

A piece of n-type silicon is doped with  $N_D = 2 \times 10^{16} \text{ cm}^{-3}$ .

- What is the probability of finding a hole in the valence band?
- What is the probability of finding an electron in the valence band?
- What is the probability of finding an electron 0.1 meV above the conduction band?
- What is the probability of finding a hole in the conduction band?

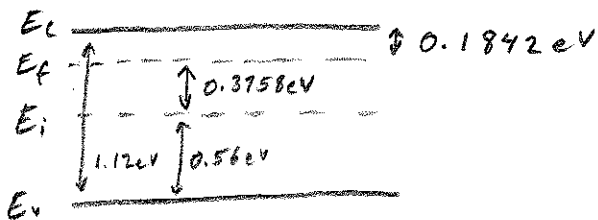
n-type material, so  $n \approx N_D = 2 \times 10^{16} \text{ cm}^{-3}$

First, we must find the Fermi level

$$n = n_i e^{(E_f - E_i)/kT} \rightarrow E_f - E_i = kT \ln\left(\frac{n}{n_i}\right)$$

$$= (0.0259 \text{ eV}) \ln\left(\frac{2 \times 10^{16} \text{ cm}^{-3}}{10^{10} \text{ cm}^{-3}}\right) = 0.3758 \text{ eV}$$

Band Diagram



$$a) 1 - f(E_v) = 1 - \frac{1}{1 + e^{(E_v - E_f)/kT}} = \frac{e^{(E_v - E_f)/kT}}{1 + e^{(E_v - E_f)/kT}} = \frac{1}{e^{-(E_v - E_f)/kT} + 1}$$

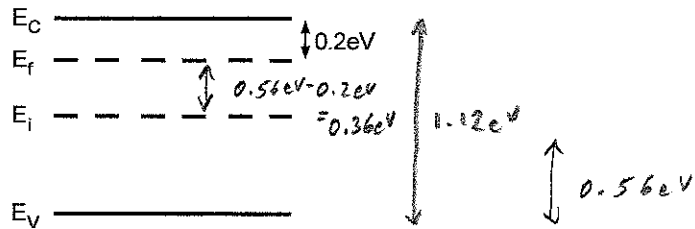
$$\therefore 1 - f(E_v) = \frac{1}{1 + e^{(E_c - E_v)/kT}} = \frac{1}{1 + e^{(0.9358)/0.0259}} = 2.0342 \times 10^{16} \quad (\text{not zero!})$$

$$b) f(E_v) = \frac{1}{1 + e^{(E_v - E_f)/kT}} = \frac{1}{1 + e^{(-0.9358 \text{ eV})/0.0259 \text{ eV}}} = 1$$

$$c) f(E_c + 0.1 \text{ meV}) = \frac{1}{1 + e^{(0.1843 \text{ eV})/0.0259 \text{ eV}}} = 8.1149 \times 10^{-9}$$

$$d) 1 - f(E_c) = \frac{1}{1 + e^{(E_c - E_f)/kT}} = \frac{1}{1 + e^{-0.1842 \text{ eV}/0.0259 \text{ eV}}} = 0.9992$$

Use the band diagram for silicon for the following parts of this problem.



- Is this n-type or p-type material? Why?
- What is the majority carrier and the majority carrier concentration?
- What is the minority carrier and the minority carrier concentration?
- What is the resistivity of this material?

a) n-type because the Fermi level is close to the conduction band (above  $E_i$ )

b) Majority carrier  $\rightarrow$  electrons (n-type material)

$$n = n_i e^{(E_f - E_i)/kT}$$

$$n = (10^{10} \text{ cm}^{-3}) e^{(0.36 \text{ eV}) / (0.0259 \text{ eV})} = 1.0877 \times 10^{16} \text{ cm}^{-3}$$

c) Minority carrier  $\rightarrow$  holes

$$p = \frac{n_i^2}{n} = \frac{(10^{10} \text{ cm}^{-3})^2}{1.0877 \times 10^{16} \text{ cm}^{-3}} = 9.1934 \times 10^3 \text{ cm}^{-3}$$

d) Resistivity

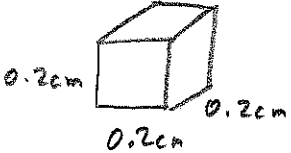
$$\rho = \frac{1}{q(M_n n + M_p p)} = \frac{1}{(1.602 \times 10^{-19} \text{ C}) [(1360 \text{ cm}^2/\text{Vs})(1.0877 \times 10^{16} \text{ cm}^{-3}) + (460 \text{ cm}^2/\text{Vs})(9.1934 \times 10^3 \text{ cm}^{-3})]}$$

$$= 0.4220 \text{ } \Omega \text{ cm}$$

$$\text{Also } \rho \approx \frac{1}{q M_n n} = 0.4220 \text{ } \Omega \text{ cm}$$

A silicon cube (2mm on each side) has been doped with  $N_D = 1 \times 10^{16} \text{ cm}^{-3}$  and  $N_A = 5 \times 10^{16} \text{ cm}^{-3}$ .

- Is this n-type or p-type material? Why?
- What is the majority carrier and the majority carrier concentration?
- What is the minority carrier and the minority carrier concentration?
- Now, assume that a voltage of 5V is placed across two opposing sides of this material. What are the values of the hole current and the electron current?
- Instead of a voltage across this material, this silicon cube is exposed to light and undergoes photogeneration. Determine the diffusion length of the minority carrier in this material, assuming low-level injection.



a)  $N_A > N_D, n_i \therefore$  p-type material

b) majority carriers  $\rightarrow$  holes (p-type material)

$$p = \frac{N_A N_D}{2} + \left[ \left( \frac{N_A - N_D}{2} \right)^2 + n_i^2 \right]^{1/2} =$$

$$= \frac{4 \times 10^{16} \text{ cm}^{-3}}{2} + \left[ \left( \frac{4 \times 10^{16} \text{ cm}^{-3}}{2} \right)^2 + (1 \times 10^{10} \text{ cm}^{-3})^2 \right]^{1/2} = 4 \times 10^{16} \text{ cm}^{-3}$$

c) minority carriers  $\rightarrow$  electrons

$$n = \frac{n_i^2}{p} = \frac{(1 \times 10^{10} \text{ cm}^{-3})^2}{4 \times 10^{16} \text{ cm}^{-3}} = 2500 \text{ cm}^{-3}$$

d) 5V place across the silicon

$$\therefore \text{Electric field} \rightarrow E = \frac{V}{x} = \frac{5V}{0.2\text{cm}} = 25 \text{ V/cm}$$

no diffusion current because no concentration gradient

$\therefore$  all current is drift current

cross-sectional area =  $0.2\text{cm} \times 0.2\text{cm}$

$$I_p = J_p A = \mu_p q p E A = (460 \text{ cm}^2/\text{Vs}) (1.602 \times 10^{-19} \text{ C}) (4 \times 10^{16} \text{ cm}^{-3}) (25 \text{ V/cm}) (0.04 \text{ cm}^2) = \boxed{2.9477 \text{ A}} \text{ hole current}$$

$$I_n = J_n A = \mu_n q n E A = (1360 \text{ cm}^2/\text{Vs}) (1.602 \times 10^{-19} \text{ C}) (2500 \text{ cm}^{-3}) (25 \text{ V/cm}) (0.04 \text{ cm}^2) = \boxed{5.4468 \times 10^{-13} \text{ A}} \text{ Electron current}$$

e)  $L_n = \sqrt{D_n \tau_n} \rightarrow L_n = \sqrt{\frac{\mu_n kT}{q} \tau_n} = \sqrt{(1360 \text{ cm}^2/\text{Vs}) (0.0159 \text{ V}) (1 \times 10^{-4} \text{ s})} = 5.9350 \times 10^{-2} \text{ cm} = 0.05935 \text{ cm}$

$$\frac{D_n}{\mu_n} = \frac{kT}{q} \rightarrow D_n = \frac{\mu_n kT}{q}$$

A silicon p-n junction with a cross sectional area of  $10^{-4} \text{ cm}^2$  has been doped on the p-type side with  $N_A = 10^{17} \text{ cm}^{-3}$  and on the n-type side with  $N_D = 3 \times 10^{17} \text{ cm}^{-3}$ .

- Determine the built-in potential.
- Determine the equilibrium width of the depletion region.
- Determine the maximum electric field in the p-n junction in equilibrium.
- Determine the zero-bias junction capacitance.
- Determine the junction capacitance if the junction is reverse biased with 3V.

For Parts f-h, a forward bias of 0.1V has been applied to the p-n junction diode.

- Determine the width of the depletion region under bias.
- Determine the maximum electric field under bias.
- Determine the total current that flows under bias.

$$a) V_{bi} = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) = 0.0259 \text{ V} \ln \left( \frac{(10^{17} \text{ cm}^{-3})(3 \times 10^{17} \text{ cm}^{-3})}{(10^{10} \text{ cm}^{-3})^2} \right) = 0.8634 \text{ V}$$

$$b) W = \left[ \frac{2K_S \epsilon_0}{q} \frac{N_A + N_D}{N_A N_D} (V_{bi} - V_A) \right]^{1/2} = \frac{N_A + N_D}{N_A N_D} = \frac{1}{N_A} + \frac{1}{N_D}$$

$$= \left[ \frac{(2)(11.9)(9.854 \times 10^{-14} \text{ F/cm})}{(1.602 \times 10^{-19} \text{ C})} \left( \frac{1}{10^{17} \text{ cm}^{-3}} + \frac{1}{3 \times 10^{17} \text{ cm}^{-3}} \right) (0.8634 \text{ V}) \right]^{1/2}$$

$$= 1.2254 \times 10^{-5} \text{ cm} = 0.1225 \mu\text{m}$$

$$c) E_{max} = - \frac{q N_A}{K_S \epsilon_0} x_p$$

$$x_p = \left[ \frac{2K_S \epsilon_0}{q} \frac{N_D}{N_A(N_A + N_D)} (V_{bi} - V_A) \right]^{1/2} = 9.1903 \times 10^{-6} \text{ cm} = 0.091903 \mu\text{m}$$

$$E_{max} = - \frac{q N_A}{K_S \epsilon_0} x_p = - \frac{(1.602 \times 10^{-19} \text{ C})(10^{17} \text{ cm}^{-3})}{(11.9)(9.854 \times 10^{-14} \text{ F/cm})} (9.1903 \times 10^{-6} \text{ cm}) =$$

$$= -1.4092 \times 10^5 \text{ V/cm}$$

Alternatively,

$$E_{max} = - \frac{2(V_{bi} - V_A)}{W} = - \frac{2V_{bi}}{W} = \frac{(-2)(0.8634 \text{ V})}{1.2254 \times 10^{-5} \text{ cm}} = -1.409 \times 10^5 \text{ V/cm}$$

$$d) C_{j0} = \sqrt{\frac{q K_s \epsilon_0 N_A N_D}{2 V_{bi} (N_A + N_D)}} =$$

$$= \sqrt{\frac{(1.602 \times 10^{-19} \text{ C})(11.8)(8.854 \times 10^{-14} \text{ F/cm})(10^{17} \text{ cm}^{-3})(3 \times 10^{17} \text{ cm}^{-3})}{(2)(0.8634 \text{ V})(4 \times 10^{17} \text{ cm}^{-3})}} =$$

$$= 8.5261 \times 10^{-8} \text{ F/cm}$$

$$e) C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_A}{V_{bi}}}} = \frac{8.5261 \text{ F/cm}^2}{\sqrt{1 + \frac{3 \text{ V}}{0.8634 \text{ V}}}} = 4.0306 \times 10^8 \text{ F/cm}$$

f) Under bias ( $V_A = 0.1 \text{ V}$  forward bias)

$$w = \left[ \frac{2 K_s \epsilon_0}{q} \left( \frac{1}{N_A} + \frac{1}{N_D} \right) (V_{bi} - V_A) \right]^{1/2} = 1.1522 \times 10^{-5} \text{ cm} = 0.011522 \times 10^{-5} \text{ m}$$

$$= 0.11522 \mu\text{m}$$

$$g) E_{\text{max}} = \frac{-2(V_{bi} - V_A)}{w} = \frac{(-2)(0.8634 \text{ V} - 0.1 \text{ V})}{1.1522 \times 10^{-5} \text{ cm}} = -1.3251 \times 10^5 \text{ V/cm}$$

$$h) I = qA \left[ \underbrace{\frac{D_n n_i^2}{L_n N_A}}_{\text{min. carriers in p-type side}} + \underbrace{\frac{D_p n_i^2}{L_p N_D}}_{\text{min. carriers in n-type side}} \right] \left[ e^{V_A/nV_T} - 1 \right]$$

$$\tau_n = \tau_p = 100 \mu\text{s}, \quad L_n = \sqrt{D_n \tau_n} = \sqrt{\mu_n \frac{kT}{q} \tau_n} = \sqrt{(1360 \text{ cm}^2/\text{Vs})(0.0259 \text{ V})(100 \times 10^{-6} \text{ s})}$$

$$= 5.9350 \times 10^{-2} \text{ cm}$$

$$L_p = \sqrt{D_p \tau_p} = \sqrt{\mu_p \frac{kT}{q} \tau_p} = 3.4517 \times 10^{-2} \text{ cm}$$

$$D_n = \mu_n \frac{kT}{q} = (1360 \text{ cm}^2/\text{Vs})(0.0259 \text{ V}) = 35.2240 \text{ cm}^2/\text{s}$$

$$D_p = \mu_p \frac{kT}{q} = (460 \text{ cm}^2/\text{Vs})(0.0259 \text{ V}) = 11.9140 \text{ cm}^2/\text{s}$$

$$I = (1.602 \times 10^{-19} \text{ C})(10^{-4} \text{ cm}^2) \left[ \frac{35.2240 \text{ cm}^2/\text{s}}{5.9350 \times 10^{-2} \text{ cm}} \frac{(10^{10} \text{ cm}^{-3})^2}{10^{19} \text{ cm}^{-3}} + \frac{11.9140 \text{ cm}^2/\text{s}}{3.4517 \times 10^{-2} \text{ cm}} \frac{(10^{10} \text{ cm}^{-3})^2}{3 \times 10^{17} \text{ cm}^{-3}} \right] \times$$

$$\times \left[ e^{0.1 \text{ V}/(0.0259 \text{ V})} - 1 \right] = 5.2797 \times 10^{-16} \text{ A}$$

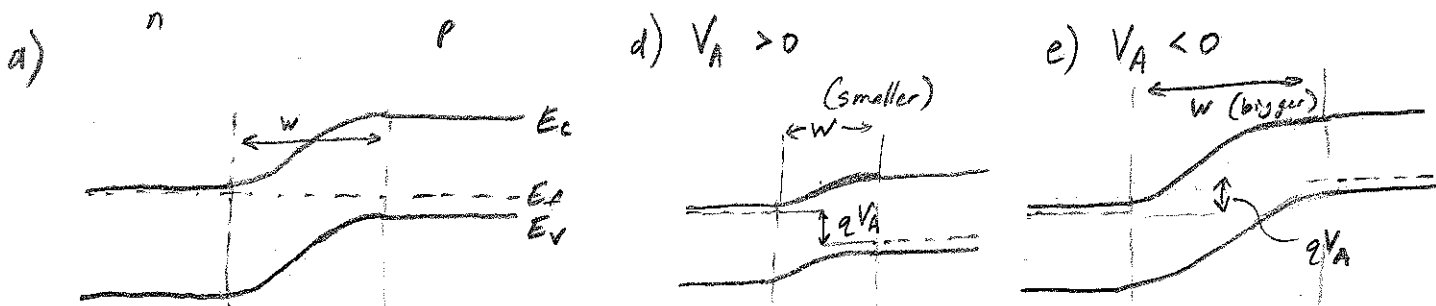
A silicon p-n junction has been doped on the p-type side with  $N_A = 10^{17} \text{cm}^{-3}$  and on the n-type side with  $N_D = 10^{18} \text{cm}^{-3}$ . Draw the following to scale. Be sure to label all important points and intercepts (exact values are not needed, but expressions are required).

- Band diagram in equilibrium.
- Charge density in equilibrium versus position along the p-n junction.
- Electric field in equilibrium versus position along the p-n junction.
- Repeat Parts a-c for a forward biased p-n junction. Emphasize the differences from the equilibrium conditions.
- Repeat Parts a-c for a reverse biased p-n junction. Emphasize the differences from the equilibrium conditions.

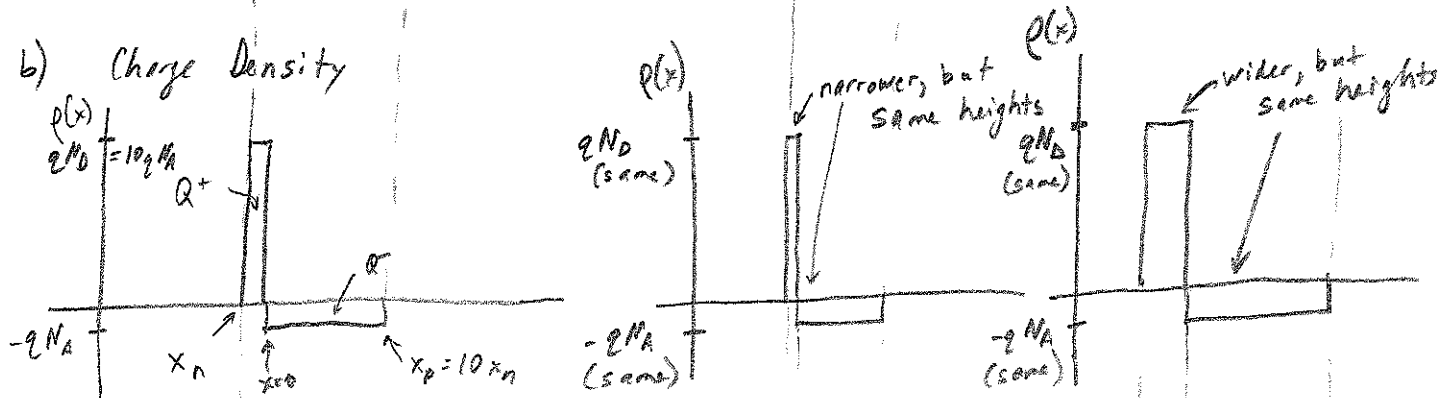
$N_D = 10 N_A \therefore E_F$  will be very close to  $E_C$  on the n-type side

$$\left| E_F - E_C \right|_{\text{n-type side}} < \left| E_F - E_V \right|_{\text{p-type side}}$$

Draw the band diagrams for each side, and then line up the Fermi levels for equilibrium conditions



b) Charge Density



c) Electric Field

