

RULES

This is a closed book, closed notes test. You are, however, allowed one piece of paper (front side and half of back side, hand-written) for notes and definitions. You must turn in your equations when you hand your test in.

You are permitted to use a calculator.

You have 50 minutes to complete the test. Please read through the entire test before starting, and read through the directions carefully. To receive partial credit, you must show your work.

If you have any questions, please raise your hand, and I will come to you to answer them. Do not hesitate to ask questions.

Circle/box all of your answers.

Use the following parameters unless otherwise specified

Transistor Parameters

$$\beta = 100, V_{BE,ON} = 0.7V, V_{CE,SAT} = 0.2V$$

$$V_{EB,ON} = 0.7V, V_{EC,SAT} = 0.2V$$

$$V^+ = 10V, V^- = -10V$$

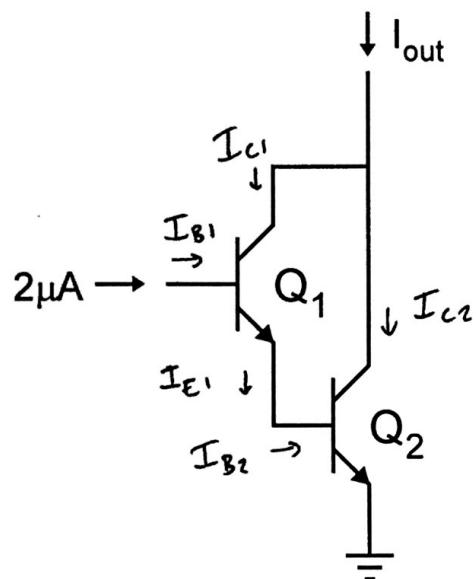
$$U_T = 25mV, V_A = 100V$$

Problem	Value	Score
1	10	
2	30	
3	30	
4	30	
Total	100	

PROBLEM 1

(10 Points)

Assume both transistors operate in the forward-active region. Calculate the value of I_{out} .



$$I_{in} = I_{B1}$$

$$I_{B2} = I_{E1} = (1 + \beta) I_{B1}$$

$$I_{C1} = \beta I_{B1}$$

$$I_{C2} = \beta I_{B2} = \beta [(1 + \beta) I_{B1}]$$

$$I_{out} = I_{C1} + I_{C2}$$

$$I_{out} = \beta I_{B1} + (\beta)(1 + \beta) I_{B1}$$

$$I_{out} = \beta I_{in} + (\beta)(1 + \beta) I_{in}$$

$$I_{out} = (I_{in})(\beta + \beta + \beta^2)$$

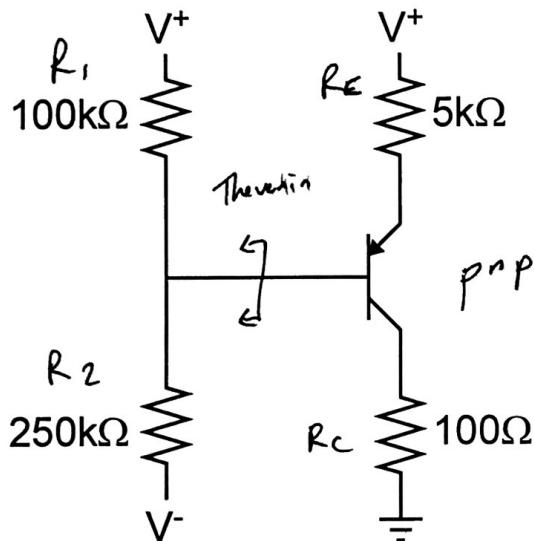
$$I_{out} = (I_{in})(2\beta + \beta^2)$$

$$I_{out} = (2\mu A) ((2)(100) + (100)^2) =$$

$$I_{out} = 20.4 \text{ mA}$$

PROBLEM 2

For the following two circuits, determine which region of operation the transistor is in and determine its operating point (all terminal voltages and currents). Write your answers on the lines that have been provided.



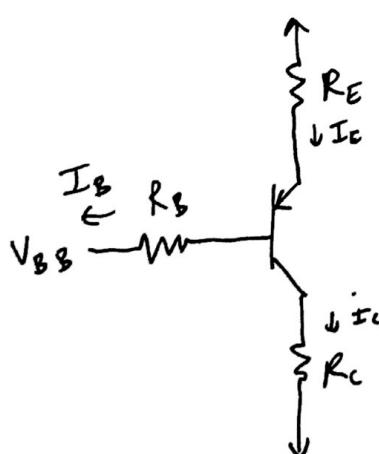
Region of Operation	<u>Forward Active</u>
V_B =	<u>4.9071 V</u>
V_E =	<u>5.6071 V</u>
V_C =	<u>86.989 mV</u>
I_B =	<u>8.6989 μA</u>
I_E =	<u>878.5889 μA</u>
I_C =	<u>869.89 μA</u>

Thevenin seen looking out of the base

$$V_{BB} = V^+ \frac{R_2}{R_1 + R_2} + V^- \frac{R_1}{R_1 + R_2} = (10V) \frac{250k\Omega}{100k\Omega + 250k\Omega} + (-10V) \frac{100k\Omega}{100k\Omega + 250k\Omega}$$

$$V_{BB} = 4.2857 V$$

$$R_B = R_1 \parallel R_2 = 100k\Omega \parallel 250k\Omega = 71.4286 k\Omega$$



Assume Forward Active

KVL around B-E loop

$$V^+ = V_{BB} + I_B R_B + V_{EB,ON} + I_E R_E$$

$$V^+ = V_{BB} + I_B R_B + V_{EB,ON} + (1+\beta) I_B R_E$$

$$I_B = \frac{V^+ - V_{BB} - V_{EB,ON}}{R_B + (1+\beta) R_E}$$

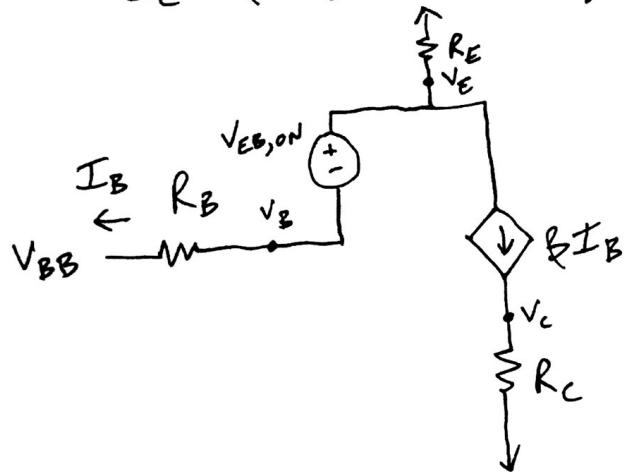
$$I_B = \frac{10V - 4.2857V - 0.7V}{71.4286 k\Omega + (1+100)(5 k\Omega)} = 8.6989 \mu A$$

3 ∴ Not Cutoff

(Extra Work Page for Problem 2)

Continue Assuming Forward-Active Operation

$$I_C = \beta I_B = 869.89 \mu A$$



$$V_C = 0V + I_C R_C$$

$$V_C = 0V + (869.89 \mu A)(100\Omega) = 86.989 mV$$

$$I_E = (1+\beta) I_B = (1+100)(869.89 \mu A) =$$

$$I_E = 878.5889 \mu A$$

$$V_E = V^+ - I_E R_E = 10V - (878.5889 \mu A)(5k\Omega) =$$

$$V_E = 5.6071V$$

$$\therefore V_{EC} = V_E - V_C = 5.6071V - (-9.9130V)$$

$$V_{EC} = 15.5201 > V_{EC, SAT}$$

∴ Forward Active

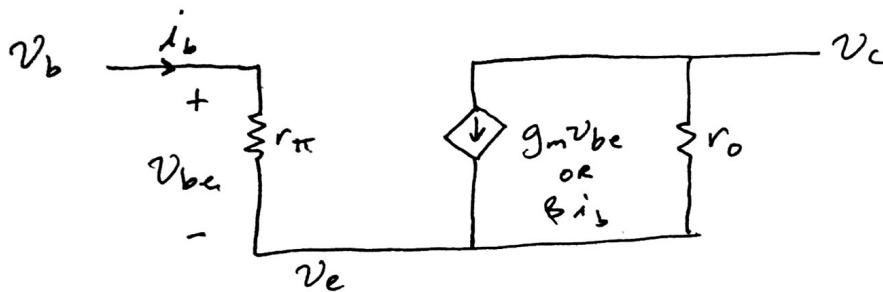
$$V_B = V_E - V_{EB,ON} = 5.6071V - 0.7V =$$

$$V_B = 4.9071V$$

PROBLEM 3

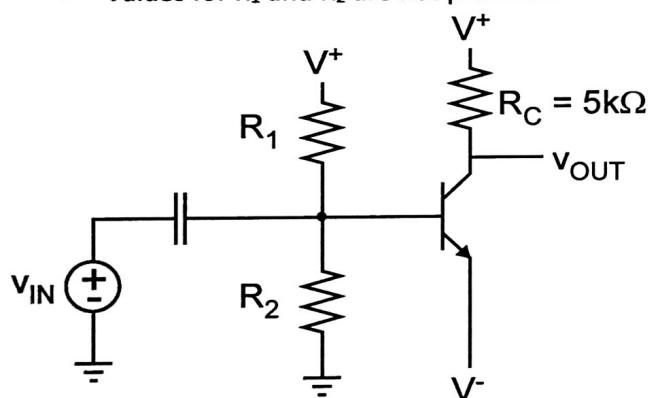
A. Draw the hybrid- π small-signal model of an npn BJT. Label everything.

(30 Points)
(5 Points)



For the remainder of this problem, use the circuit shown below.

- The DC value of the output voltage is 5V.
- Assume the transistor is in forward-active mode.
- Values for R_1 and R_2 are not provided.



$$I_C = \frac{V^+ - V_{OUT}}{R_C} = \frac{10V - 5V}{5k\Omega} =$$

$$I_C = 1mA$$

B. Calculate the transconductance of the transistor.

(5 Points)

$$g_m = \frac{I_C}{V_T} = \frac{1mA}{25mV} = \boxed{40mS}$$

C. Calculate r_π term of the transistor.

(5 Points)

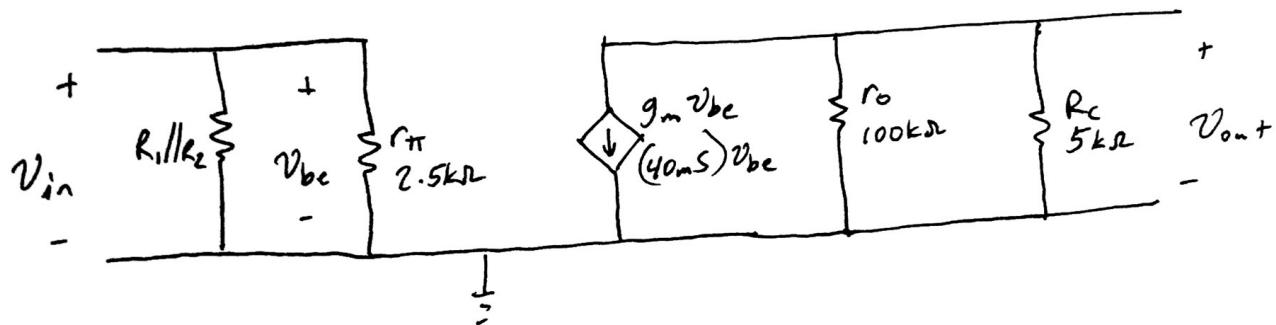
$$r_\pi = \frac{\beta V_T}{I_C} = \frac{(100)(25mV)}{1mA} = \boxed{2.5k\Omega}$$

D. Calculate r_o term of the transistor.

(5 Points)

$$r_o = \frac{V_A}{I_C} = \frac{100V}{1mA} = \boxed{100k\Omega}$$

C. Draw the small-signal equivalent circuit for this complete circuit. Hint. Use the hybrid- π model).
(5 Points)



D. Calculate the small-signal voltage gain for this circuit.
(5 Points)

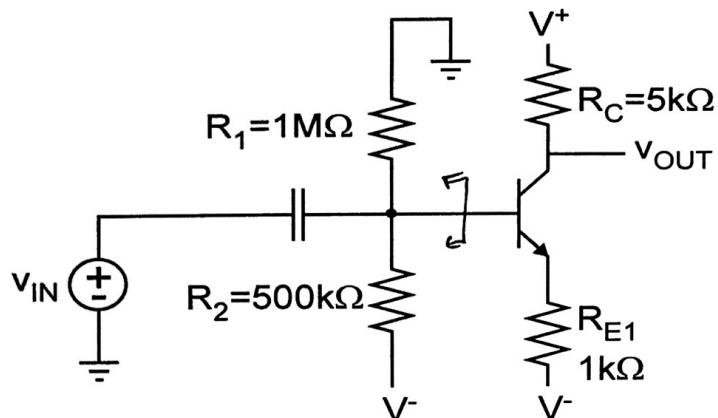
$$a_v = \frac{V_{out+}}{V_{in}} = -g_m r_o // R_c$$

$$a_v = - (40mS) \left(100k\Omega // 5k\Omega \right) = \boxed{-190.4762}$$

PROBLEM 5

(30 Points)

For the following circuit, calculate the small-signal voltage gain. Let $V_A = \infty$ for this problem.



Thevenin equivalent looking out of the base:

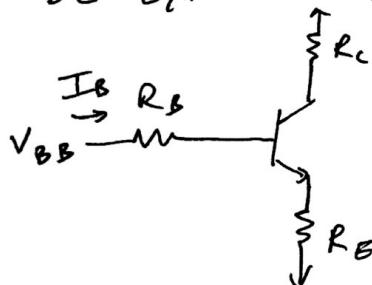
$$V_{BB} = V^- - \frac{R_1}{R_1 + R_2} = (-10\text{V}) \frac{1\text{M}\Omega}{500\text{k}\Omega + 1\text{M}\Omega} = -6.67\text{V}$$

$$V_{BB} = -6.67\text{V}$$

$$R_B = R_1 // R_2 = 1\text{M}\Omega // 500\text{k}\Omega =$$

$$R_B = 333.33\text{ k}\Omega$$

DC Equivalent Circuit



Assume Forward Active

$$V_{BB} = I_B R_B + V_{BE,ON} + (1+\beta) I_B R_E + V^-$$

$$I_B = \frac{V_{BB} - V_{BE,ON} - V^-}{R_B + (1+\beta) R_E} =$$

$$I_B = \frac{-6.67\text{V} - 0.7\text{V} - (-10\text{V})}{333.33\text{k}\Omega + (1+100)(1\text{k}\Omega)} = 6.0629\mu\text{A} \quad (\text{No cutoff})$$

Check for Forward Active

$$I_C = \beta I_B = 606.3\mu\text{A} \quad I_E = (1+\beta) I_B = 612.36\mu\text{A}$$

$$V_C = V^+ - I_C R_C = 10\text{V} - (606.3\mu\text{A})(5\text{k}\Omega) = 6.97\text{V}$$

$$V_E = V^- + I_E R_E = -10\text{V} + (612.36\mu\text{A})(1\text{k}\Omega) = -9.39\text{V}$$

$\therefore V_{CE} > V_{CE,SAT} \Rightarrow$ Forward-Active operation

Small-Signal Parameters

$$g_m = \frac{I_c}{V_T} = \frac{606.3\mu\text{A}}{25\text{mV}} = 24.25 \text{ mS} \quad r_o = \frac{V_A}{I_c} \rightarrow \infty \text{ since } V_A \rightarrow \infty$$

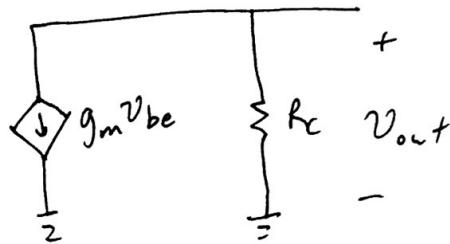
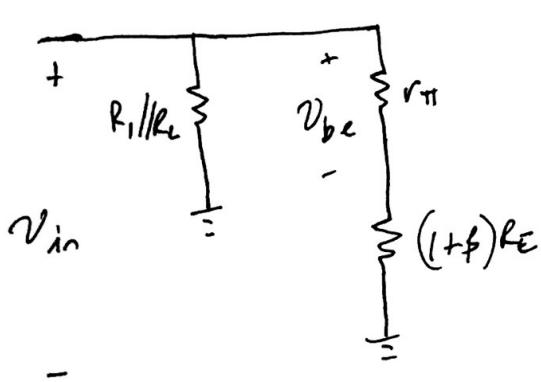
$$r_{\pi} = \frac{\beta V_T}{I_c} = \frac{(100)(25\text{mV})}{606.3\mu\text{A}} = 4.1234\text{k}\Omega$$

(Extra Work Page for Problem 4)

Since $r_o \rightarrow \infty$

$$r_{ib} = r_\pi + (1+\beta) R_E = 4.1234 \text{ k}\Omega + (1+100)(1 \text{ k}\Omega) = 105.12 \text{ k}\Omega$$

Small-Signal Equivalent Model (Can use simplified version since $r_o \rightarrow \infty$)



$$a_v = \frac{V_{out}}{V_{in}} = \frac{V_{be}}{V_{in}} \cdot \frac{V_{out}}{V_{be}} = \left(\frac{r_\pi}{r_\pi + (1+\beta) R_E} \right) \left(-g_m R_C \right) =$$

$$a_v = \left(\frac{4.1234 \text{ k}\Omega}{4.1234 \text{ k}\Omega + (1+100)(1 \text{ k}\Omega)} \right) \left(-(24.25 \text{ mS})(5 \text{ k}\Omega) \right) = \boxed{-4.7560}$$

Quick Sanity Check

$$a_v \approx -\frac{R_C}{R_E} = -5 \Rightarrow \text{Close to the actual gain}$$