Given $R_1 = 1\,k\Omega$

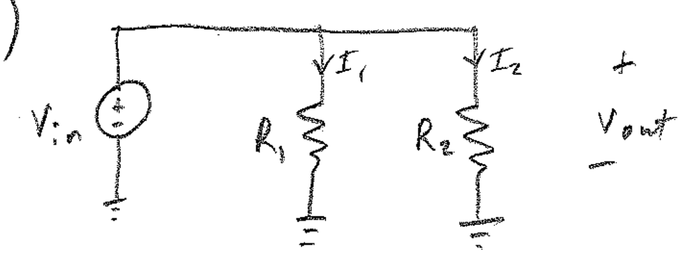
$R_2 = 10\,k\Omega$

$R_3 = 10\,k\Omega$

$I_{in} = 1\,mA$

$V_{in} = 1\,V$

a)



Find V_{out}, I_1, I_2

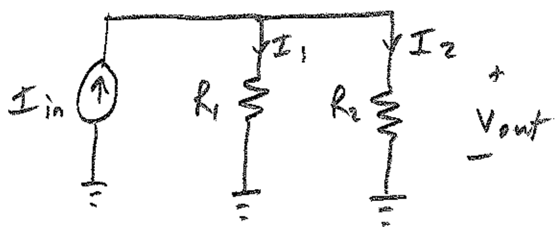
$\boxed{V_{out} = V_{in}} = 1\,V$ → R_1 and R_2 are in parallel with the voltage source

Ohm's Law → $V = IR$

$$\therefore \quad I_1 = \frac{V_{out}}{R_1} = \frac{1V}{1k\Omega} = 1mA$$

$$I_2 = \frac{V_{out}}{R_2} = \frac{1V}{10k\Omega} = 0.1mA$$

b)



Find V_{out}, I_1, I_2

Derive the expression for I_1 + I_2

Current Divider Derivation

Using KCL

$$I_{in} = I_1 + I_2 = I_1 + \frac{V_{out}}{R_2}$$

$$I_1 = I_{in} - \frac{V_{out}}{R_2}$$

$$V_{out} = I_{in} \, R_1 /\!/ R_2 = \boxed{I_{in} \frac{R_1 R_2}{R_1 + R_2}} = (1mA)\frac{(1k\Omega)(10k\Omega)}{(1k\Omega + 10k\Omega)} = \boxed{0.9091 \text{ V}}$$

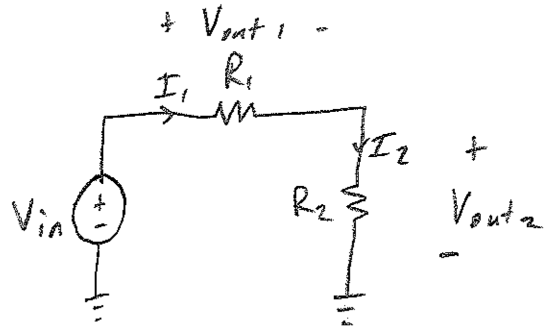
$$\therefore I_1 = I_{in} - \frac{I_{in}}{R_2} \frac{R_1 R_2}{R_1 + R_2} = I_{in}\left(1 - \frac{R_1}{R_1 + R_2}\right) =$$

$$= I_{in}\left(\frac{R_1 + R_2 - R_1}{R_1 + R_2}\right)$$

$$\boxed{I_1 = I_{in} \frac{R_2}{R_1 + R_2}} = (1mA)\left(\frac{10k\Omega}{11k\Omega}\right) = 0.9091 \text{ mA}$$

$$\boxed{I_2 = I_{in} \frac{R_1}{R_1 + R_2}} = (1mA)\left(\frac{1k\Omega}{11k\Omega}\right) = 0.0909 \text{ mA}$$

This is the <u>current divider</u> equation set c)



Find V_{out1}, V_{out2}, I_1, and I_2

Derive expressions for V_{out1} & V_{out2}

Voltage Divider Derivations

For V_{out2}, use KVL

$$V_{in} = I_1 R_1 + I_2 R_2 \quad , \quad I_1 = I_2 \quad \text{(same loop)}$$

$$V_{in} = I_1 R_1 + V_{out2} \qquad \boxed{\therefore I_1 = \frac{V_{in}}{R_1 + R_2}} = \frac{1V}{1k\Omega + 10k\Omega} = \boxed{0.0909mA}$$

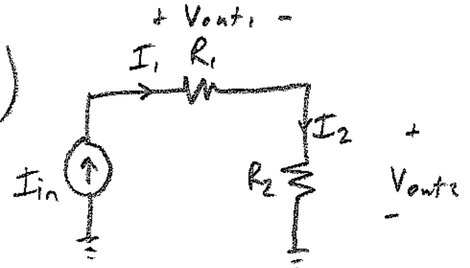
$$V_{out2} = V_{in} - I_1 R_1 = V_{in} - \frac{V_{in}}{R_1 + R_2} R_1 = V_{in}\left(1 - \frac{R_1}{R_1 + R_2}\right)$$

$$= V_{in}\left(\frac{R_1 + R_2 - R_1}{R_1 + R_2}\right) = V_{in}\frac{R_2}{R_1 + R_2}$$

$$\therefore \boxed{\begin{array}{l} V_{out2} = V_{in}\dfrac{R_2}{R_1 + R_2} \\ V_{out1} = V_{in}\dfrac{R_1}{R_1 + R_2} \end{array}} \begin{array}{l} = (1V)\dfrac{10k\Omega}{11k\Omega} = \boxed{0.9091V} \\ = (1V)\dfrac{1k\Omega}{11k\Omega} = \boxed{0.0909V} \end{array}$$

This is the <u>voltage divider</u> equation set

d)



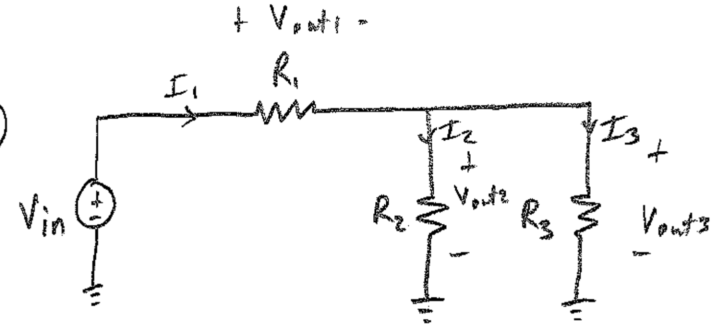
Find V_{out1}, V_{out2}, I_1, and I_2

One loop, so $\boxed{I_1 = I_2 = I_{in} = 1mA}$

$$V_{out2} = I_2 R_2 = (1mA)(10k\Omega) = 10V$$

$$V_{out1} = I_1 R_1 = (1mA)(1k\Omega) = 1V$$

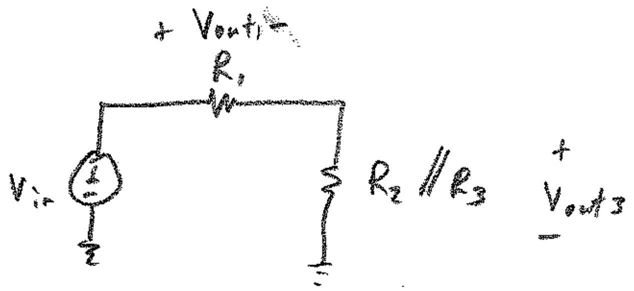
e)



Find the Voltage across each resistor, and also find the current through each resistor

$V_{out2} = V_{out}$ (in parallel)

$R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3} = 5\,k\Omega$



$$V_{out1} = V_{in} \frac{R_1}{R_1 + R_2 \parallel R_3}$$

$$= (1V)\frac{1k\Omega}{1k\Omega + 5k\Omega} = 0.1667V$$

$$V_{out2} = V_{in} \frac{R_2 \parallel R_3}{R_1 + R_2 \parallel R_3} =$$

$$= (1V)\frac{5k\Omega}{6k\Omega} = 0.8333\,V$$

$$I_1 = \frac{V_{out1}}{R_1} = \frac{V_{in}}{R_1 + R_2 \parallel R_3}$$

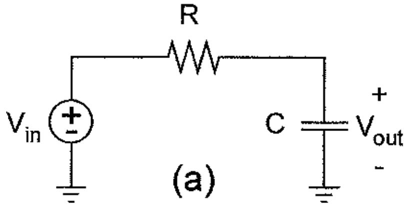
$$= \frac{1V}{1k\Omega + 5k\Omega} = 0.1667\,mA$$

$I_2 = I_3$ (same resistance values)

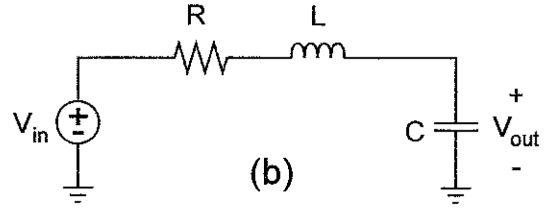
$I_2 = I_1 \frac{R_3}{R_2 + R_3}$ Current Divider

$$I_2 = \frac{V_{in}}{R_1 + R_2 \parallel R_3} \frac{R_3}{R_2 + R_3} = \frac{1V}{6k\Omega} \frac{10k\Omega}{20k\Omega} = 0.0833\,mA$$

Solve for the Laplace-domain transfer function of each of the following circuits. If the circuit performs a filtering operation, determine the type of filtering operation and also the order of the filter.

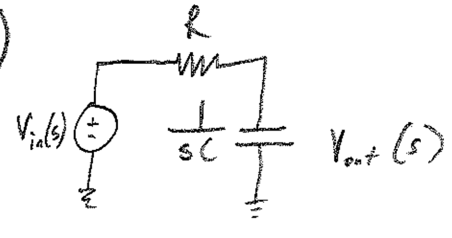


(a)



(b)

a)



$$V_{out}(s) = V_{in}(s)\frac{\frac{1}{sC}}{R+\frac{1}{sC}} = \frac{1}{RCs+1}$$

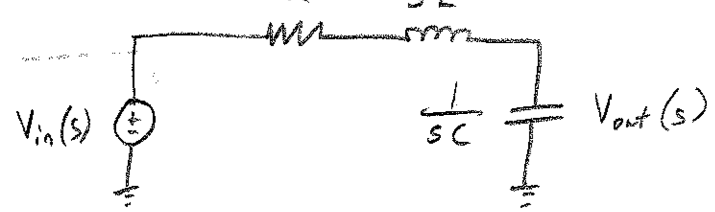
$$\therefore H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{RCs+1}$$

Yes, this is a filter (the transfer function is dependent upon frequency)

→ Lowpass filter

→ First-order filter (first-order transfer function)

b)



$$V_{out}(s) = V_{in}(s)\frac{\frac{1}{sC}}{R+sL+\frac{1}{sC}} = V_{in}(s)\frac{1}{LCs^2+RCs+1}$$

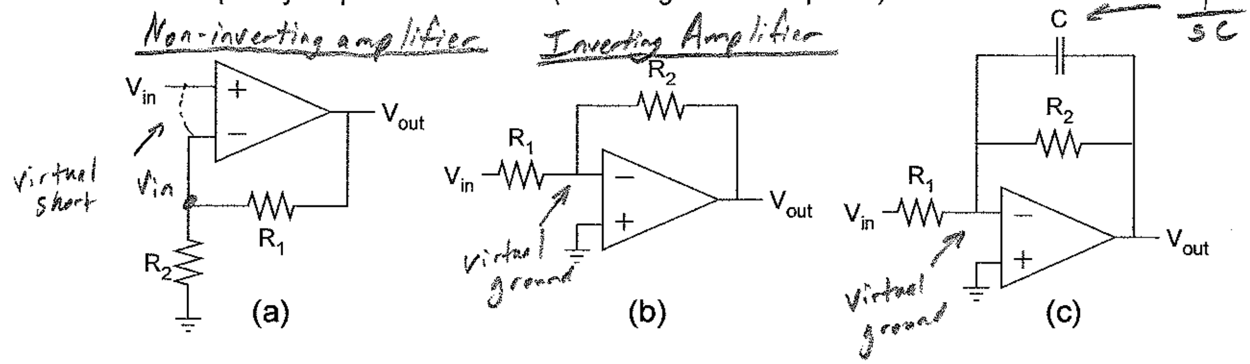
$$\therefore H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1}{LCs^2+RCs+1}$$

Yes, this is a filter

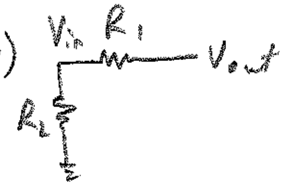
→ Lowpass filter

→ Second-order filter (second-order transfer function)

For each of the following circuits, solve for the transfer function. Assume all opamps are idea. If any of the circuits perform filtering, determine the order of the filter and the type of filtering operation, and then sketch the frequency response of the filter (both magnitude and phase).



a)



$$\rightarrow V_{in} = V_{out}\frac{R_2}{R_1+R_2} \rightarrow \frac{V_{out}}{V_{in}} = \frac{R_1+R_2}{R_2} = 1 + \frac{R_1}{R_2}$$

No filtering (no capacitors or inductors

b) Use KCL at the virtual ground

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2} = 0 \rightarrow \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}$$, No filtering

c) Use KCL at the virtual ground

$$\frac{V_{in}}{R_1} + \frac{V_{out}}{R_2 // \frac{1}{sC}} = 0 \rightarrow \frac{V_{in}}{R_1} + \frac{V_{out}}{\frac{R_2}{R_2Cs+1}} = 0$$

$$\therefore \frac{V_{out}}{V_{in}} = \frac{-R_2}{R_1}\frac{1}{R_2Cs+1}$$

Yes, this is a first-order, lowpass filter (with inverting gain)

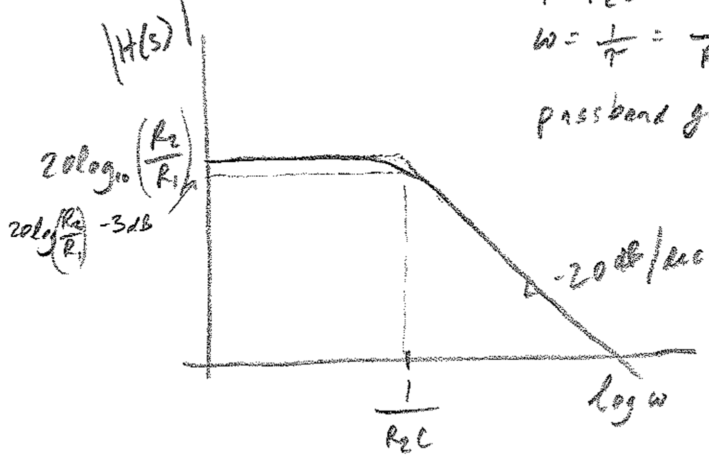
$\tau = R_2C$

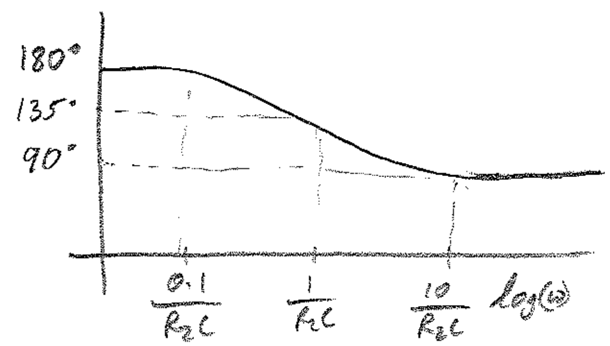
$\omega = \frac{1}{\tau} = \frac{1}{R_2C}$ (the corner frequency)

passband gain $= -\frac{R_2}{R_1}$

First-Order LPF

$H(s) = \frac{A_v}{\tau s+1}$



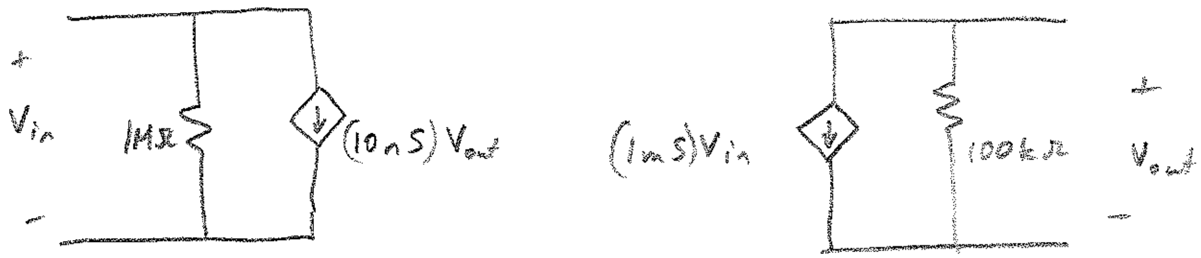


Draw the two-port amplifier model that is described below

- Forward transconductance = 1mS
- Reverse transconductance = 10nS
- Input impedance = 1MΩ
- Output impedance = 100kΩ

Determine the forward voltage gain of this amplifier.

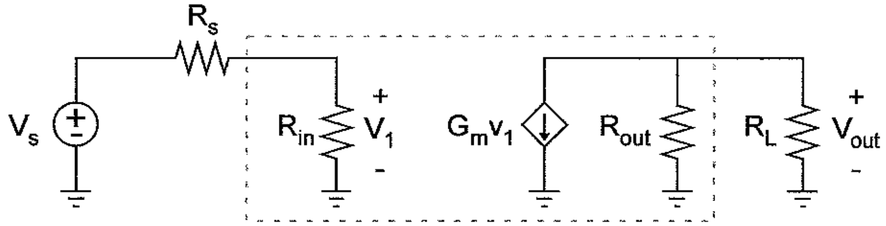
Two-Port Model



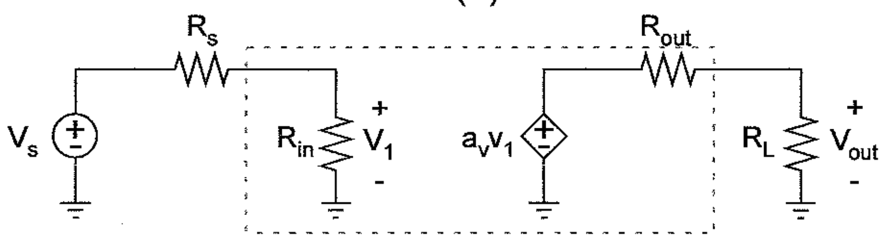
$$\frac{V_{out}}{V_{in}} = -(1mS)(100k\Omega) = 100$$

For each circuit below, the amplifier is drawn within the dotted lines. Perform the following.

- Determine the unloaded voltage gain of the amplifier (i.e. ideal voltage source and load impedance)
- Determine the actual voltage gain of the amplifier (i.e. using the circuit shown below)
- Assuming we have control over the input/output impedances of the amplifier, but not the source and load, how can we design the amplifier to maximize the voltage gain
- Convert the amplifier of Part a to a voltage output (i.e. Thevenin). Convert the amplifier of Part b to a current output (i.e. Norton)



(a)



(b)

a) Unloaded gain $\rightarrow \dfrac{V_{out}}{V_{in}} = -G_m R_{out}$

Loaded gain $\rightarrow \dfrac{V_{out}}{V_S} = \dfrac{V_1}{V_S} \cdot \dfrac{V_{out}}{V_S} = \left(\dfrac{R_{in}}{R_{in}+R_s} \dfrac{\cancel{V_s}}{\cancel{V_s}}\right)\left(\dfrac{-G_m \cancel{V_1}\, R_{out} /\!/ R_L}{\cancel{V_1}}\right) =$

$= \left(\dfrac{R_{in}}{R_{in}+R_s}\right)\left(-G_m\, R_{out} /\!/ R_L\right) =$

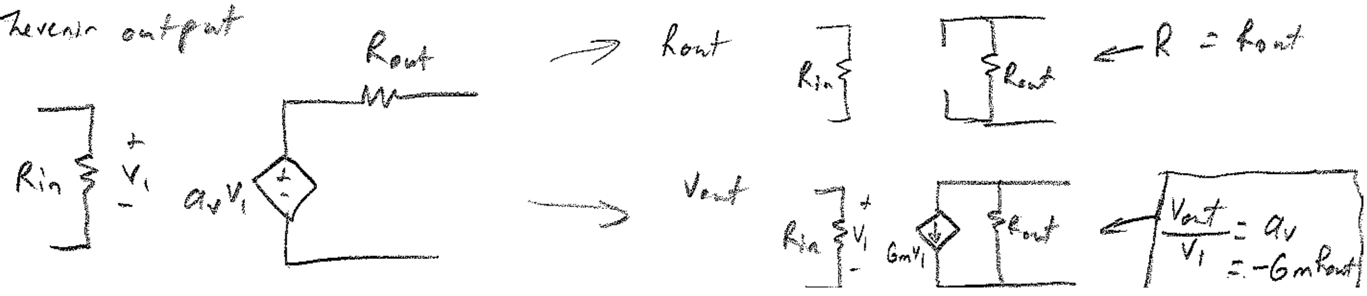
$= \left(\dfrac{R_{in}}{R_{in}+R_s}\right)\left(-G_m \dfrac{R_{out} R_L}{R_{out}+R_L}\right)$

We can maximize the voltage gain if

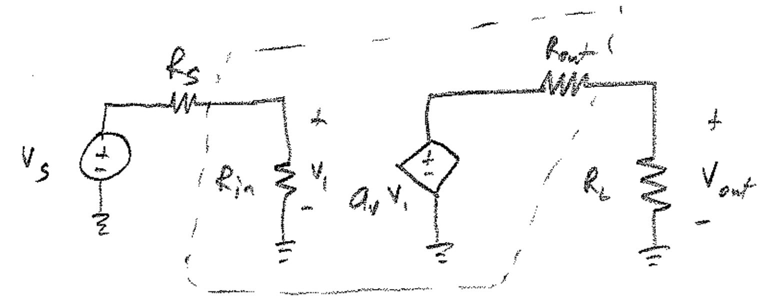
$\left(\dfrac{R_{in}}{R_{in}+R_s}\right) \rightarrow 1 \Rightarrow \therefore \boxed{R_{in} \rightarrow \infty}$

$-G_m\left(R_{out}/\!/R_L\right) \rightarrow -G_m R_L$ if $\boxed{R_{out} \rightarrow \infty}$, Also $\boxed{\text{increase } G_m}$

Thevenin output



b) Unloaded Gain $\rightarrow$



$$\frac{V_{out}}{V_1} = \frac{a_v V_1}{V_1} = a_v$$

Loaded Gain

$$\frac{V_{out}}{V_S} = \frac{V_1}{V_S} \cdot \frac{V_{out}}{V_1} = \left(\frac{R_{in}}{R_{in}+R_S}\frac{V_S}{V_S}\right)\left(\frac{R_L}{R_L+R_{out}}\frac{a_v V_1}{V_1}\right) =$$

$$= \left(\frac{R_{in}}{R_{in}+R_S}\right)\left(\frac{R_L}{R_L+R_{out}} a_v\right)$$

To maximize the gain

$$\left(\frac{R_{in}}{R_{in}+R_S}\right) \rightarrow 1 \qquad \therefore\ R_{in} \rightarrow \infty$$

$$\frac{R_L}{R_L+R_{out}} a_v \rightarrow \text{large} \qquad \therefore\ a_v \text{ as large as possible}$$

$$R_{out} \rightarrow 0$$

Norton Output

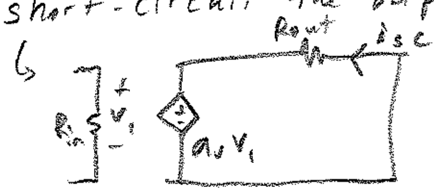
Looking into the output of the original amplifier

set $V_1 = 0$

$\hookrightarrow R_N = R_{out}$

short-circuit the output and measure the current

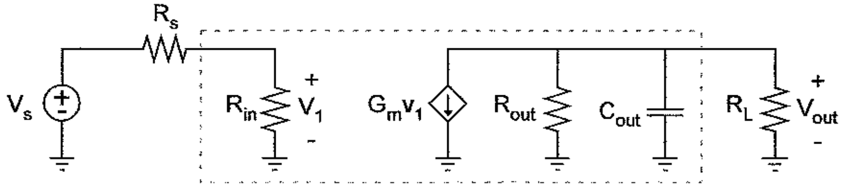
$\hookrightarrow$



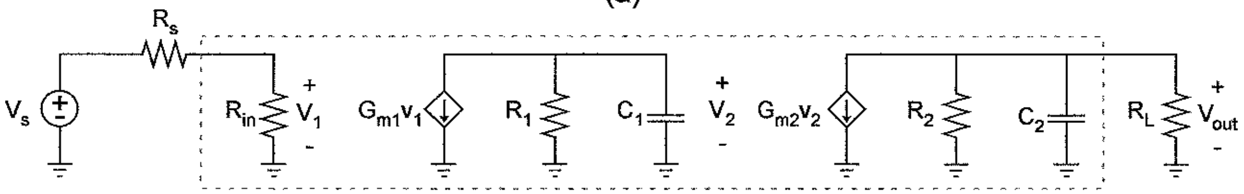
$$i_{sc} = -\frac{a_v V_1}{R_{out}} = G_m V_1$$

$$\therefore\ G_m = -\frac{a_v}{R_{out}}$$

For each circuit below, the amplifier is drawn within the dotted lines. Determine the unloaded and loaded voltage gains of each amplifier. How can we design the amplifier to maximize the voltage gain and the corner frequency?



(a)



(b)

a) Unloaded Gain

$$\frac{V_{out}}{V_1} = \frac{-G_m V_1 \; R_{out} \parallel \frac{1}{sC_{out}}}{V_1} = -G_m R_{out} \parallel \frac{1}{sC_{out}} =$$

$$= -G_m \frac{\frac{R_{out}}{sC_{out}}}{R_{out} + \frac{1}{sC_{out}}} = \frac{-G_m R_{out}}{R_{out} C_{out} s + 1}$$

To maximize the frequency, look to

$$\frac{A_v}{(R_{out} + R_L) C_{out} s + 1}$$

$$\omega = \frac{1}{(R_{out} + R_L) C_{out}}$$ corner frequency

$\therefore \omega \uparrow$ as $R_{out} \downarrow$, $\boxed{C_{out} \downarrow}$

Loaded Gain

$$\frac{V_{out}}{V_s} = \frac{V_1}{V_s} \cdot \frac{V_{out}}{V_s} = \left(\frac{R_{in}}{R_{in} + R_s} \frac{V_s}{V_s}\right)\left(\frac{-G_m V_1 \; R_{out} \parallel R_L \parallel \frac{1}{sC_{out}}}{V_1}\right) =$$

$$= \left(\frac{R_{in}}{R_{in} + R_s}\right)\left(-G_m R_{out} \parallel R_L \parallel \frac{1}{sC_{out}}\right) = \left(\frac{R_{in}}{R_{in} + R_s}\right)\left(-G_m \frac{\frac{R_{out} R_L}{sC_{out}}}{R_{out} R_L + \frac{R_{out}}{sC_{out}} + \frac{R_L}{sC_{out}}}\right) =$$

$$= \left(\frac{R_{in}}{R_{in} + R_s}\right)\left(\frac{-G_m R_{out} R_L}{sC_{out} R_{out} R_L + (R_{out} + R_L)}\right) = \left(\frac{R_{in}}{R_{in} + R_s}\right)\left(\frac{-G_m R_{out} \parallel R_L}{sC_{out} R_{out} \parallel R_L + 1}\right)$$

To maximize the loaded gain

$$\frac{R_{in}}{R_{in} + R_s} \to 1 \Rightarrow \therefore R_{in} \to \infty \quad , \quad \frac{G_m R_{out} \parallel R_L}{sC_{out} R_{out} \parallel R_L + 1}$$

for low frequencies, make $R_{out} \to \infty$ for high gain

C_{out} small for high corner frequency

b) Unloaded Gain

$$\frac{V_{out}}{V_1} = \frac{V_2}{V_1} \cdot \frac{V_{out}}{V_2} = \left(\frac{-G_{m1} V_1 R_1 // \frac{1}{sC_1}}{V_1} \right) \left(\frac{-G_{m2} V_2 R_2 // \frac{1}{sC_2}}{V_2} \right) =$$

$$= \left(G_{m1} \; R_1 // \frac{1}{sC_1} \right) \left(G_{m2} \; R_2 // \frac{1}{sC_2} \right) =$$

$$= \frac{G_{m1} G_{m2} \; R_1 R_2}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}$$

Loaded Gain

$$\frac{V_{out}}{V_S} = \frac{V_1}{V_S} \cdot \frac{V_2}{V_1} \cdot \frac{V_{out}}{V_2} =$$

$$= \left(\frac{R_{in}}{R_{in} + R_S} \right) \left(-G_{m1} \; R_1 // \frac{1}{sC_1} \right) \left(-G_{m2} \; R_2 // R_L // \frac{1}{sC_2} \right) =$$

$$= \frac{G_{m1} G_{m2} \; R_{in} \; R_1 \; R_2 // R_L}{R_{in} + R_S} \; \frac{1}{(R_1 C_1 s + 1)((R_2 // R_L) C_2 s + 1)}$$

To maximize the loaded gain (low frequency)

$\frac{G_{m1} G_{m2} \; R_{in} \; R_1 \; R_2 // R_L}{R_{in} + R_S}$ term, make $R_{in} \to \infty$
Increase G_{m1}, G_{m2}, R_1, R_2

To increase the two corner frequencies

$$\omega_1 = \frac{1}{R_1 C_1}$$

Could decrease the resistances, but this would cause the gain to decrease

$$\omega_2 = \frac{1}{(R_2 // R_L) C_2}$$

$\therefore$ Decrease the capacitances