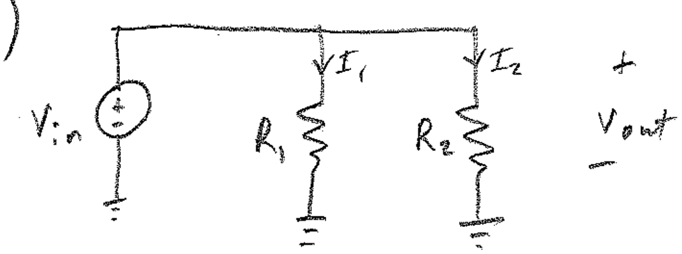
Given $R_1 = 1\,k\Omega$
$R_2 = 10\,k\Omega$
$R_3 = 10\,k\Omega$
$I_{in} = 1\,mA$
$V_{in} = 1\,V$

a)



Find V_{out}, I_1, I_2

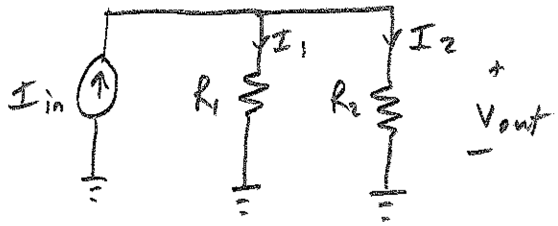
$\boxed{V_{out} = V_{in}} = 1\,V$ → R_1 and R_2 are in parallel with the voltage source

Ohm's Law → $V = IR$

$$\therefore \quad I_1 = \frac{V_{out}}{R_1} = \frac{1V}{1k\Omega} = 1mA$$

$$I_2 = \frac{V_{out}}{R_2} = \frac{1V}{10k\Omega} = 0.1mA$$

b)



Find V_{out}, I_1, I_2

Derive the expression for I_1 + I_2

Current Divider Derivation

Using KCL

$$I_{in} = I_1 + I_2 = I_1 + \frac{V_{out}}{R_2}$$

$$I_1 = I_{in} - \frac{V_{out}}{R_2}$$

$$V_{out} = I_{in}\, R_1 /\!/ R_2 = \boxed{I_{in} \frac{R_1 R_2}{R_1 + R_2}} = (1mA)\frac{(1k\Omega)(10k\Omega)}{(1k\Omega + 10k\Omega)} =$$

$$= \boxed{0.9091\ V}$$

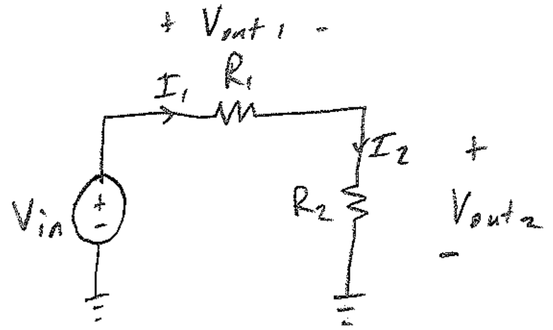
$$\therefore I_1 = I_{in} - \frac{I_{in}}{R_2}\frac{R_1 R_2}{R_1 + R_2} = I_{in}\left(1 - \frac{R_1}{R_1 + R_2}\right) =$$

$$= I_{in}\left(\frac{R_1 + R_2 - R_1}{R_1 + R_2}\right)$$

$$\boxed{I_1 = I_{in}\frac{R_2}{R_1 + R_2}} = (1mA)\left(\frac{10k\Omega}{11k\Omega}\right) = 0.9091\ mA$$

$$\boxed{I_2 = I_{in}\frac{R_1}{R_1 + R_2}} = (1mA)\left(\frac{1k\Omega}{11k\Omega}\right) = 0.0909\ mA$$

This is the <u>current divider</u> equation set c)



Find V_{out1}, V_{out2}, I_1, and I_2

Derive expressions for V_{out1} & V_{out2}

Voltage Divider Derivations

For V_{out2}, use KVL

$$V_{in} = I_1 R_1 + I_2 R_2 \quad , \quad I_1 = I_2 \quad \text{(same looop)}$$

$$V_{in} = I_1 R_1 + V_{out2} \qquad \boxed{\therefore I_1 = \frac{V_{in}}{R_1 + R_2}} = \frac{1V}{1k\Omega + 10k\Omega} = \boxed{0.0909 mA}$$

$$V_{out2} = V_{in} - I_1 R_1 = V_{in} - \frac{V_{in}}{R_1+R_2} R_1 = V_{in}\left(1 - \frac{R_1}{R_1+R_2}\right)$$

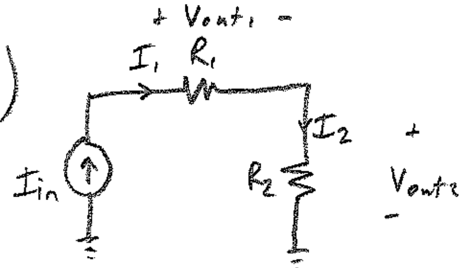
$$= V_{in}\left(\frac{R_1+R_2-R_1}{R_1+R_2}\right) = V_{in}\frac{R_2}{R_1+R_2}$$

$$\therefore \boxed{V_{out2} = V_{in}\frac{R_2}{R_1+R_2}} = (1V)\frac{10k\Omega}{11k\Omega} = \boxed{0.9091 V}$$

$$\boxed{V_{out1} = V_{in}\frac{R_1}{R_1+R_2}} = (1V)\frac{1k\Omega}{11k\Omega} = \boxed{0.0909V}$$

This is the <u>voltage divider</u> equation set

d)



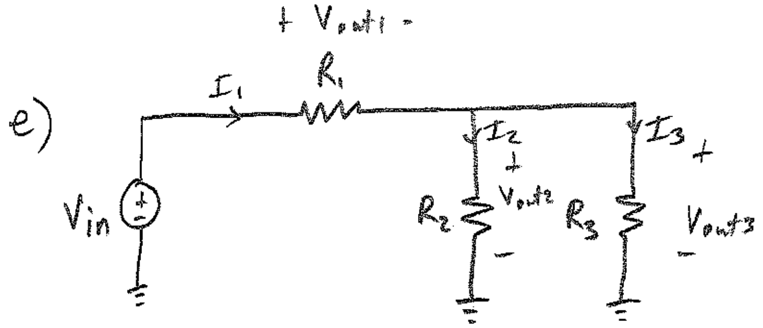
Find V_{out1}, V_{out2}, I_1, and I_2

One loop, so $\boxed{I_1 = I_2 = I_{in} = 1mA}$

$$V_{out2} = I_2 R_2 = (1mA)(10k\Omega) = 10V$$

$$V_{out1} = I_1 R_1 = (1mA)(1k\Omega) = 1V$$

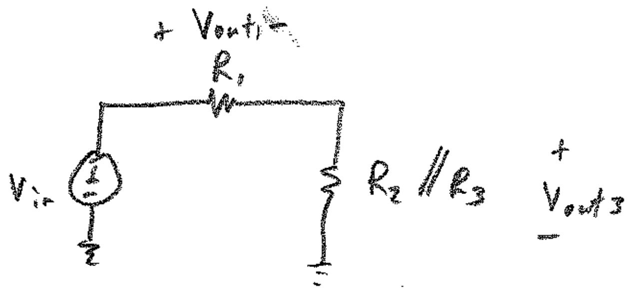
e)



Find the Voltage across each resistor, and also find the current through each resistor

$V_{out2} = V_{out}$ (in parallel)

$$R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = 5\,k\Omega$$



$$V_{out1} = V_{in} \frac{R_1}{R_1 + R_2 \parallel R_3}$$

$$= (1V) \frac{1k\Omega}{1k\Omega + 5k\Omega} = 0.1667V$$

$$V_{out2} = V_{in} \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3} =$$

$$= (1V) \frac{5k\Omega}{6k\Omega} = 0.8333V$$

$$I_1 = \frac{V_{out1}}{R_1} = \frac{V_{in}}{R_1 + R_2 \parallel R_3}$$

$$= \frac{1V}{1k\Omega + 5k\Omega} = 0.1667\,mA$$

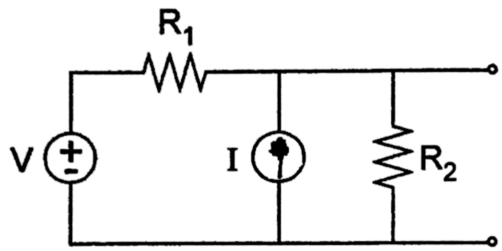
$I_2 = I_3$ (same resistance values)

$$I_2 = I_1 \frac{R_3}{R_2 + R_3}$$ Current Divider

$$I_2 = \frac{V_{in}}{R_1 + R_2 \parallel R_3} \frac{R_3}{R_2 + R_3} = \frac{1V}{6k\Omega} \frac{10k\Omega}{20k\Omega} = 0.0833\,mA$$

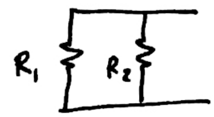
For the following circuit, determine both Thevenin and Norton equivalents. Use the following values.

$R_1 = 1k\Omega$, $R_2 = 3k\Omega$, $V = 2V$, and $I = 1mA$



Find $R_{th} = R_N$

→ Turn off all sources

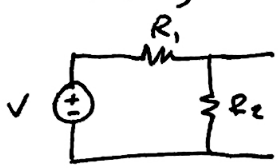


$R_{th} = R_1 // R_2 = 750\Omega$

Find the Thevenin voltage

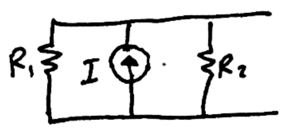
⇒ Use Superposition

V → ON, I → OFF



$V_{oc} = V \frac{R_2}{R_1 + R_2}$

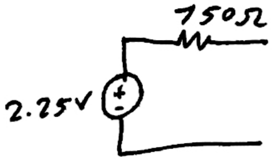
V → OFF, I → ON



$V_{o.c.} = I(R_1 // R_2)$

$\therefore V_{th} = V \frac{R_2}{R_1 + R_2} + I(R_1 // R_2) = 2.25V$

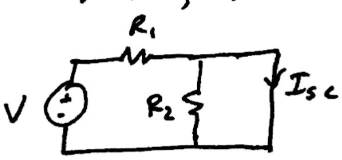
Thevenin Equivalent



Find Norton short-circuit current

⇒ Use Superposition

V → ON, I → OFF

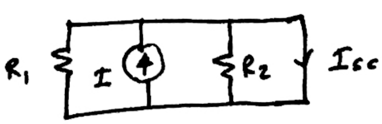


Note → R_2 is shorted out

$\therefore I_{sc} = \frac{V}{R_1}$

⇒ $I_N = \frac{V}{R_1} + I = 3mA$

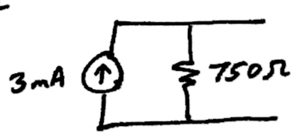
V → OFF, I → ON



Note → Both resistors are shorted out

$\therefore I_{sc} = I$

Norton Equivalent



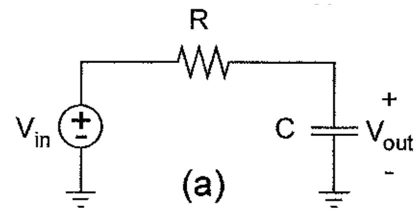
Double Check

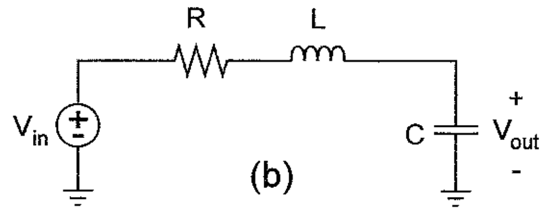
→ Voltage of the output of the Norton Equivale

$(3mA)(750\Omega) = 2.25V$

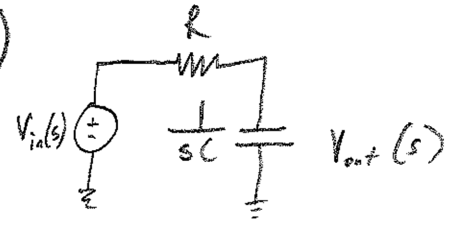
Agrees!

Solve for the Laplace-domain transfer function of each of the following circuits.





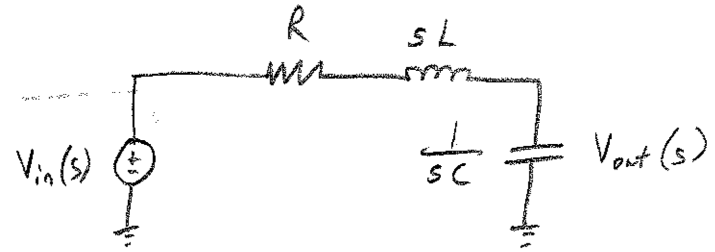
a)



$$V_{out}(s) = V_{in}(s)\frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{RCs+1}$$

$$\therefore\ H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{RCs+1}$$

b)

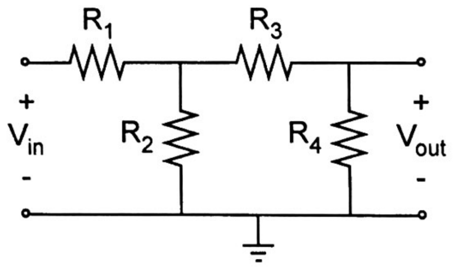


$$V_{out}(s) = V_{in}(s)\frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}} = V_{in}(s)\frac{1}{LCs^2+RCs+1}$$

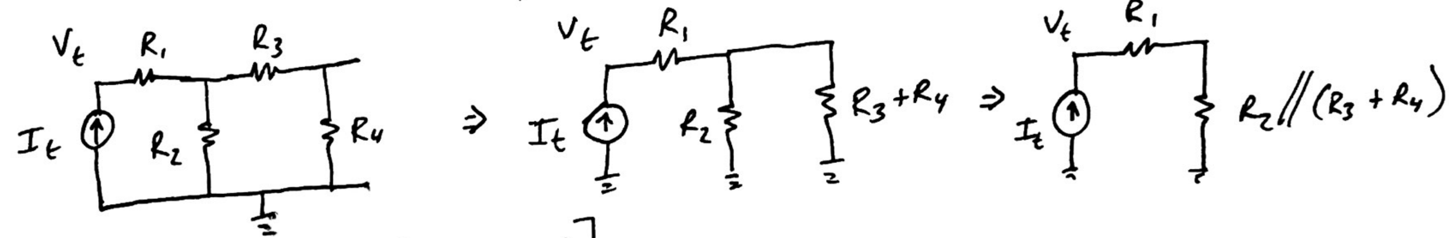
$$\therefore\ H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{LCs^2+RCs+1}$$

Determine an expression for both the input impedance and output impedance for the circuit shown below. Also, solve for the numerical values of the input and output impedances given that

$R_1 = 200\Omega$, $R_2 = 200\Omega$, $R_3 = 100\Omega$, $R_4 = 200\Omega$



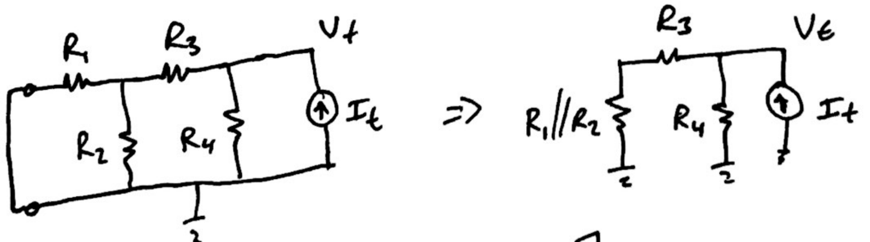
Input Impedance → Apply a test current source at the input while the output floats. Measure the change in the input voltage



$$V_t = I_t\left[R_1 + R_2 // (R_3 + R_4)\right]$$

$$R_{in} = \frac{V_t}{I_t} = \boxed{R_1 + R_2 // (R_3 + R_4)} = 200\Omega + (200\Omega)//(300\Omega) = \boxed{320\Omega}$$

Output Impedance → Apply a test current source at the output while the input is shorted to ground. Measure the change in the output voltage $\Rightarrow R_{out} = \left.\frac{V_{out}}{I_{out}}\right|_{V_{in}=0}$



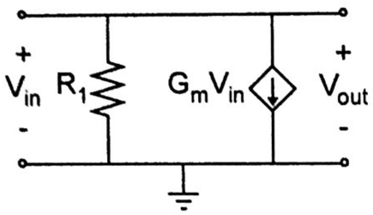
$$V_t = (I_t)\left[R_4 // (R_3 + R_1//R_2)\right]$$

$$R_{out} = \left.\frac{V_t}{I_t}\right|_{V_{in}=0} = \boxed{R_4 // (R_3 + R_1//R_2)} = (200\Omega)//(100\Omega + 200\Omega//200\Omega) =$$

$$\boxed{R_{out} = 100\Omega}$$

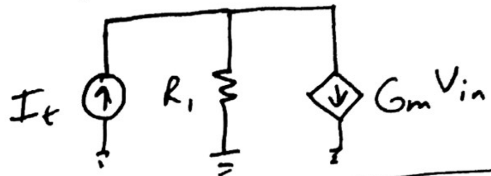
Determine an expression for both the input impedance and output impedance for the circuit shown below. Also, solve for the numerical values of the input and output impedances given that

$R_1 = 200\Omega$ and $G_m = 0.1S$



Input Impedance $\quad R_{in} = \dfrac{V_t}{I_t}$

$V_t : V_{in}$



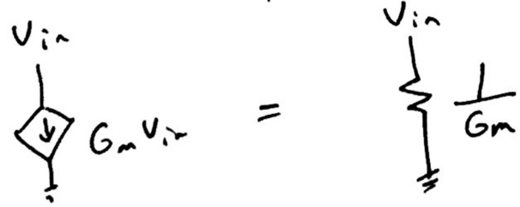
KCL

$$I_t = \frac{V_t}{R_1} + G_m V_t$$

$$\boxed{\therefore\ R_{in} = \frac{V_t}{I_t} = \frac{1}{\frac{1}{R_1} + G_m} = 9.52\Omega}$$

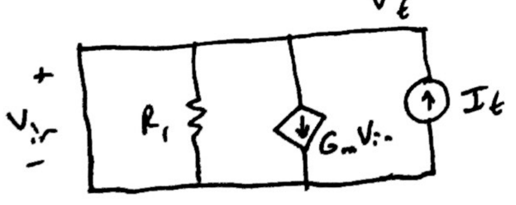
Interesting Point

$\Rightarrow$ This is a voltage-controlled current source whose current is proportional to the voltage across it. This has the same properties of a resistor!

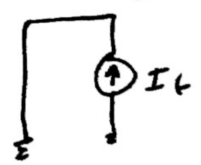


$$\Rightarrow R_{in} = R_1 // \frac{1}{G_m} = 9.52\Omega$$

Output Impedance $\quad R_{out} = \dfrac{V_{out}}{I_{out}}\Big|_{V_{in}=0}$



$V_{in} = 0 \quad \therefore$



$$\therefore\ \boxed{R_{out} = 0\Omega}$$