BJT Parameters

β=100, V^+=10V, V^- = −10V, $V_{BE,ON}$=0.7V, $V_{CE,SAT}$=0.2V, V_A=50V
I_s=10fA, k=1.38x10^{-23}J/K, T=300K, q=1.602x10^{-19}C

Determine the small-signal parameters (r_π, g_m, and r_o) for the following BJTs, assuming they operate in forward-active mode.

a. npn BJT with I_C = 25mA
b. pnp BJT with V_{BE} = 0.71V

$$U_T = \frac{kT}{q} = \frac{(1.38\times10^{-23}\ J/K)(300K)}{1.602\times10^{-19}\ C} \approx 25.84m$$

A.

$\downarrow I_c = 25mA$

$$r_\pi = \frac{\beta U_T}{I_c} = \frac{(100)(25.84mV)}{25mA} = 103.36\ \Omega$$

$$g_m = \frac{I_c}{U_T} = \frac{25mA}{25.84mV} = 0.9675\ S$$

$$r_o = \frac{V_A}{I_c} = \frac{50V}{25mA} = 2\ k\Omega$$

B.

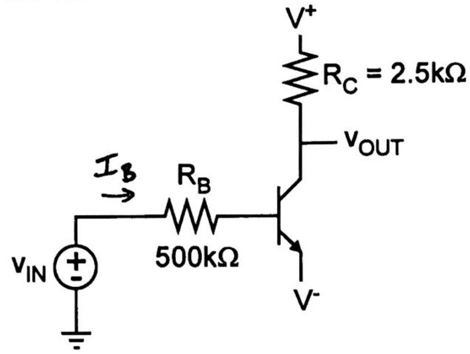
$$I_c = I_S e^{V_{EB}/U_T} = (10fA)\, e^{0.71V/25.84mV} =$$
$$= 8.57mA$$

$$r_\pi = \frac{\beta U_T}{I_c} = \frac{(100)(25.84mV)}{8.57mA} = 301.52\ \Omega$$

$$g_m = \frac{I_c}{U_T} = \frac{8.57mA}{25.84mV} = 0.3317\ S$$

$$r_o = \frac{V_A}{I_c} = \frac{50V}{8.57mA} = 5.8343\ k\Omega$$

For the following circuits, determine the small-signal voltage gain. You may assume the transistor operates in forward-active mode. $V^+ = 15V$ and $V^- = -15V$ for this problem. Let the DC value of the input voltage be 1V.



KVL $\Rightarrow V_{IN} = I_B R_B + V_{BE,ON} + V^-$

$$I_B = \frac{V_{IN} - V_{BE,ON} - V^-}{R_B} = \frac{1V - 0.7V - (-15V)}{500k\Omega} = 30.6\mu A$$

$\therefore I_C = \beta I_B = 3.06mA$

Note $\rightarrow V^+ - I_C R = 15V - (3.06mA)(2.5k\Omega) = 7.35V$

$V_{CE} = 7.35V - (-15V) > V_{CE,SAT}$

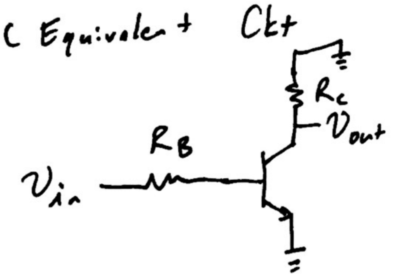
$\therefore$ Forward Active

$$r_\pi = \frac{\beta U_T}{I_C} = \frac{(100)(25.84mV)}{3.06mA} = 844.44\Omega$$

$$g_m = \frac{I_C}{U_T} = \frac{3.06mA}{25.84mV} = 118.42mS$$

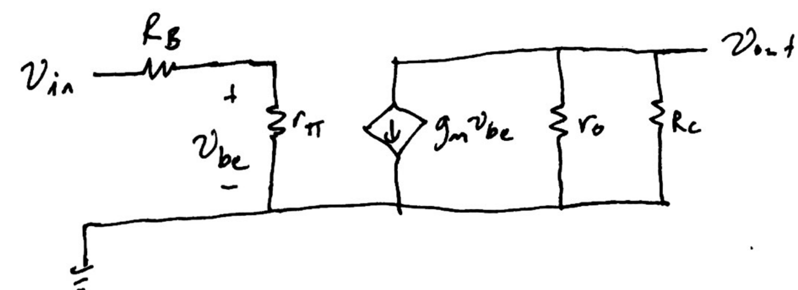
$$r_o = \frac{V_A}{I_C} = \frac{50V}{3.06mA} = 16.34k\Omega$$

AC Equivalent Ckt



$\Rightarrow$

Small-Signal Model



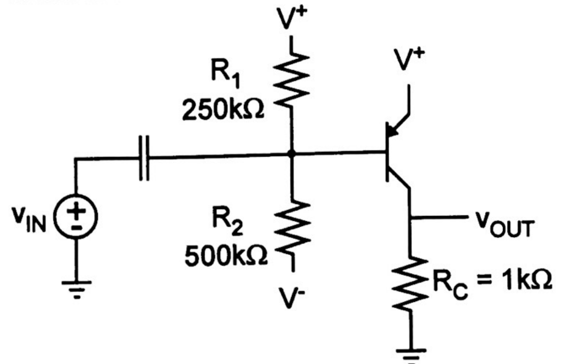
$$\frac{v_{out}}{v_{in}} = \frac{v_{be}}{v_{in}} \cdot \frac{v_{out}}{v_{be}} = \left(\frac{r_\pi}{r_\pi + R_B}\right)\left(-g_m\, r_o // R_C\right) =$$

$$= \left(\frac{844.44\Omega}{844.44\Omega + 500k\Omega}\right)\left(-(118.42mS)\,(16.34k\Omega)//(2.5k\Omega)\right) =$$

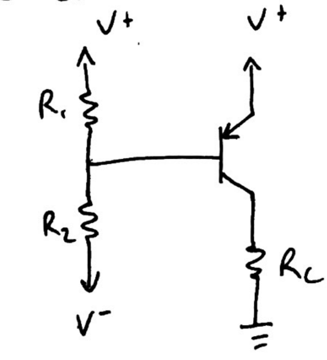
$$= \boxed{-0.4329}$$

Not very high gain! r_π is small

For the following circuit, determine the small-signal voltage gain. You may assume the transistor operates in forward-active mode. $V^+ = 15V$ and $V^- = -15V$ for this problem.



DC Circuit



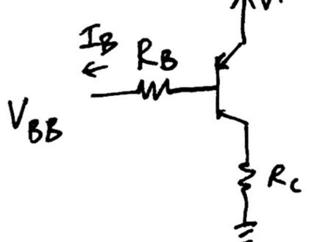
Thevenin Equivalent looking out of the base

$R_B = R_1 // R_2 = 166.67k\Omega$

$V_{BB} = V^+ \frac{R_2}{R_1+R_2} + V^- \frac{R_1}{R_1+R_2}$

$= 5V$

Redraw DC Circuit



KVL around E-B Loop

$V^+ = V_{BB} + I_B R_B + V_{EB,ON}$

$I_B = \frac{V^+ - V_{BB} - V_{EB,ON}}{R_B} = \frac{9.3V}{166.67k\Omega} = 55.80\mu A$

$\therefore I_C = \beta I_B = 5.58 mA$

$V_C = 0V + I_C R_C = 5.58V$

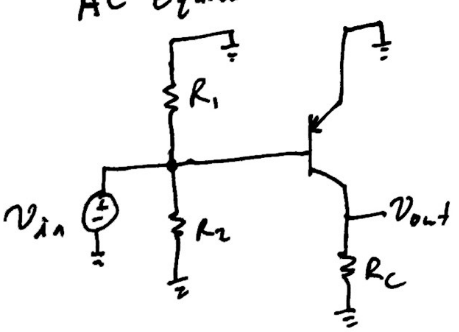
$V_{CE} = V^+ - V_C = 9.42V > V_{EC,SAT}$

$r_\pi = \frac{\beta U_T}{I_C} = \frac{(100)(25.84mV)}{5.58 mA} = 463.08\Omega$

$g_m = \frac{I_C}{U_T} = 215.94 mS$

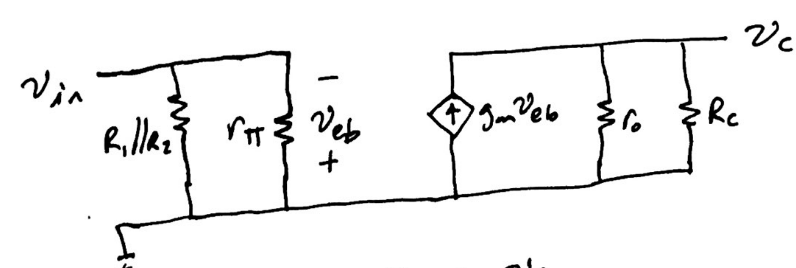
$r_o = \frac{V_A}{I_C} = 8.96 k\Omega$

AC Equivalent Circuit



$\Rightarrow$

Small-Signal Model

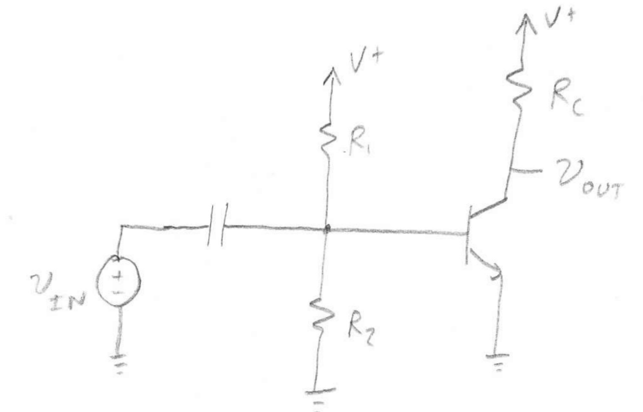


Note → $v_{eb} = -v_{in}$

No voltage divider at the input

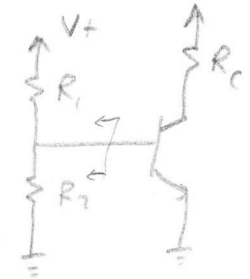
$v_{out} = g_m v_{eb}\, r_o // R_C = -g_m v_{in}\, r_o // R_C$

$\therefore \frac{v_{out}}{v_{in}} = -g_m\, r_o // R_C = \boxed{-194.26}$



Need to find the DC bias to determine $I_c \rightarrow$ for r_π, g_m, r_o

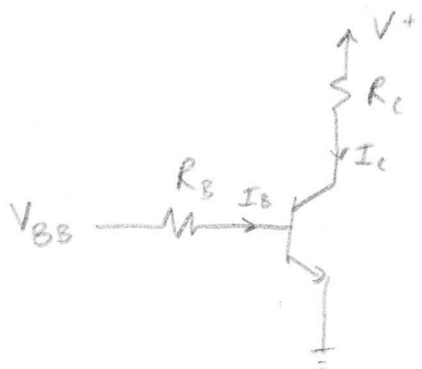
DC Circuit



Let
$V_{BB} = V^+ \frac{R_2}{R_1 + R_2} = 2\,V$

$R_B = R_1 // R_2 = 40\,k\Omega$

Redraw DC Bias Circuit



KVL $\Rightarrow$ $V_{BB} = I_B R_B + V_{BE,ON}$

$$I_B = \frac{V_{BB} - V_{BE,ON}}{R_B} = 32.5\,\mu A$$

$I_c = \beta I_B = 3.25\,mA$ (Assuming Forward Active)

$V_{CE} = V^+ - I_c R_c = 3.5 > V_{CE,SAT}$

$\therefore$ It is, indeed, in forward active

$$U_T = \frac{kT}{q} = 26\,mV$$

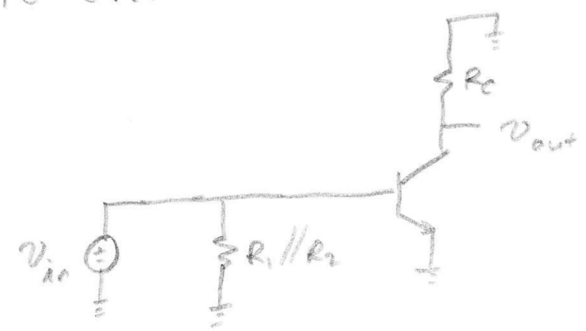
Small-Signal Parameters

$$r_\pi = \frac{\beta U_T}{I_c} = 800\,\Omega$$

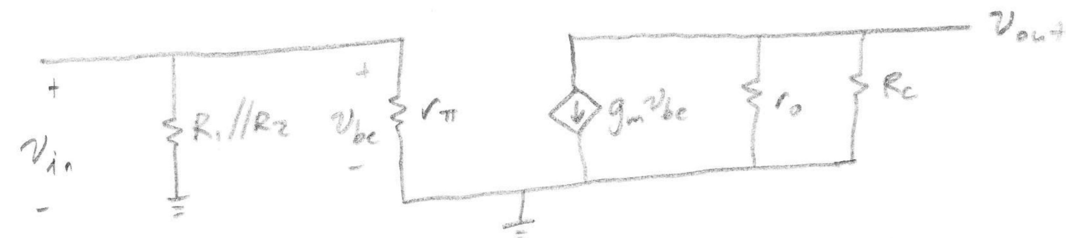
$$g_m = \frac{I_c}{U_T} = 0.125\,S = 125\,mS$$

$$r_o = \frac{V_A}{I_c} = 15.4\,k\Omega$$

AC Circuit



Small-Signal Model



Note $v_{be} = v_{in}$

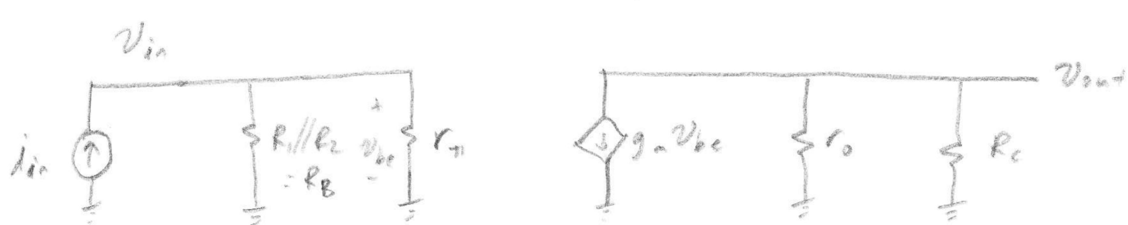
KVL Loop

Small-Signal Gain

$$a_v = \frac{v_{out}}{v_{in}} = -g_m \, r_o // R_C = -221$$

Two-Port Parameters $\Rightarrow R_{in}, R_{out}, G_m$

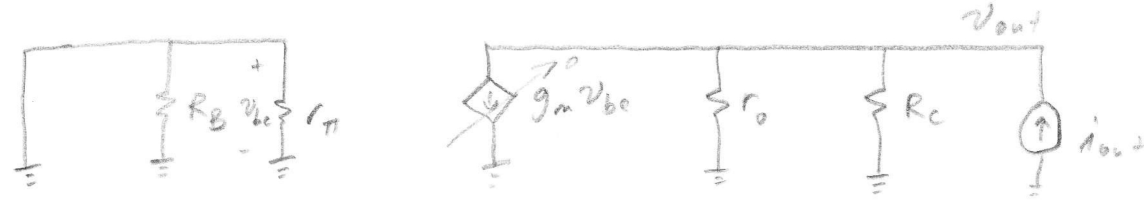
$$R_{in} = \frac{v_{in}}{i_{in}} \Rightarrow \text{Use a test current source}$$



$$v_{in} = i_{in} \left(R_B // r_\pi \right)$$

$$R_{in} = \frac{v_{in}}{i_{in}} = R_B // r_\pi = 784.3\ \Omega$$

$$R_{out} = \left.\frac{v_{out}}{i_{out}}\right|_{v_{in}=0}$$

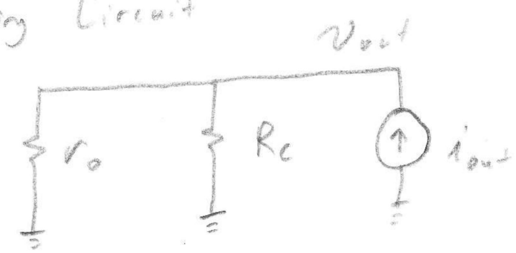


↖ v_{in} = AC ground

$\therefore\ v_{be} = 0$

$\Rightarrow g_m v_{be} = 0$

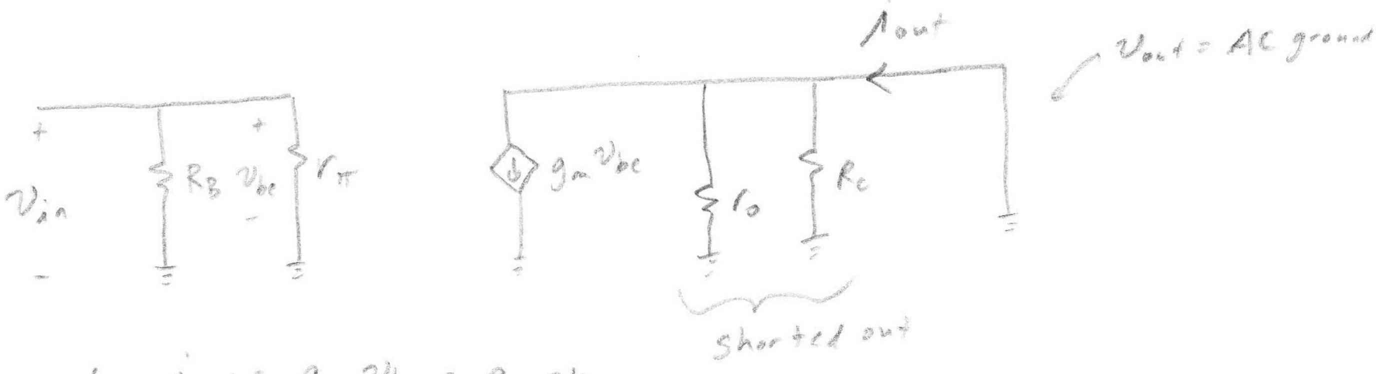
Resulting Circuit



$v_{out} = i_{out}\ r_o \| R_C$

$R_{out} = r_o \| R_C = 1.77\ k\Omega$

$$G_m = \left.\frac{i_{out}}{v_{in}}\right|_{v_{out}=0}$$

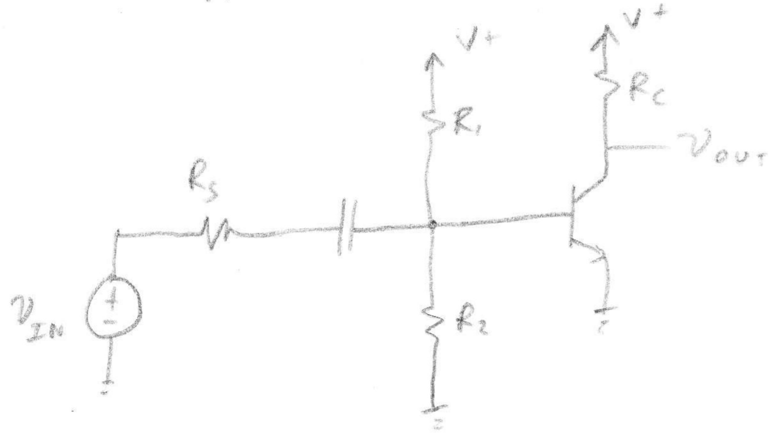


$\therefore\ i_{out} = g_m v_{be} = g_m v_{in}$

$\Rightarrow G_m = \frac{i_{out}}{v_{in}} = g_m$

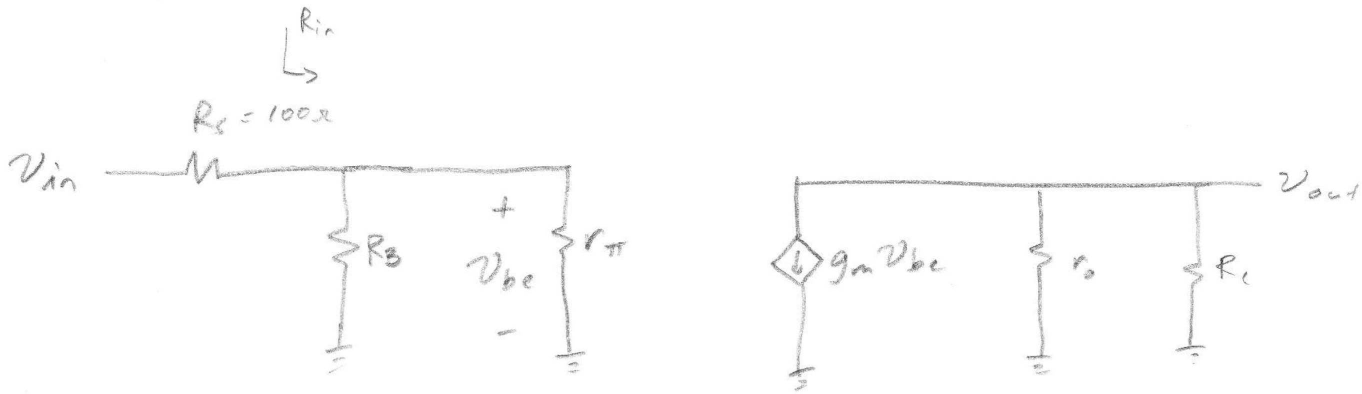
Note
$a_v = -G_m R_{out} = -g_m\ r_o \| R_C$
$\Rightarrow$ Agrees with our previous solution for the gain

What happens if the input voltage source is not ideal?



Note → The DC circuit is exactly the same as it was previously
Therefore I_C is unchanged
⇒ r_π, r_o, g_m are also unchanged

Small-Signal Model



Note → $v_{be} = v_{in} \dfrac{R_B // r_\pi}{R_B // r_\pi + R_S} = 0.887 v_{in}$

$$a_v = \frac{v_{be}}{v_{in}} \cdot \frac{v_{out}}{v_{be}} = (0.887)\left[-g_m\, r_o // R_C\right] = -196$$

→ smaller than before

R_{out} is definitely unchanged

R_{in} (from the right side of the voltage source) is unchanged
If you include R_S, then $R_{in} = R_S + R_B // r_\pi = 843.3\,\Omega$

G_m (from the right side of the voltage source) is unchanged
If you include R_S, then G_m is decreased by the factor of the voltage divider (0.887)

$$G_m = (0.887)(g_m) = 0.111\ S$$