z) = -z\frac{d}{dz}\left[\frac{4z}{z-2}\right] = -z\left[\frac{4}{z-2} - \frac{4z}{(z-2)^2}\right] =$$

$$= -z\left[\frac{4z - 8 - 4z}{(z-2)^2}\right] = \frac{8z}{(z-2)^2}$$

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PROBLEM 5   (30 Points)

A system is defined by the following transfer function.

$$H(z) = \frac{z^2 + 2z - 3}{z^2 + 1}$$

This system receives the following input.

$$X(z) = \frac{1}{z + \frac{1}{2}}$$

A. Determine if this system is stable, marginally stable, or unstable. You must justify your answer to received full credit. (5 Points)

poles of $H(z) \rightarrow (z^2+1) \Rightarrow \frac{-0 \pm \sqrt{0-4}}{2} = \pm j$

poles are on the unit circle

$\therefore$ Marginally stable

B. Given the transfer function above, determine the difference equation that describes this system. (5 Points)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^2 + 2z - 3}{z^2 + 1} \cdot \frac{z^{-2}}{z^{-2}} = \frac{1 + 2z^{-1} - 3z^{-2}}{1 + z^{-2}}$$

$$Y(z)\left(1 + z^{-2}\right) = X(z)\left(1 + 2z^{-1} - 3z^{-2}\right)$$

$$y[n] + y[n-2] = x[n] + 2x[n-1] - 3x[n-2]$$

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C. Determine the z-domain output, $Y(z)$, of the system, $H(z)$, in response to the input, $X(z)$. (5 Points)

$$Y(z) = H(z)\,X(z)$$

$$Y(z) = \frac{z^2+2z-3}{(z^2+1)(z+\frac{1}{2})} = \frac{z^2+2z-3}{(z+j)(z-j)(z+\frac{1}{2})} \qquad \begin{matrix}\deg(\text{num})=2\\ \deg(\text{den})=3\end{matrix}$$

$$\deg(\text{den}) > \deg(\text{num}) \checkmark$$

D. Determine the complete solution for the time-domain response, $y[n]$. (More room on the next page.) (15 Points)

$$Y(z) = \frac{z^2+2z-3}{(z+j)(z-j)(z+\frac{1}{2})} = \frac{k_1}{z+j} + \frac{k_1^*}{z-j} + \frac{k_2}{z+\frac{1}{2}}$$

$$k_1 = Y(z)(z+j)\Big|_{z=-j} = \frac{z^2+2z-3}{(z-j)(z+\frac{1}{2})}\Bigg|_{z=-j} = \frac{-1-2j-3}{(-2j)(-j+\frac{1}{2})}$$

$$= \frac{-4-2j}{-2-j} = 2\,\frac{(-2-j)}{(-2-j)} = 2$$

$$k_1^* = 2$$

$$k_2 = Y(z)(z+\tfrac{1}{2})\Big|_{z=-\frac{1}{2}} = \frac{z^2+2z-3}{z^2+1}\Bigg|_{z=-\frac{1}{2}} =$$

$$= \frac{\frac{1}{4}-1-3}{\frac{1}{4}+1} = \frac{\frac{1}{4}-4}{\frac{1}{4}+1} = \frac{-\frac{15}{4}}{\frac{5}{4}} = -3$$

$$\therefore\ Y(z) = \frac{2}{z+j} + \frac{2}{z-j} - \frac{3}{z+\frac{1}{2}}$$

$$= 2z^{-1}\frac{z}{z+j} + 2z^{-1}\frac{z}{z-j} - 3z^{-1}\frac{z}{z+\frac{1}{2}}$$

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– Problem 5 Work Page –

$|K_1| = 2$

$\angle K_1 = 0$

$|\text{pole}| = 1$

$\angle \text{pole} = \frac{\pi}{2}$

delay

$$y[n] = (2)\cos\left(\frac{\pi}{2}(n-1)\right) u[n-1] \quad - 3\left(-\frac{1}{2}\right)^{n-1} u[n-1]$$